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ABSOLUTELY CONTINUOUS INVARIANT MEASURES FOR CIRCLE MAPS WITH CRITICAL AND SINGULAR POINTS

HIROKI TAKAHASI

Motivated by a global study of the dynamics of periodically perturbed dissipative double homoclinic loops [5], we consider a two-parameter family $(f_{a,L})$ of maps on the circle $S^1 = \mathbb{R}/\mathbb{Z}$ given by

$$f_{a,L}\colon \theta \mapsto \theta + a + L\log|\Phi(\theta)|, \quad a\in[0,1), L>0.$$

The $\Phi\colon S^1 \to \mathbb{R}$ is a Morse function, with its graph intersecting the θ-axis transversely. The value of $f_{a,L}$ is undefined at $S=\{\theta\colon \Phi(\theta)=0\}$, which is a finite set. All the θ-derivatives blow up to infinity at S. The $f_{a,L}$ has a finite number of critical points.

Main Theorem. [2] *For all large L, there exists a set $A_L^{(\infty)}$ in $[0,1)$ with positive Lebesgue measure such that for all $a\in A_L^{(\infty)}$, the corresponding $f_{a,L}$ admits a unique absolutely continuous invariant probability measure μ. Lebesgue almost every $\theta\in S^1$ is μ-generic, that is,*

$$\lim_{n\to\infty}\frac{1}{n}\sum_{i=0}^{n-1}\varphi(f_{a,L}^i\theta) = \int \varphi d\mu \quad \textit{for all continuous } \varphi\colon S^1\to\mathbb{R}.$$

Moreover, the Lebesgue measure of $A_L^{(\infty)}$ satisfies $\lim_{L\to\infty}\mathrm{Leb}(A_L^{(\infty)})=1$.

For the construction of the parameter set $A_L^{(\infty)}$, we perform an inductive parameter exclusion in the spirit of Benedicks and Carleson. To deal with the effect of the singular set, and to get a good estimate of the measure as in the last line of the statement, some additional considerations are necessary. For the construction of the acip, we follow a standard inducing argument. The uniqueness of acip and the genericity follow from the assumption that L is large.