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Choice of Alternative Environmental Policies with Quadratic Costs under Uncertainty[†]

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1 Introduction

When we consider many environmental problems, such as acid rain and global warming, we face many uncertainties including, for example, demographic change, economic development, and technological progress. Decision makers must then consider these uncertainties when they develop and implement environmental policy. For instance, Pindyck (2000, 2002) investigate an environmental policy designed to reduce the emission of a pollutant under uncertainty. See also Barrieu and Chesney (2003), Ohyama and Tsujimura (2006, 2008), Wirl (2006a,2006b), and Lin, Ko and Yeh (2007)

In this paper, we also investigate environmental policy under uncertainty. We consider that an economic agent benefits from an economic activity that emits a certain pollutant. Simultaneously, the agent suffers from the pollutant. Importantly, we assume that how much damage the agent suffers from the pollutant is uncertain, represented by a stochastic differential equation. We also assume that implementing the policy is irreversible. Given there is uncertainty and irreversibility in the implementation of environmental policy, it is important to decide the timing of its implementation. Further, the agent has two policy options distinguished by the amount of emission reductions possible and their associated costs, such that one policy reduces emissions and costs less than the alternative policy. We then assume we can divide the costs to implement the chosen environmental policy into its fixed, proportional, and quadratic adjustment components. The agent must then decide which policy to implement and when to implement the chosen policy to maximize the benefit. To solve the agent's problem, we formulate it as an optimal stopping problem.

Our analysis differs and relates to previous work in several respects. For instance, while Pindyck (2000, 2002) investigated environmental policy when the agent has just a single policy option, we analyze the outcomes when the agent has two policy options. We refer to the former as the single environmental policy and the latter as the alternative environmental policies. In related work, Décamps, Mariotti and Villeneuve (2006) explore the investment decision problem of two alternative projects. They then show the value of the flexibility when the agent can choose between the alternative projects.

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The remainder of the paper is organized as follows. In Section 2 we investigate the single environmental policy under uncertainty. Section 3, examines the alternative environmental policies under uncertainty. Section 4 presents the numerical analysis. Section 5 concludes the paper.

2 Single Environmental Policy

Suppose that an economic agent benefits from an economic activity that emits a pollutant. At the same time, the agent suffers from the pollutant. The agent then considers when it is optimal to implement the environmental policy designed to reduce the emission of the pollutant. There are two environmental policies, 1 and 2, available, where Environmental Policy 1 (EP1) reduces emissions and costs less than Environmental Policy 2 (EP2). The agent then considers when it is optimal to implement the chosen policy. In this section, we assume that the agent has either EP1 or EP2 available as policy options, but not both. Pindyck (2000, 2002) investigate a similar problem.

2.1 Agent's Problem

Let Q_t be the level of economic activity at time $t \geq 0$. The dynamics of the process of Q_t, $Q = \{Q_t\}_{t\geq 0}$ are given by:

$$dQ_t = \alpha Q_t dt, \qquad Q_0 = q, \tag{2.1}$$

where $\alpha > 0$ is the constant growth rate of economic activity. The agent benefit is assumed to be given by pQ_t, where p is a parameter that converts the level of economic activity to a money amount. If Q_t represents the amount of production, p is the price of the product. Let $\gamma^0 Q_t$ be the emission flow of the pollutant when the agent has not implemented the policy. If the agent has implemented the policy i ($i = 1, 2$), it reduces the emission flow to $\gamma^i Q_t$ with $\gamma^0 > \gamma^1 > \gamma^2 > 0$. Then, the dynamics of the stock of the pollutant Y_t is given by:

$$dY_t^i = (\gamma^i Q_t - \delta Y_t^i)dt, \qquad Y_0^i = y, \tag{2.2}$$

where $\delta \in (0,1)$ is the rate of natural decay of the stock of the pollutant. Let $X_t Y_t^2$ be the damage the agent suffers from the stock of the pollutant. X_t is a variable that stochastically shifts over time to reflect the damage owing to the pollutant and assumed to be governed by:

$$dX_t = \mu X_t dt + \sigma X_t dW_t, \qquad X_0 = x, \tag{2.3}$$

where $\mu > 0$, $\sigma > 0$, and W_t is a standard Brownian motion on a filtered probability space $(\Omega, \mathcal{F}, \mathbb{P}, \{\mathcal{F}_t\}_{t\geq 0})$ satisfying the usual conditions[2]. Here $\mathcal{F}_t$ is generated by W_t in $\mathbb{R}$, i.e., $\mathcal{F}_t = \sigma(W_s, s \leq t)$. . The net benefit $B(Q_t, X_t, Y_t)$ from economic activity is given by:

$$B^i(Q_t, X_t, Y_t^i) = pQ_t - X_t(Y_t^i)^2. \tag{2.4}$$

Let $K^i(Q_t)$ be the cost function of policy i and be given by:

$$K^i(Q_t) = k_0 + k_1(\gamma^0 - \gamma^i)Q_t + k_2(\gamma^0 - \gamma^i)^2 Q_t^2, \tag{2.5}$$

where $k_0 > 0$ is the fixed cost, $k_1 > 0$ is the proportional cost parameter, and $k_2 > 0$ is the adjustment cost parameter. Given $\gamma^0 > \gamma^1 > \gamma^2$, we have $K^1 < K^2$. Then, the agent's expected total discounted benefit associated with policy i is given by:

$$J^i(q, x, y; \tau_S^i) = \mathbb{E}\left[\int_0^\infty e^{-rt} B^i(Q_t, X_t, Y_t^i)dt - e^{-r\tau_S^i} K^i(Q_{\tau_S^i})\right], \tag{2.6}$$

[2]See, for example, Karatzas and Shreve (1991).

where $r > 0$ is the discount rate, $\tau_S^i \in \mathcal{T}$ is the implementation time of the policy i, and $\mathcal{T}$ is the set of all admissible implementation times. Furthermore, we assume the following condition:

$$\mathbb{E}\left[\int_0^{\infty} \mathrm{e}^{-rt}|B(Q_t, X_t, Y_t)|\mathrm{d}t\right] < \infty. \tag{AS.1}$$

Therefore, the agent's problem is to choose the timing of implementing the policy i to maximize J^i:

$$V^i(q,x,y) = \sup_{\tau_S^i \in \mathcal{T}} J^i(q,x,y;\tau_S^i) = J^i(q,x,y;\tau_S^{i*}), \tag{2.7}$$

where V^i is the value function of the agent's problem and τ_S^{i*} is the optimal timing to implement the policy i.

2.2 Optimal Environmental Policy

The agent's problem (2.7) is formulated as an optimal stopping problem. As is well known, optimal stopping problems are solved by variational inequalities. See, for example, Hu and Øksendal (1998), Dupuis and Wang (2002), Øksendal (2003).

To define the variational inequalities, we rewrite (2.6) as:

$$\begin{aligned} J^i(q,x,y;\tau_S^i) &= \mathbb{E}\left[\int_0^{\infty} \mathrm{e}^{-rt} B(Q_t, X_t, Y_t)\mathrm{d}t - \mathrm{e}^{-r\tau_S^i} K^i(Q_t)\right] \\ &= \mathbb{E}\left[\int_0^{\tau_S^i} \mathrm{e}^{-rt} B^0(Q_t, X_t, Y_t^0)\mathrm{d}t \right. \\ &\qquad \left. + \mathrm{e}^{-r\tau_S^i}\left(\int_{\tau_S^i}^{\infty} \mathrm{e}^{-r(t-\tau_S^i)} B^i(Q_t, X_t, Y_t^i)\mathrm{d}t - K^i(Q_t)\right)\right] \\ &= \mathbb{E}\left[\int_0^{\tau_S^i} \mathrm{e}^{-rt} B^0(Q_t, X_t, Y_t^0)\mathrm{d}t + \mathrm{e}^{-r\tau_S^i} G^i(Q_{\tau_S^i}, X_{\tau_S^i}, Y_{\tau_S^i}^i)\right], \end{aligned} \tag{2.8}$$

where $G^i(Q_t, X_t, Y_t^i)$ is given by:

$$G^i(Q_t, X_t, Y_t^i) = \int_t^{\infty} \mathrm{e}^{-r(s-t)} B^i(Q_s, X_s, Y_s^i)\mathrm{d}s - K^i(Q_t). \tag{2.9}$$

The region where the agent has not implemented the environmental policy i is defined by:

$$H_S^i = \{(x,y); V^i(q,x,y) > G^i(q,x,y)\}. \tag{2.10}$$

That is, H_S^i is the continuation region and yields the timing of implementing the environmental policy i, τ_S^i, given by:

$$\tau_S^i = \inf\{t > 0; (x,y) \notin H_S^i\}. \tag{2.11}$$

We now define the variational inequalities of the agent's problem (2.7).

Definition 2.1 (Variational Inequalities). *The following relations are the variational inequalities of the agent's problem (2.7):*

$$\mathcal{L}V^i(q,x,y) + B^0(q,x,y) \le 0, \tag{2.12}$$

$$V^i(q,x,y) \ge G^i(q,x,y), \tag{2.13}$$

$$[\mathcal{L}V^i(q,x,y) + B^0(q,x,y)][V^i(q,x,y) - G^i(q,x,y)] = 0, \tag{2.14}$$

where $\mathcal{L}$ is the partial differential operator defined by:

$$\mathcal{L} := \frac{1}{2}\sigma^2 x^2 \frac{\partial^2}{\partial x^2} + \mu x \frac{\partial}{\partial x} + (\gamma^i q - \delta y)\frac{\partial}{\partial y} + \alpha q \frac{\partial}{\partial q} - r. \tag{2.15}$$

(2.14) is the complementary condition and can be rewritten as follows. If $(x, y) \in H_S^i$, we then have:

$$\mathcal{L}V^i(q, x, y) + B^0(q, x, y) = 0. \tag{2.16}$$

Alternatively, if $(x, y) \notin H_S^i$, we have:

$$V^i(q, x, y) - G^i(q, x, y) = 0. \tag{2.17}$$

Let $\phi^i(q, x, y)$ be a candidate function of the value function $V^i(q, x, y)$. We can now prove that an environmental policy derived by the variational inequalities is optimal. The following theorem is the well-known verification theorem. See, for example, Theorem 10.4.1 in Øksendal (2003). The theorem also states if a candidate function satisfies the variational inequalities, the candidate function is equal to the value function. See also Hu and Øksendal (1998), Dupuis and Wang (2002).

Theorem 2.1. *1. Let $\phi^i(q, x, y)$ be a solution of the variational inequalities (2.12)-(2.14) that satisfies the following:*
The family $\{\phi^i(Q_{\tau_S^i}, X_{\tau_S^i}, Y_{\tau_S^i})\}_{\tau_S^i \in \hat{T}}$ is uniformly integrable with respect to $\mathbb{P}$, where $\hat{T}$ is the set of all bounded stopping times. Then we obtain that:

$$\phi^i(q, x, y) \geq V^i(q, x, y). \tag{2.18}$$

2. When $(x, y) \in H_S^i$, we have (2.16). Furthermore, the timing of implementing the policy i, τ_S^i, is given by (2.11). Then, the candidate function ϕ^i is equal to the value function V^i:

$$\phi^i(q, x, y) = V^i(q, x, y). \tag{2.19}$$

In addition, τ_S^i is optimal.

Proof. We omit the proof as it is similar to Øksendal (2003, Theorem 10.4.1). □

Next, we investigate whether the candidate function $\phi^i(q, x, y)$ is a solution to the variational inequalities. From the formulation of the agent's problem (2.7), we conjecture the optimal environmental policy as follows. For a given pollutant stock level y, if the process of $X = \{X_t\}_{t \geq 0}$ reaches some threshold $x_S^i(y)$, the agent implements the environmental policy i, and otherwise does not. Thus, the optimal timing of implementing the policy i is given by:

$$\tau_S^i := \tau_S^i(y) = \inf\{t > 0; x \geq x_S^i(y)\}. \tag{2.20}$$

The variational inequalities imply that (2.16) holds for $x < x_S^i(y)$. We conjecture a solution to (2.16) is:

$$\phi^i(q, x, y) = C_{S1}^i(y)x^{\beta_1} + C_{S2}^i(y)x^{\beta_2} + \frac{pq}{r - \alpha} - \frac{xy^2}{\rho_1} - \frac{2xy\gamma^0 q}{\rho_1\rho_2} - \frac{2x(\gamma^0)^2 q^2}{\rho_1\rho_2\rho_3}, \tag{2.21}$$

where $C_{S1}^i(y)$ and $C_{S2}^i(y)$ are unknowns to be determined. $\rho_1 = r - \mu + 2\delta$, $\rho_2 = r - \mu + \delta - \alpha$, and $\rho_3 = r - \mu - 2\alpha$. β_1 and β_2 are the solutions to the following characteristic equation:

$$\frac{1}{2}\sigma^2\beta(\beta - 1) + \mu\beta - r = 0, \tag{2.22}$$

and are calculated as:

$$\begin{aligned}\beta_1 &= \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1,\\ \beta_2 &= \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} < 0.\end{aligned} \tag{2.23}$$

If $X_t = 0$, the agent does not suffer from the pollutant. Then, we obtain the following boundary condition of the agent's problem:

$$\phi^i(q, 0, y) = \frac{pq}{r - \alpha}. \tag{2.24}$$

It follows from (2.21) and (2.24) that we put $C^i_{S2}(y) = 0$. Then, (2.21) becomes:

$$\phi^i(q, x, y) = C^i_{S1}(y)x^{\beta^i_1} + \frac{pq}{r - \alpha} - \frac{xy^2}{\rho_1} - \frac{2xy\gamma^0 q}{\rho_1\rho_2} - \frac{2x(\gamma^0)^2 q^2}{\rho_1\rho_2\rho_3}. \tag{2.25}$$

The first term on the right-hand side of (2.25) represents the value from where the agent can choose the timing of implementing the policy. The second to fifth terms represent the expected discounted value of B^0 where the agent does not perpetually implement the environmental policy i. We calculate these as follows:

$$\begin{aligned}&\mathbb{E}\left[\int_0^\infty \mathrm{e}^{-rt}(pQ_t - X_t(Y^i_t)^2)\mathrm{d}t\right]\\ &= \int_0^\infty \mathrm{e}^{-rt} pq\mathrm{e}^{\alpha t}\mathrm{d}t - \int_0^\infty \mathrm{e}^{-rt} x\mathrm{e}^{\mu t}\left\{\mathrm{e}^{-\delta t}\left(y - \frac{\gamma^0 q}{\alpha + \delta}\right) + \mathrm{e}^{\alpha t}\frac{\gamma^0 q}{\alpha + \delta}\right\}^2 \mathrm{d}t\\ &= \frac{pq}{r - \alpha} - \frac{xy^2}{\rho_1} - \frac{2xy\gamma^0 q}{\rho_1\rho_2} - \frac{2x(\gamma^0 q)^2}{\rho_1\rho_2\rho_3}.\end{aligned} \tag{2.26}$$

The unknown $C^i_{S1}(y)$ and threshold $x^i_S(y)$ are calculated by the following simultaneous equations:

$$\phi^i(q, x^i_S(y), y) = G^i(q, x^i_S(y), y), \tag{2.27}$$

$$\phi^i_x(q, x^i_S(y), y) = G^i_x(q, x^i_S(y), y). \tag{2.28}$$

These respective equations are well known as the value-matching and smooth-pasting conditions. Then, we obtain that:

$$C^i_{S1}(y) = \left(\frac{2(\rho_3 y\Gamma^i + \Upsilon^i)}{\beta_1\rho_1\rho_2\rho_3}\right)^{\beta_1}\left(\frac{\beta_1 - 1}{K^i(q)}\right)^{\beta_1 - 1}, \tag{2.29}$$

$$x^i_S(y) = \left(\frac{\beta_1}{\beta_1 - 1}\right)\left(\frac{\rho_1\rho_2\rho_3}{2(\rho_3 y\Gamma^i + \Upsilon^i)}\right)K^i(q), \tag{2.30}$$

where $\Gamma^i = (\gamma^0 - \gamma^i)q$, $\Upsilon^i = ((\gamma^0)^2 - (\gamma^i)^2)q^2$. In what follows, owing to the tractability of notation, $C^i_{S1} := C^i_{S1}(y)$. From $\gamma^0 > \gamma^1 > \gamma^2$, $K^1 < K^2$ and (2.30), the threshold of EP1 is smaller than the threshold of EP2:

$$x^1_S(y) < x^2_S(y). \tag{2.31}$$

3 Alternative Environmental Policies

In this section, we consider that the agent has two environmental policy options and assume that the agent implements either EP1 or EP2. We follow the framework in Décamps, Mariotti and Villeneuve (2006) who investigate the choice problem between two alternative investment projects. Let τ_A be the timing of implementing EP1 or EP2 given by:

$$\tau_A = \min\left[\tau_A^1, \tau_A^2\right], \tag{3.1}$$

where τ_A^i $(i = 1, 2)$ is the timing of implementing the policy i where the agent has two environmental policy options. Notice that these timings depend on the stock of the pollutant y. Owing to the tractability of the notations, we omit the dependency on y. Then, the agent's expected total discounted benefit J is:

$$\begin{aligned} J(q,x,y;\tau_A) &= \mathbb{E}\bigg[\int_0^{\tau_A^1\wedge\tau_A^2} \mathrm{e}^{-rt}B^0(Q_t,X_t,Y_t^0)\mathrm{d}t \\ &\quad + \mathbf{1}_{\{\tau_A^1\le\tau_A^2\}}\mathrm{e}^{-r\tau_A^1}\Big(\int_{\tau_A^1}^{\infty} \mathrm{e}^{-r(t-\tau_A^1)}B^1(Q_t,X_t,Y_t^1)\mathrm{d}t - K^1(Q_t)\Big) \\ &\quad + \mathbf{1}_{\{\tau_A^1>\tau_A^2\}}\mathrm{e}^{-r\tau_A^2}\Big(\int_{\tau_A^2}^{\infty} \mathrm{e}^{-r(t-\tau_A^2)}B^2(Q_t,X_t,Y_t^2)\mathrm{d}t - K^2(Q_t)\Big)\bigg] \\ &= \mathbb{E}\bigg[\int_0^{\tau_A^1\wedge\tau_A^2} \mathrm{e}^{-rt}B^0(Q_t,X_t,Y_t^0)\mathrm{d}t \\ &\quad + \mathbf{1}_{\{\tau_A^1\le\tau_A^2\}}\mathrm{e}^{-r\tau_A^1}G^1(Q_{\tau_A^1},X_{\tau_A^1},Y_{\tau_A^1}^1) + \mathbf{1}_{\{\tau_A^1>\tau_A^2\}}\mathrm{e}^{-r\tau_A^2}G^2(Q_{\tau_A^2},X_{\tau_A^2},Y_{\tau_A^2}^2)\bigg]. \end{aligned} \tag{3.2}$$

Therefore, the agent's problem is to choose the timing of implementing the policy to maximize their expected total discounted benefit J:

$$V(q,x,y) = \sup_{\tau_A\in\mathcal{T}} J(q,x,y;\tau_A) = J(q,x,y;\tau_A^*). \tag{3.3}$$

From (2.10) the region where the agent implements neither EP1 nor EP2 is defined by:

$$H_A(y) = \{(x,y); V(q,x,y) > \max[G^1(q,x,y), G^2(q,x,y)]\}. \tag{3.4}$$

That is, $H_A(y)$ is the continuation region. Then, τ_A is given by:

$$\tau_A = \inf\{t > 0; x \notin H_A(y)\}. \tag{3.5}$$

As in Section 2, the agent's problem is formulated as an optimal stopping problem and is solved via the variational inequalities. The variational inequalities of the agent's problem (3.3) are as follows:

$$\mathcal{L}V(q,x,y) + B^0(q,x,y) \le 0, \tag{3.6}$$

$$V(q,x,y) \ge \max[G^1(q,x,y), G^2(q,x,y)], \tag{3.7}$$

$$\left[\mathcal{L}V(q,x,y) + B^0(q,x,y)\right]\left[V(q,x,y) - \max\left[G^1(q,x,y), G^2(q,x,y)\right]\right] = 0. \tag{3.8}$$

Let $\tilde{x}$ be the value of the shift variable such that $G^1(q,x,y) = G^2(q,x,y)$. Then, $\tilde{x}$ is calculated as:

$$\tilde{x} = (K^2(q) - K^1(q))\left[\frac{\rho_1\rho_2\rho_3}{2(y\rho_3\Gamma + \Upsilon)}\right], \tag{3.9}$$

where $\Gamma = (\gamma^1 - \gamma^2)q$, $\Upsilon = ((\gamma^1)^2 - (\gamma^2)^2)q^2$. The value function V smoothly pastes neither the function G^1 nor the function G^2 at $x = \tilde{x}$. Then, we obtain the following result.

Proposition 3.1. *When the shift variable is $\tilde{x}$, the agent implements neither policy.*

Décamps, Mariotti and Villeneuve (2006) provide a rigorous treatment in their Proposition 2.2. Furthermore, Décamps, Mariotti and Villeneuve (2006) obtain the following result in their Theorem 2.1.

Theorem 3.1. *Assume that:*

$$\frac{(\rho_3 y\Gamma^1 + \Upsilon^1)^{\beta_1}}{K^1(q)^{\beta_1 - 1}} > \frac{(\rho_3 y\Gamma^2 + \Upsilon^2)^{\beta_1}}{K^2(q)^{\beta_1 - 1}}. \tag{3.10}$$

Let $x^i_A(y)$ $(i = 1, 2)$ be the threshold of implementing the policy i when the agent has two policy options: EP1 and EP2. The timing of implementing EP1, τ^1_A, is given by:

$$\tau^1_A = \inf\{t > 0; x^1_S(y) \leq X_t \leq x^1_A(y)\}. \tag{3.11}$$

Conversely, the timing of implementing EP2, τ^2_A, is given by:

$$\tau^2_A = \inf\{t > 0; X_t \geq x^2_A(y)\}. \tag{3.12}$$

The continuation region $H_A(y)$ is redefined by:

$$H_A(y) = \{x; x < x^1_S(y),\ x^1_A(y) < x < x^2_A(y)\}. \tag{3.13}$$

From (3.11), the region where EP1 is implemented is defined by:

$$I_1(y) = \{x; x^1_S(y) \leq x \leq x^1_A(y)\}. \tag{3.14}$$

Similarly, from (3.12), the region where EP2 is implemented is defined by:

$$I_2(y) = \{x; x \geq x^2_A(y)\}. \tag{3.15}$$

The continuation region H_A is divided into two regions. The first region is defined by:

$$H_{A1}(y) = \{x; x < x^1_S(y)\}, \tag{3.16}$$

where H_{A1} is the continuation region when the agent has only EP1. The second region is defined by:

$$H_{A12}(y) = \{x; x^1_A(y) < x < x^2_A(y)\}. \tag{3.17}$$

This region arises from the flexibility where the agent can choose between EP1 and EP2.

Let $\phi(q, x, y)$ be a candidate function of the value function $V(q, x, y)$. From the variational inequalities (3.6)–(3.8), for $x \in H_A$ we have:

$$\frac{1}{2}\sigma^2 x^2 \phi_{xx} + \mu x \phi_x + (\gamma^0 q - \delta y)\phi_y + \alpha q \phi_q - r\phi + B^0 = 0. \tag{3.18}$$

For $x < x^1_S$, when x reaches x^1_S, the agent implements EP1. Then, we have ϕ^1 given by (2.25). For $x^1_A < x < x^2_A$, when x reaches x^1_A before x^2_A, the agent implements EP1. Alternatively, when x reaches x^2_A before x^1_A, the agent implements EP2. Thus, the agent has two types of flexibility in this region. The candidate function is then:

$$\phi(q, x, y) = C_{A1}x^{\beta_1} + C_{A2}x^{\beta_2} + \frac{pq}{r - \alpha} - \frac{xy^2}{\rho_1} - \frac{2xy\gamma^0 q}{\rho_1\rho_2} - \frac{2x(\gamma^0)^2 q^2}{\rho_1\rho_2\rho_3}, \tag{3.19}$$

where $C_{A1} := C_{A1}(y)$ and $C_{A2} := C_{A2}(y)$ are unknowns to be determined. The first term on the right-hand side is the value of the flexibility from where the agent chooses the timing of implementing EP1. The second term is the value of the flexibility from where the agent chooses EP2. Then, ϕ is divided by the level of x as follows:

$$\phi(q,x,y)=\begin{cases} C_{S1}^{1}x^{\beta_1}+\frac{pq}{r-\alpha}-\frac{xy^2}{\rho_1}-\frac{2xy\gamma^0 q}{\rho_1\rho_2}-\frac{2x(\gamma^0)^2q^2}{\rho_1\rho_2\rho_3}, & x<x_S^1(y),\\ G^1(q,x,y), & x_S^1(y)\le x\le x_A^1(y),\\ C_{A1}x^{\beta_1}+C_{A2}x^{\beta_2}+\frac{pq}{r-\alpha}-\frac{xy^2}{\rho_1}-\frac{2xy\gamma^0 q}{\rho_1\rho_2}-\frac{2x(\gamma^0)^2q^2}{\rho_1\rho_2\rho_3}, & x_A^1(y)<x<x_A^2(y),\\ G^2(q,x,y), & x\ge x_A^2(y). \end{cases} \tag{3.20}$$

As in Section 2, we have to determine the unknowns: C_{A1}, C_{A2} and the thresholds: $x_A^1(y)$, $x_A^2(y)$. These are calculated using simultaneous equations:

$$\phi(q,x_A^1(y),y)=G^1(q,x_A^1(y),y). \tag{3.21}$$

$$\phi(q,x_A^2(y),y)=G^2(q,x_A^2(y),y). \tag{3.22}$$

$$\phi_x(q,x_A^1(y),y)=G_x^1(q,x_A^1(y),y). \tag{3.23}$$

$$\phi_x(q,x_A^2(y),y)=G_x^2(q,x_A^2(y),y). \tag{3.24}$$

Unfortunately, as we cannot analytically derive these thresholds, in the following section we numerically calculate their values.

4 Numerical Analysis

In this section, we numerically calculate the thresholds: $x_S^1(y)$, $x_S^2(y)$, $x_A^1(y)$, and $x_A^2(y)$ and investigate the effects of changes in the parameters on the thresholds. The basic parameter values are set out in Table 1.

The value function V where the agent has two environmental policy options is illustrated in Figure 1. The threshold values are calculated as $x_S^1 = 0.1494$, $x_S^2 = 0.2423$, $x_A^1 = 0.2176$, $x_A^2 = 0.2793$ in the base case. The indifference value of the shift variable is $\tilde{x} = 0.2476$.

We provide the results of the comparative static analysis of the thresholds in Figures 2–14. Figure 2 shows that the continuation region H_A is increasing in the discount rate r. While the implementation region of EP1, I_1, is increasing in r, the implementation region of EP2, I_2, is decreasing in r. That is, the higher the discount rate, the smaller the present value of the damage. Then, the agent postpones implementing the environmental policy. For $r < 0.03316$, assumption (3.10) does not hold.

Figure 3 shows that the continuation region H_A is decreasing in the expected growth rate of economic activity, α. However, while the implementation region of EP1 I_1 is decreasing in α, the implementation region of EP2 I_2 is increasing in α. That is, the higher the expected growth rate of economic activity, the larger the present value of the damage. Accordingly, the agent hastens implementation of the environmental policy. For $\alpha > 0.02075$, assumption (3.10) does not hold.

Figure 4 shows that the continuation region H_A is decreasing in the parameter of economic activity, q. However, while the region I_1 is increasing in q, region I_2 is decreasing in q. For $q < 1.72152$, assumption (3.10) does not hold. Figure 5 shows that the continuation region H_A is decreasing in the expected growth rate of the shift variable, μ. However, while region I_1 is decreasing in μ, region I_2 is increasing in μ. That is, the higher the expected growth rate of the shift variable, the larger the present value of the damage. For $\mu > 0.02075$, assumption (3.10) does not hold.

Figure 6 shows that the continuation region H_A is increasing in the volatility of the shift variable, σ. Regions I_1 and I_2 are both decreasing in σ. For $\sigma > 0.27923$, assumption (3.10) does not hold. Figure 7 shows the continuation region H_A is decreasing in the stock of the pollutant, y. However, while region I_1 is decreasing in y, region I_2 is increasing in y. Figure 8 shows that region H_{A1} is increasing in the emission conversion factor γ^0. In contrast, H_{A12} is decreasing in γ^0. Combining these effects, H_A is increasing in γ^0. While region I_1 is increasing in γ^0, region I_2 is decreasing in γ^0. For $\gamma^0 < 0.0396$, assumption (3.10) does not hold.

Figures 9 and 10 show that region H_{A1} is decreasing in the emission conversion factor γ^1. In contrast, H_{A12} is increasing in γ^1. Combining these effects, H_A is not monotonic with respect to γ^1. To start with, H_A is decreasing and then increasing in γ^1. However, while region I_1 is decreasing in γ^1, region I_2 is increasing in γ^1. For $\gamma^1 > 0.0420$, assumption (3.10) does not hold. For $\gamma^1 = 0.02$, threshold x_A^1 equals threshold x_A^2 and threshold x_S^1 equals threshold x_S^2.

Figure 11 shows that region H_{A1} does not change by varying the emission conversion factor γ^2. This is because H_{A12} is decreasing in γ^2. Then, H_A is decreasing in γ^2. While region I_1 is decreasing in γ^2, region I_2 is increasing in γ^2. For $\gamma^2 = 0.03$, threshold x_A^1 equals threshold x_A^2 and threshold x_S^1 equals threshold x_S^2.

Figure 12 shows that regions H_{A1} and H_{A12} are increasing in the fixed cost to implement the environmental policy, k_0. Then, region H_A is increasing in k_0 and regions I_1 and I_2 are decreasing in k_0. For $k_0 > 51.0730$, assumption (3.10) does not hold. Figure 13 shows that region H_{A1} and H_{A12} are increasing in the proportional cost parameter k_1. Then, region H_A is increasing in k_1 and regions I_1 and I_2 are decreasing in k_1. For $k_1 > 1291.7$, assumption (3.10) does not hold.

Figure 14 shows that region H_{A1} and H_{A12} are increasing in the adjustment cost parameter k_2. Then, region H_A is increasing in k_2. However, while region I_1 is increasing in k_2, region I_2 is decreasing in k_2. For $k_2 < 1622.2$, assumption (3.10) does not hold.

5 Conclusion

In this paper, we investigate environmental policy under uncertainty. We consider that an economic agent benefits from the economic activity and suffers from the pollutant so emitted. As the agent has two policy options, the agent must decide which policy to implement and when to implement the chosen policy in order to maximize its benefit. To solve the agent's problem, we formulate it as an optimal stopping problem. We first investigate the single environmental policy and obtain the closed form of the threshold. Next, we investigate the alternative environmental policies. Unfortunately, the thresholds of the policies are not explicitly derived. Therefore, we conduct numerical and comparative static analysis. The representative findings indicate that the continuation region increases in volatility, that is, with uncertainty, while the policy implementing regions decrease in volatility. Further, the continuation region increases in the emission conversion factor where the agent has implemented neither EP1 nor EP2. That is, while the region for implementing EP1 increases in the conversion factor, the region for implementing EP2 decreases in the conversion factor.

To conclude the paper, we suggest a number of possible extensions for our model. First, to future work we defer examination of the effect of technological progress, particularly as this plays an important role in environmental policy. Second, in this paper, we assume that the dynamics of economic activity are deterministic. As economic development is also uncertain in the real world, the dynamics of economic activity (or the price) should incorporate stochastic differential equations. We leave these topics for future research.

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Table 1: The base case values of the parameters and variables

Variable	Symbol	Value
Parameters		
Discount rate	r	0.05
Growth rate of economic activity	α	0.01
Price	p	10
Growth rate of shift variable	μ	0.01
Volatility of shift variable	σ	0.2
Rate of natural decay	δ	0.01
Emission conversion factor for policy 0	γ^0	0.05
Emission conversion factor for policy 1	γ^1	0.03
Emission conversion factor for policy 2	γ^2	0.02
Fixed cost	k_0	5
Proportional cost	k_1	100
Adjustment cost	k_2	10000
Variables		
Economic activity	q	5
Stock of pollutant	y	0.1

Policy 0 is when the agent has implemented neither EP1 nor EP2.

Figure 1:
Value function of alternative environmental policies.

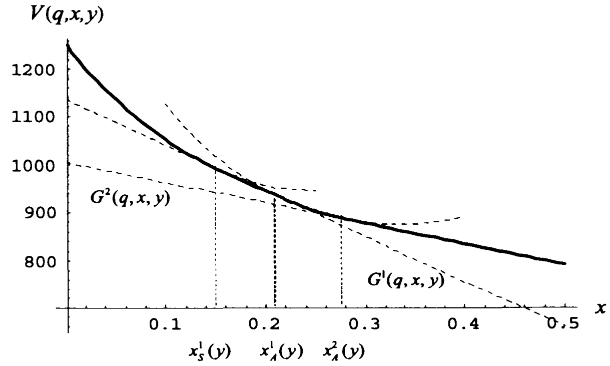

Figure 2:
Comparative statics of thresholds with respect to r.

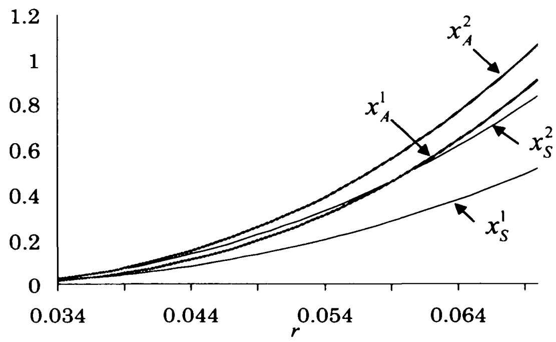

Figure 3:
Comparative statics of thresholds with respect to α.

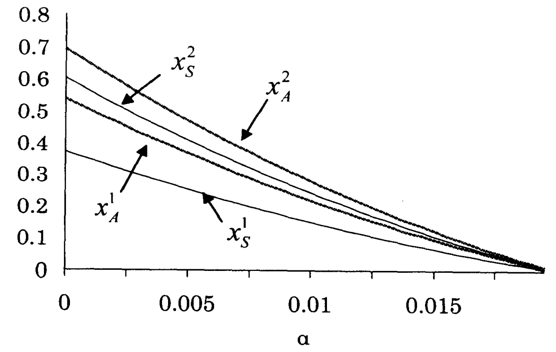

Figure 4:
Comparative statics of thresholds with respect to q.

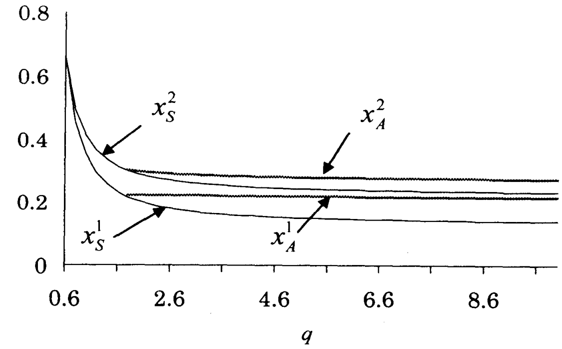

Figure 5:
Comparative statics of thresholds with respect to μ.

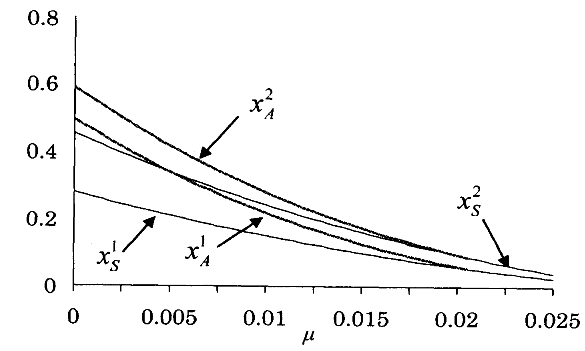

Figure 6:
Comparative statics of thresholds with respect to σ.

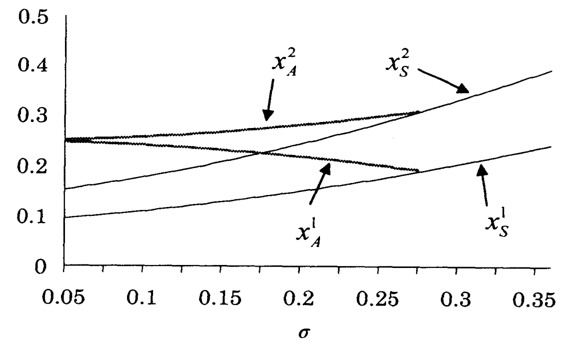

Figure 7:
Comparative statics of thresholds with respect to y.

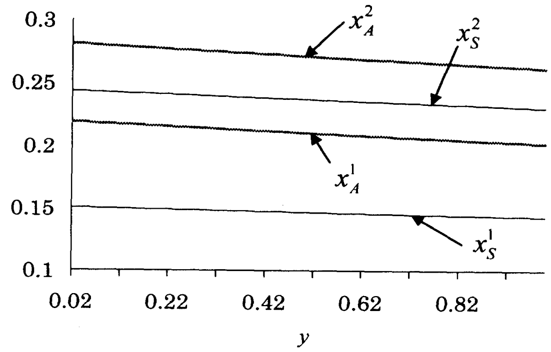

Figure 8:
Comparative statics of thresholds with respect to γ^0.

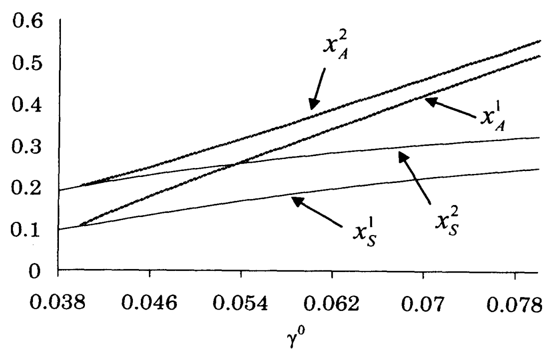

Figure 9:
Comparative statics of thresholds with respect to γ^1.

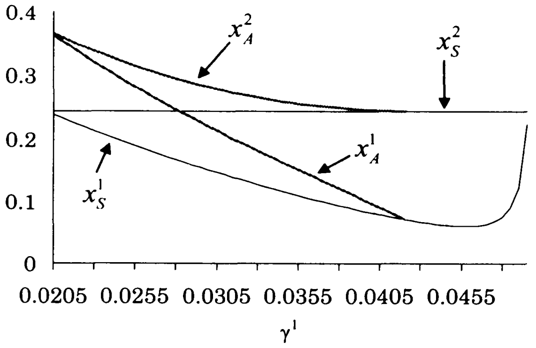

Figure 10:
Comparative statics of continuation regions H_{A1}, H_{A12}, and H_A, respect to γ^1.

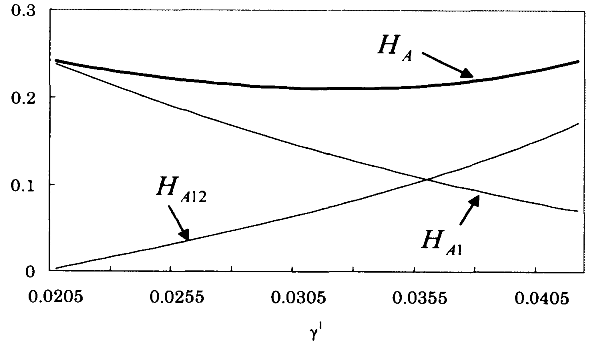

Figure 11:
Comparative statics of thresholds with respect to γ^2.

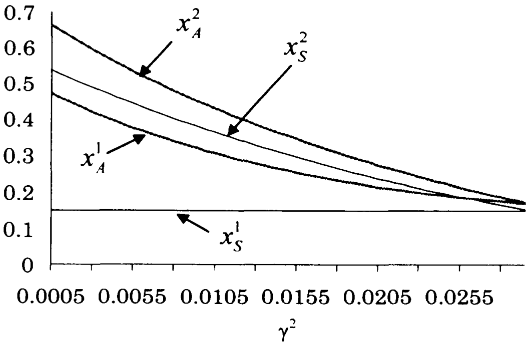

Figure 12:
Comparative statics of thresholds with respect to k_0.

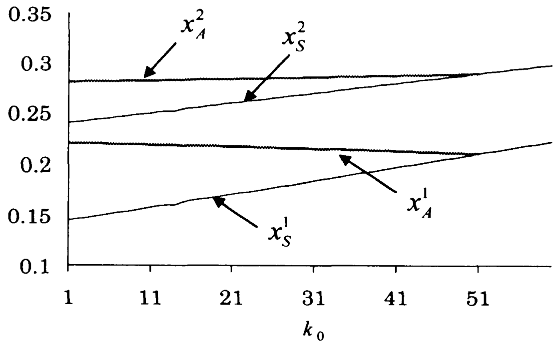

Figure 13:
Comparative statics of thresholds with respect to k_1.

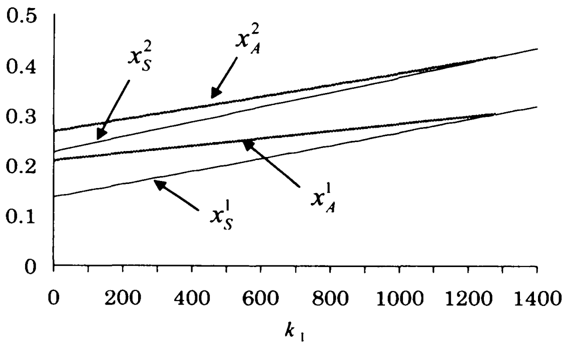

Figure 14:
Comparative statics of thresholds with respect to k_2.