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A CONJECTURE IN REPRESENTATION THEORY OF FINITE GROUPS

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1. Introduction

Let G be a finite group and p a prime dividing the order of G. There are several conjectures connecting the representation theory of G with the representation theory of certain p-local subgroups (i.e. the p-subgroups and their normalizers) of G. For example, it seems to be true, that if P is a Sylow p-subgroup of G, then the number of complex irreducible characters of G of degree coprime with p equals the same number for the normalizer $N_G(P)$.

This conjecture, called McKay conjecture [55], and its block-theoretic version due to Alperin [1] were generalized by various authors. In [50], Isaacs and Navarro proposed the following refinement of the McKay conjecture: If k is a residue class modulo p different from zero, then the two numbers above should still be equal when we count only those characters having a degree in the residue classes k or $-k$.

In a series of papers [30], [31], [32], Dade developed several conjectures expressing the number of complex irreducible characters with a fixed defect in a given p-block of G in terms of an alternating sum of related values for p-blocks of certain p-local subgroups of G. The ordinary conjecture is the simplest one among others, and the most complicated one is called the inductive form, which implies all the other. If G has a trivial Schur multiplier and a cyclic outer automorphism group, it follows that Dade's inductive conjecture is also true for G in this case. Dade claimed that, if the inductive form is true for all finite simple groups, then it is true for all finite groups. In [31], Dade proved that his (projective) conjecture implies the McKay conjecture. Motivated by the Isaacs-Navarro conjecture [50], Uno [60] suggested a further refinement of Dade's conjecture including the p'-parts of character degrees.

In [51], Isaacs, Malle and Navarro reduced the McKay conjecture to a question about finite simple group. In particular, they showed that every finite group will satisfy the McKay conjecture if every finite non-abelian simple group is "good".

This note is organised as follows: In Section 2, we fix notation and state Dade's and Uno's invariant conjectures in detail. In Section 3, we sketch the proof of Dade's and Uno's invariant conjecture for some exceptional groups in the defining characteristic. In Section 4, we deal with the McKay conjecture for the Big Ree groups ${}^2F_4(q)$ in characteristic 2. In Section 5, we present some new results on Dade's conjecture.

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2. Conjectures of Dade and Uno

Let R be a p-subgroup of a finite group G. Then R is *radical* if $O_p(N(R)) = R$, where $O_p(N(R))$ is the largest normal p-subgroup of the normalizer $N(R) := N_G(R)$. Denote by $\mathrm{Irr}(G)$ the set of all irreducible ordinary characters of G, and by $\mathrm{Blk}(G)$ the set of p-blocks. If $H \leq G$, $\widetilde{B} \in \mathrm{Blk}(G)$, and d is an integer, we denote by $\mathrm{Irr}(H, \widetilde{B}, d)$ the set of characters $\chi \in \mathrm{Irr}(H)$ satisfying $\mathrm{d}(\chi) = d$ and $b(\chi)^G = \widetilde{B}$ (in the sense of Brauer), where $\mathrm{d}(\chi) = \log_p(|H|_p) - \log_p(\chi(1)_p)$ is the p-defect of χ and $b(\chi)$ is the block of H containing χ.

Given a p-subgroup chain

$$C : P_0 < P_1 < \cdots < P_n$$

of G, define the length $|C| := n$, $C_k : P_0 < P_1 < \cdots < P_k$ and

$$N(C) = N_G(C) := N_G(P_0) \cap N_G(P_1) \cap \cdots \cap N_G(P_n).$$

The chain C is said to be *radical* if it satisfies the following two conditions:

(a) $P_0 = O_p(G)$ and
(b) $P_k = O_p(N(C_k))$ for $1 \leq k \leq n$.

Denote by $\mathcal{R} = \mathcal{R}(G)$ the set of all radical p-chains of G.

Suppose $1 \to G \to E \to \overline{E} \to 1$ is an exact sequence, so that E is an extension of G by $\overline{E}$. Then E acts on $\mathcal{R}$ by conjugation. Given $C \in \mathcal{R}$ and $\psi \in \mathrm{Irr}(N_G(C))$, let $N_E(C, \psi)$ be the stabilizer of (C, ψ) in E, and

$$N_{\overline{E}}(C, \psi) := N_E(C, \psi)/N_G(C).$$

For $\widetilde{B} \in \mathrm{Blk}(G)$, an integer $d \geq 0$ and $U \leq \overline{E}$, we define

$$\mathrm{Irr}(N_G(C), \widetilde{B}, d, U) := \{\psi \in \mathrm{Irr}(N_G(C), \widetilde{B}, d) \mid N_{\overline{E}}(C, \psi) = U\}.$$

Dade's invariant conjecture can be stated as follows:

Dade's Invariant Conjecture ([32]) *If $O_p(G) = 1$ and $\widetilde{B} \in \mathrm{Blk}(G)$ with defect group $D(\widetilde{B}) \neq 1$, then*

$$\sum_{C \in \mathcal{R}/G} (-1)^{|C|} \, |\mathrm{Irr}(N_G(C), \widetilde{B}, d, U)| = 0,$$

where $\mathcal{R}/G$ is a set of representatives for the G-orbits of $\mathcal{R}$.

Let H be a subgroup of G, $\varphi \in \mathrm{Irr}(H)$, and let $r(\varphi) = r_p(\varphi)$ be the integer $0 < r(\varphi) \leq (p-1)$ such that the p'-part $(|H|/\varphi(1))_{p'}$ of $|H|/\varphi(1)$ satisfies

$$\left(\frac{|H|}{\varphi(1)}\right)_{p'} \equiv r(\varphi) \bmod p.$$

Given $1 \leq r < (p+1)/2$, let $\mathrm{Irr}(H, [r])$ be the subset of $\mathrm{Irr}(H)$ consisting of those characters φ with $r(\varphi) \equiv \pm r \bmod p$. For $\widetilde{B} \in \mathrm{Blk}(G)$, $C \in \mathcal{R}$, an integer $d \geq 0$ and $U \leq \overline{E}$, we define

$$\mathrm{Irr}(N_G(C), \widetilde{B}, d, U, [r]) := \mathrm{Irr}(N_G(C), \widetilde{B}, d, U) \cap \mathrm{Irr}(N_G(C), [r]).$$

The following refinement of Dade's conjecture is due to Uno.

Uno's Invariant Conjecture ([60], Conjecture 3.2) *If* $O_p(G) = 1$ *and* $\widetilde{B} \in \mathrm{Blk}(G)$ *with defect group* $D(\widetilde{B}) \neq 1$, *then for all integers* $d \geq 0$ *and* $1 \leq r < (p+1)/2$,

$$\sum_{C\in\mathcal{R}/G} (-1)^{|C|}\ |\mathrm{Irr}(N_G(C), \widetilde{B}, d, U, [r])| = 0.$$

Note that if $p = 2$ or 3, then Uno's conjecture is equivalent to Dade's conjecture.

3. Dade's/Uno's invariant conjecture for some exceptional groups

In this section, we sketch the proof of Dade's/Uno's invariant conjecture for some exceptional groups in the defining characteristic. Let $\mathrm{Aut}(G)$ and $\mathrm{Out}(G)$ be the automorphism and outer automorphism groups of G, respectively. Let n be a positive integer and

$$G \in \{G_2(p^n)\,(p \geq 5),\ {}^3D_4(p^n)\,(p = 2 \text{ or odd}),\ {}^2F_4(2^{2n+1})\}.$$

Then $\mathrm{Out}(G)$ is cyclic and the Schur multiplier of G is trivial. So the invariant conjecture for G is equivalent to the inductive conjecture.

Let $O = \mathrm{Out}(G) = \langle\alpha\rangle$, where α is a field automorphism of order

$$|\alpha| = \begin{cases} n & \text{if } G = G_2(p^n)\,(p \geq 5), \\ 3n & \text{if } G = {}^3D_4(p^n), \\ 2n+1 & \text{if } G = {}^2F_4(2^{2n+1}). \end{cases}$$

We fix a Borel subgroup B and maximal parabolic subgroups P and Q of G containing B as in [15], [40], [39], [42] and [43]. In particular, we may assume that α stabilizes B, P and Q. We note that the maximal parabolic subgroups P, Q are the groups denoted by P_a, P_b respectively in [43].

By the remarks on p. 152 in [48], G has only two p-blocks, the principal block B_0 and one defect-0-block (corresponding to the Steinberg character). Hence we have to verify Dade's/Uno's conjecture only for the principal block B_0.

By a corollary of the Borel-Tits theorem [26], the normalizers of radical p-subgroups are parabolic subgroups. The radical p-chains of G (up to G-conjugacy) are given in Table 1.

Table 1 *Radical p-chains of G.*

C		$N_G(C)$	$N_A(C)$	Parity
C_1	$\{1\}$	G	A	$+$
C_2	$\{1\} < O_p(P)$	P	$P \rtimes \langle\alpha\rangle$	$-$
C_3	$\{1\} < O_p(P) < O_p(B)$	B	$B \rtimes \langle\alpha\rangle$	$+$
C_4	$\{1\} < O_p(Q)$	Q	$Q \rtimes \langle\alpha\rangle$	$-$
C_5	$\{1\} < O_p(Q) < O_p(B)$	B	$B \rtimes \langle\alpha\rangle$	$+$
C_6	$\{1\} < O_p(B)$	B	$B \rtimes \langle\alpha\rangle$	$-$

Since C_5 and C_6 have the same normalizers $N_G(C_5) = N_G(C_6)$ and $N_A(C_5) = N_A(C_6)$, it follows that

$$|\mathrm{Irr}(N_G(C_5), B_0, d, u, [r])| = |\mathrm{Irr}(N_G(C_6), B_0, d, u, [r])|$$

for all $d \in \mathbb{N}$, $u \mid |\alpha|$ and $1 \le r < (p+1)/2$. Thus the contribution of C_5 and C_6 in the alternating sum of Dade's/Uno's invariant conjecture is zero. So Dade's/Uno's invariant conjecture for G is equivalent to

(1)
$$|\mathrm{Irr}(G, B_0, d, u, [r])| + |\mathrm{Irr}(B, B_0, d, u, [r])| = |\mathrm{Irr}(P, B_0, d, u, [r])| + |\mathrm{Irr}(Q, B_0, d, u, [r])|$$

for all $d \in \mathbb{N}$, $u \mid |\alpha|$ and $1 \le r < (p+1)/2$.

In order to verify (1), we need to determine the character tables of parabolic subgroups of G. Up to conjugacy, G has four parabolic subgroups: G, B, P and Q. Here, we present the results on the character tables of parabolic subgroups of G:

G	$\mathrm{Irr}(G)$	$\mathrm{Irr}(B)$, $\mathrm{Irr}(P)$, $\mathrm{Irr}(Q)$
$G_2(p^n)\ (p \ge 5)$	Chang, Ree [28]	An, Huang [15]
${}^3D_4(p^n)$	Deriziotis, Michler [34]	Himstedt [39], [40]
${}^2F_4(2^{2n+1})$	Malle [54]	Himstedt, Huang [42], [43]

For $L \in \{G, B, P, Q\}$, the action of $O = \mathrm{Out}(G)$ on the conjugacy classes of elements of L induces an action of O on the sets of $\mathrm{Irr}(L)$ and then an action on the parameter sets. Using the degrees and character values on the conjugacy classes we can describe the action of O on the parameter sets. Suppose $u \mid |\alpha|$ and set $t := \frac{|\alpha|}{u}$ and $H := \langle \alpha^t \rangle$. Let $\mathrm{Irr}(L, B_0, d, [r]) = \mathrm{Irr}(L, B_0, d) \cap \mathrm{Irr}(L, [r])$. Our main task is to show that

$$\mathrm{Irr}(G, B_0, d, [r]) \cup \mathrm{Irr}(B, B_0, d, [r]) \quad \text{and} \quad \mathrm{Irr}(P, B_0, d, [r]) \cup \mathrm{Irr}(Q, B_0, d, [r])$$

are isomorphic O-sets. Our approach is similar to that in [41]: we want to use [49, Lemma (13.23)], so we have to count fixed points of subgroups $H \le O$. Then (1) is equivalent to

$$|\mathrm{Irr}(G, B_0, d, [r])^{\alpha^t}| + |\mathrm{Irr}(B, B_0, d, [r])^{\alpha^t}| = |\mathrm{Irr}(P, B_0, d, [r])^{\alpha^t}| + |\mathrm{Irr}(Q, B_0, d, [r])^{\alpha^t}|.$$

Then we compute the number of fixed points of $\mathrm{Irr}(L, B_0, d, [r])$ under the action of H and prove that above equation holds.

4. McKay conjecture for ${}^2F_4(q)$

In [51], Isaacs, Malle and Navarro reduced the McKay conjecture to a question about finite simple groups. They showed that the conjecture is true for every finite group if every finite non-abelian simple group satisfies certain conditions. In this section, we sketch the proof of Isaacs-Malle-Navarro version of McKay conjecture for $G = {}^2F_4(q)$.

Let $\mathrm{Aut}(G)$ and $\mathrm{Out}(G)$ be the automorphism and outer automorphism groups of G, respectively. Let $O = \mathrm{Out}(G)$ and $A = \mathrm{Aut}(G)$. Then $O = \langle \alpha \rangle$ and $\mathrm{Aut}(G) = G \rtimes \langle \alpha \rangle$, where α is a field automorphism of (odd) order $2n+1$. We write $\mathrm{Irr}_{2'}(B)$ and $\mathrm{Irr}_{2'}(G)$ for the set of irreducible characters of odd degree of B and G, respectively. Since B is α-invariant we get an action of O on $\mathrm{Irr}_{2'}(B)$ and $\mathrm{Irr}_{2'}(G)$. Our main task is to show that $\mathrm{Irr}_{2'}(B)$ and $\mathrm{Irr}_{2'}(G)$ are isomorphic O-sets. Our approach is similar to that in [41]: we want to use [49, Lemma (13.23)], so we have to count fixed points of $\mathrm{Irr}_{2'}(B)$ and $\mathrm{Irr}_{2'}(G)$ under the action of subgroups $H \le O$.

Theorem 4.1. ([42, Section 6]) *For $q = 2^{2n+1} \ge 8$, the group ${}^2F_4(q)$ is good for the prime 2 in the sense of* [51, Section 10].

5. Results on Dade's conjecture

So far, Dade's conjecture has been proved for the following cases:

(a) Sporadic simple groups:

M_{11}, J_1	final	Dade [30]
M_{12}	final	Dade
M_{22}	final	Huang [45]
M_{23}, M_{24}	final	Schwartz, An, Conder [13]
J_2	final	Dade
J_3	final	Kotlica [52]
McL	final	Murray [56], Entz, Pahlings [36]
Ru	final	Dade, An, O'Brien [16]
He	final	An [4]
HS	final	Hassan, Horváth [37]
Co_1	final	An, O'Brien [21]
Co_2	final	An, O'Brien [17]
Co_3	final	An [6]
Suz	final	Himstedt [38]
$O'N$	final	An, O'Brien [16], Uno, Yoshiara [61]
Th	final	Uno [60]
Ly	final	Sawabe, Uno [58]
HN	final	An, O'Brien [20]
Fi_{23}	final	An, O'Brien [18]
Fi_{22}	invar.	An, O'Brien [19]
J_4		An, O'Brien, Wilson [22]
B	p odd	An, Wilson [23]
Fi'_{24}		An, Cannon, O'Brien, Unger [12]

(b) Classical groups:

$GL_n(q)$	ord., $p \mid q$	Olsson, Uno [57]
$GU_n(q)$	ord., $p \mid q$	Ku [53]
$GL_n(q)$, $GU_n(q)$	invar., $p \nmid q$	An [9]
$Sp_{2n}(q)$, $SO^{\pm}_{m}(q)$	ord., $p \nmid q$, p, q odd	An [11]
$L_2(q)$	final	Dade [33]
$L_3(q)$	final, $p \mid q$	Dade
$L_n(q)$	ord., $p \mid q$	Sukizaki [59]

(c) Exceptional groups:

${}^2B_2(2^{2n+1})$	final	Dade [33]
${}^2G_2(3^{2n+1})$	final	$p \neq 3$ An [2], $p = 3$ Eaton [35]
$G_2(q)$	final, 2, $3 \mid q$, $p \nmid q$ $q \neq 3, 4$	An [8], [10]
${}^3D_4(q)$	final, $p \nmid q$	An [7]
${}^2F_4(2^{2n+1})$	ord, $p \neq 2$	An [5]
${}^2F_4(2)'$	final	An [3]

Here, we present some new results on Dade's conjecture for exceptional groups:

$G_2(q)$	final, $p \mid q\,(p \geq 5)$, $q = 3, 4$	Huang [46], [47]
${}^3D_4(q)$	final, $p \mid q$ ($p = 2$ or odd)	An, Himstedt, Huang [14], [41]
${}^2F_4(2^{2n+1})$	final, $p = 2$	Himstedt, Huang [44]

Together with the results in [8], [10], [7] and [5], this completes the proof of Dade's conjecture for $G_2(q)$, ${}^3D_4(q)$ and ${}^2F_4(2^{2n+1})$.,

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