数理解析研究所講究録
第 1648 巻 2009 年 216-220

On Characteristic Features of Traditional Mathematics in Ancient China

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Abstract

The traditional mathematics in ancient China had some characteristic features quite different from those represented by the euclidean system of mathematics in ancient Greece which governs the development of present-day modern mathematics.

For example, for euclidean system one usually develops a logical system of reasoning based on a set of axioms admitted to be true *ad hoc*. On the other hand the traditional mathematics of ancient China is interested in the solving of problems arising from practical needs. For this purpose, on analyzing the problems one introduces new novel concepts, arrives at methods of solving, and then formulates general principle to treat further new more delicate problems. This will run recurrently and indefinitely as shown in the diagram below:

Practical Problems
↓
Concepts and Methods of Solving
↓
General Principle
↓
Further Examples
↓
...

As an illustration we have the diagram below:

[1] taken from KyotoConference in Kyoto, March 7-8,2006.

$$\begin{array}{c}\text{Rectangular Area Determination}\\ \downarrow\\ \text{Methods}\\ \downarrow\\ \text{Out-In Complementary Principle}\\ \downarrow\\ \text{Polygonal Area Determination}\\ \downarrow\\ \cdots\end{array}$$

The most important mathematics classics of ancient China is ≪ *Nine Chapters of Arithmetic* ≫ completed in early years of Western Han Dynasty (206-8 B.C.) by some officers in charge of economic reconstructions. The *Out-In-Complementary Principle* is explitely formulated in Chap.9 of ≪ *Nine Chapters* ≫ in dealing with the simple practical problem of determining the area of rectangular agricultural fields. The Principle thus formulated permits to solve quite a lot of intricate problems including the proof of the *Sun-Height Formula* dated in quite remote time. Moreover, the recurrent application of the above diagram permits the discovery of further principles: the 2:1 Principle of LIU Hui, and the LIU-ZU Principle of LIU Hui and ZU Geng (equivalent to much later re-discovered Cavalieri Principle). These Principles had permitted the Chinese ancient scholars to deal with great success such delicate problems like the computation of π, the determination of volumes of polyhedras and curved solids like spheres, circular pyramids, etc.

In the 8th Chapter of ≪ *Nine Chapters* ≫ it is shown how to solve by some definite elimination procedure an arbitrary system of linear equations in integer coefficients. For this sake positive and negative fractional numbers together with rules of computations are introduced giving thus the complete system of rational numbers. Furthermore, in order to solve such problems of determining the side of a square field of known integral-valued area or that of a cubic-form container with known integral-valued volume, irrational numbers of square or cubic root type are introduced with a special terminology of *Mian* literally meaning *side*. Approximate integral values of such roots are shown how to compute and even some general formulae governing such irrationals are given in that classic.

Besides the classic ≪ *Nine Chapters* ≫ appeared in year 263 A.D. toward the end of the period of *Three Kingdoms* (220-265 A.D.) appeared the extremely important classic ≪ *Annotations to Nine Chapters* ≫ due to scholar LIU Hui. Among its diverse important contributions to mathematics we may cite in particular the following one. By introducing the notion of *decimal numbers* to arbitary stage as well as limit concepts and approximating procedure LIU Hui had actually completed the *Real Number System.* This system was so introduced in a natural and easily-handled way together with the important concept of *decimal numbers.* For this we may compare the much later introduced *real number systems* in 19th century by western mathamaticians in ways quite involved and seeminly not quite natural.

A further characteristic feature of China's ancient traditional mathematics is the following one: In contrast to euclidean tradition of laying emphasis on theorem-proving, the traditional mathematics of ancient China concentrated its efforts to solving of problems arising from practical needs. As the datas given and the numerical results to be found of the problems should be related by some kind of equations, naturally and usually in the form of polynomial ones, so polynomial equations-solving becomes a central theme throughout the development of China's ancient mathematics.

Thus, in the classic $\ll$ *Nine Chapters* $\gg$ besides the square- and cubic root extraction corresponding to the solving of equations $x^2 = A$ or $x^3 = A$ in integers for A a positive integer, there appears also the solving of such quadratic equations like $x^2 + 34 * x = 71000$ with solution $x = 250$, (Ch.9, Prob.20). In fact, some contemporain scholar ZHAO Shuang of LIU Hui had written some short essays about mathematics. One essay concerns a detailed analysis of the *Sun-Height Formula* and a second one deals with the general quadratic equation. Among the results involved we may cite that one which gives a general formula about the roots of a quadratic equation in terms of its coefficients, which is same as the one well-known to us in now-a-days.

The solving of polynomial equations developed incessantly up to the end of Yuan Dynasty (1271-1386 A.D.). Thus, in the time of Southern-Northern Dynasties (420-589 A.D.), there appeared the important classic $\ll$ *Zhui Shus* $\gg$ (literally meaning $\ll$ *The Art of Mending* $\gg$ due to the celebrated great mathematician and astronomer ZU Chongzhi. It is very likely a collected work of ZU (perhaps together with his son ZU Geng (fl. 5c A.D.), also a great mathematician with invaluable contributions). ZU Chongzhi, besides being a great scientist, is also the important Daming-Calender maker, reknown for some revolutionary concepts and extremely-high accuracy in numerical datas. Moreover, ZU is also an engineer and instrument-manufacturer. Thus, he had built some Constantly-Southern-Directed Vehicle and some Thousand -Li-Ship. In mathematics ZU is best known for his computation of the value of π with

$$\frac{22}{7} < \pi < \frac{355}{113}.$$

Besides, ZU had considered the problem of solving such cubic equations of the form

$$x * (x + k) * (x + l) = V, \ or$$
$$x^3 + (k + l) * x^2 + k * l * x = V.$$

In the early years of Tang Dynasty (618 - 907 A.D.) some scholar WANG Xiaotong (fl. 626 A.D.) wrote the classic $\ll$ *Continuation of Ancient Mathematics* $\gg$ devoting to cubic equations closely connected with complicate reconstructions. Wang seriously criticized the work of ZU on cubic equations as completely absurd. In year 656 A.D. of Tang Dynasty it was established some kind of official school for the training of mathematicians with ten classics as textbooks among which are the $\ll$

Nine Chapters $\gg$, the $\ll$ *Sea – Island Manuel* $\gg$ of LIU Hui, the $\ll$ *Zhui Shus* $\gg$ of ZU Chongzhi, etc. However, owing to the extreme difficulties of ZU's classic it was compelled to be removed from the study. Henceforth this classic was lost in China and no one can tell what are its contents. In view of the well-known computational results of ZU about π but little known about the way that ZU reached such elegant results, scholars in later years usually held the opinion that the difficulties of ZU's classic are caused by difficulties for the understanding of ZU's method of such computations of π. The present author cannot agree with such opinions. In fact, the principle and method of computing π was laid down by LIU Hui in his $\ll$ *Annotations* $\gg$ and LIU himself had shown in stepwise details how to carry out the computations up to certain degree. The computations of ZU follows closely the steps of LIU. Thus, no matter how ingenious and how somewhat new original artifice created *ad hoc* are introduced by ZU for his computations, it is highly unbelievable that such artifices are so difficult to be non-understandable by scholars of contemporain and later generations.

In the present author's opinion, what caused ZU's classic to be non-understandable and to be seriously criticized by WANG Xiaotong to be absurd is this: ZU had, through his study of cubic equations, introduced some kind of *new numbers* equivalent to the present-day *complex numbers*!!!

Remark that in Europe the complex numbers was not introduced through the study of such simple equation $x^2 = -1$, but through the trial of solving some kind of *cubic equations* as in the case of G.Cardano (1501-1576 A.D.)!!!

The development of Chinese traditional mathematics reached a climax in Song Dynasty (960-1279). In year 1247 A.D. the scholar QIN Jiushao wrote the gigantic classic $\ll$ *Mathematical Treatise in Nine Chapters* $\gg$. In this classic besides various contributions including the now-a-days well-known *Chinese Remainder Theorem*, we may cite in particular the following one: It concerns with a general method of solving numerically a polynomial equation of arbitrarily high degree with arbitrary numerical coefficients. Moreover, in the period of Southern Song (1127-1279 A.D.) and Yuan Dynasty (1271-1368), mathematicians introduced the novel concepts of *Heaven's Element, Earth's Element*, etc., which correspond to unknowns x, y, etc. in the present-day mathematics. With the aids of such concepts the scholars in these days had introduced the notions of *rational functions*, the *algebrization of geometry*, and the way of establishing equations from given geometrical conditions, etc. In particular, it leads the way to the solving of systems of polynomial equations. Such developments reached the climax in the hand of some scholar ZHU Shijie (fl. 1299 A.D.). In fact, in the classic $\ll$ *Jade Mirror of Four Elements* $\gg$ Zhu gave a general method of solving up to 4 polynomial equations in 4 unknowns. To simplify the matter let us consider the case of 3 polynomial equations $P_i(x_1, x_2, x_3) = 0$ in 3 unknowns x_1, x_2, x_3 with known numerical coefficients. Let us arrange the 3 unknowns x_i in an arbitrary order say $x_1 \prec x_2 \prec x_3$. Let us eliminate x_3 from pairs of equations $P_i = 0$ to get a set of polynomial equations in x_1, x_2 alone: $Q_j(x_1, x_2) = 0, j = 1, 2, \cdots$. Next, let us eliminate x_2 pairwise from these equations $Q_j = 0$ to get a set of polynomial equations in x_1 alone: $R_k(x_1) = 0, k = 1, 2, \cdots$. Let us now choose one from each of the above set of equations, say:

$$R_1(x_1) = 0,\ Q_1(x_1, x_2) = 0,\ P_1(x_1, x_2, x_3) = 0.$$

Let us now solve the polynomial equation $R_1 = 0$ for x_1 which we know how to do by various known methods, e.g. that of QIN Jiushao. Substitute this value of $x_1 = x_1^0$ in $Q_1 = 0$ and solve it for x_2 to get a solution $x_2 = x_2^0$, then substitute the values of x_1^0, x_2^0 thus found in $P_1 = 0$ and solve it for the sole unknown x_3 to get solution x_3^0. Combining together, we get a solution $(x_1, x_2, x_3) = (x_1^0, x_2^0, x_3^0)$ of the original given set of equations $P_i = 0, i = 1, 2, 3$. From the above we see that the line of thought and the procedure is quite clear and it is available for an arbitrary number of variables even equations. In the latter part of our article we shall exploit it in more details together with its variety of applications.

We leave aside other contributions of our ancient traditional mathematics, but we should point out the fact below: In contrast to the modern mathematics inherited from the ancient Euclidean tradition for which the results are expressed in the form of *theorems*, our traditional mathematics expresses its results usually in the form of *shui*, literally meaning *(stepwise) method of solving*, which corresponds in essence to the present-day **algorithm**.