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Lee's homology and Rasmussen invariant

Tetsuya Abe
Department of Mathematics, Osaka City University
Sugimoto, Sumiyoshi-ku Osaka 558-8585, Japan
Email: t-abe@sci.osaka-cu.ac.jp

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Abstract

In this note, we consider some cycles for Lee's complex which represent canonical classes of Lee's homology of a knot. We also consider the Rasmussen invariant of a homogeneous knot and its application.

1 Introduction

In [19], Rasmussen introduced a smooth concordance invariant of a knot K, now called the Rasmussen invariant $s(K)$, which is defined by cycles of Lee's complex. There are many results on the Rasmussen invariant However little is known on cycles of Lee's complex. In this note, we consider some cycles for Lee's complex which represent canonical classes of Lee's homology of a knot. We also consider the Rasmussen invariant of a homogeneous knot and its application.

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2 Lee's homology of a knot

Lee [13] constructed a homology theory which is closely related to Khovanov homology theory. We review the results in [13].

2.1 The construction of Lee's homology of a knot

In this subsection, we recall the construction of Lee's homology of a knot.

Let K be a knot, D a diagram of K, $c_1, \cdots, c_n$ the crossings of D and $n_-(D)$ the number of negative crossings of D. A *state* $s = (s_1, \cdots, s_n)$ for D is a vertex of the n-dimensional cube $[0,1]^n$, that is, an element of $\{0,1\}^n$. The *grading* of s is the sum $\sum_{i=1}^{n} s_i - n_-(D)$ and denote it by $|s|$. A *0-smoothing* and a *1-smoothing* are local moves on a link diagram as in Figure 1. We denote by D_s the loops which are obtained from D by applying s_i-smoothing at c_i $(i = 1, \cdots, n)$ and by $|D_s|$ the number of components of D_s. Let $V = \mathbb{Q}[x]/(x^2 - 1)$ be a vector space, which is spanned by 1

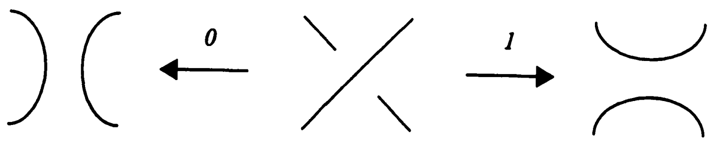

Figure 1: 0- and 1-smoothings

and x. The object of Lee's complex is defined as follows,

$$C^i_{Lee}(D) = \bigoplus_{s\in\{0,1\}^n:\ |s|=i} V^{\otimes|D_s|} \quad \text{and} \quad C^*_{Lee}(D) = \bigoplus_{i\in\mathbf{Z}} C^i_{Lee}(D).$$

The multiplication $m : V \otimes V \to V$ and the comultiplication $\Delta : V \to V \otimes V$ are defined by

$$\begin{aligned} m(1\otimes 1) &= m(x\otimes x) = 1, \quad \Delta(1) = 1\otimes x + x\otimes 1, \\ m(1\otimes x) &= m(x\otimes 1) = x, \quad \Delta(x) = x\otimes x + 1\otimes 1. \end{aligned}$$

Let $\xi = (\xi_1, \cdots, \xi_i, \cdots, \xi_n)$ be an edge of the n-dimensional cube $[0,1]^n$, that is, an element of $\{0,*,1\}^n$ with just one $*$. Suppose that $\xi_i = *$. Then we define to be $|\xi| = \xi_1 + \cdots + \xi_{i-1}$, $\xi(0) = (\xi_1, \cdots, \xi_{i-1}, 0, \xi_{i+1}, \cdots, \xi_n)$, $\xi(1) = (\xi_1, \cdots, , \xi_{i-1}, 1, \xi_{i+1}, \cdots, \xi_n)$ and $\xi(*) = i$. For example, suppose that $n = 5$ and $\xi = (1,1,*,0,1)$. Then $|\xi| = 2$, $\xi(0) = (1,1,0,0,1)$, $\xi(1) = (1,1,1,0,1)$ and $\xi(*) = 3$.

For an edge ξ, we associate the cobordism S_ξ from $D_{\xi(0)}$ to $D_{\xi(1)}$ as follows: we remove a neighborhood of the $\xi(*)$-th crossing, assign a product cobordism, and fill the saddle cobordism between the 0- and 1-smoothings around the $\xi(*)$-th crossing. The cobordism is either of the following two types: (i) two circles of $D_{\xi(0)}$ merge into one circle of $D_{\xi(1)}$, or (ii) one circle of $D_{\xi(0)}$ splits into two circles of $D_{\xi(1)}$. Furthermore, we associate the map $d_\xi \colon V^{\otimes|D_{\xi(0)}|} \longrightarrow V^{\otimes|D_{\xi(1)}|}$ as follows: the homeomorphism d_ξ is induced by the map m if the cobordism S_ξ is of type (i) and by the map Δ if the cobordism S_ξ is of type (ii). Note that we set d_ξ to be the identity on the tensor factors corresponding to the loops that do not participate. For an element $x \in V^{\otimes|D_s|} \subset C^i_{Lee}(D)$, we define d^i as follows,

$$d^i(x) = \sum_{\xi\in\{0,*,1\}^n:\ \xi(0)=s} (-1)^{|\xi|} d_\xi(x),$$

where s is a state for D. Let d be $\bigoplus_{i\in\mathbf{Z}} d^i$. We obtain $d^2 = 0$. The complex $C^*_{Lee}(D) = (C^*_{Lee}(D), d)$ is called Lee's complex. The Lee's homology of K, $H^*_{Lee}(K)$, is defined to be the homology group of $C^*_{Lee}(D)$. By the following lemma, $H^*_{Lee}(K)$ does not depend on the choice of diagrams of K.

Lemma 2.1 ([13]). *Let D and D' be diagrams of a knot K. Then $C^*_{Lee}(D)$ and $C^*_{Lee}(D')$ are chain homotopic.*

2.2 The basis of Lee's homology of a knot

It is known that Lee's homology of a knot is very simple as a vector space. Indeed, Lee [13] showed that $\dim H^*_{Lee}(K) = 2$ and described a basis of Lee's homology of a knot K. In this subsection, we explain these results. We also recall the notion of an enhanced state.

It is useful to use the basis $a = 1 + x$, $b = 1 - x$ of V. Then

$$\begin{aligned} m(a\otimes a) &= 2a, m(b\otimes b) = 2b, \quad \Delta(a) = 2a\otimes a, \\ m(a\otimes b) &= 0, m(b\otimes a) = 0, \quad \Delta(b) = -2b\otimes b. \end{aligned}$$

For a state s for D, we define *col*(D_s) to be the set of coloring maps from the components of D_s to V. Note that an element of $\mathrm{col}(D_s)$ is naturally identified with an element of $V^{\otimes|D_s|} \subset C^{|s|}_{Lee}(D)$. Hereafter we always identify an element of $\mathrm{col}(D_s)$ with the element of $V^{\otimes|D_s|} \subset C^{|s|}_{Lee}(D)$. We call an element of $\mathrm{col}(D_s)$ an *enhanced state.*

Let o be the orientation of D and s_o the state for D corresponding to o, that is, the state whose i-th element is 0 if the sign of c_i is positive and 1 if the sign of c_i is negative. Then, by definition, D_{s_o} are the Seifert circles and $|s_o| = 0$. Let $f_o(D) \in \mathrm{col}(D_{s_o})$ be the enhanced state whose values of any adjacent Seifert circles are a and b respectively and the outer most right-handed Seifert circle is a and the outer most left-handed Seifert circle is b (see Figure 3). Let $\overline{o}$ be the reversed orientation of D. Then $f_o(D)$ and $f_{\overline{o}}(D)$ are cycles of $C^0_{Lee}(D)$ and we obtain the following.

Theorem 2.2 ([13]). *Let K be a knot and D a diagram of K. Then*

$$H^i_{Lee}(K) = \begin{cases} \mathbb{Q} \oplus \mathbb{Q} & i = 0, \\ 0 & i \neq 0. \end{cases}$$

Here, $[f_o(D)]$ and $[f_{\overline{o}}(D)]$ form a basis of $H^0_{Lee}(K)$.

Remark 2.3. The two cycles $f_o(D)$ and $f_{\overline{o}}(D)$ are determined up to multiplication of 2^c, where c is an integer (see [13]). Therefore we call $[f_o(D)]$ and $[f_{\overline{o}}(D)]$ the *canonical classes* of $H^*_{Lee}(K)$.

3 State cycles which represent canonical classes

In this section, we recall the notion of a state cycle, which is a cycle of $C^0_{Lee}(D)$ and a result on state cycles (Theorem 3.2).

We recall some terms. A Seifert circle of a diagram is *strongly negative* if signs of the adjacent crossings to it are all negative. Let D be a diagram of a knot. An enhanced state $g \in \mathrm{col}(D_{s_o})$ is *state cycle* if $f_o(l) = g(l)$ for any Seifert circle l which is not strongly negative. We define *col*$_o(D_{s_o})$ to be the subset of $\mathrm{col}(D_{s_o})$ which consists of state cycles. Note that the cycle $f_o(D)$ is a state cycle. Any state cycles are, indeed, cycles of $C^0_{Lee}(D)$ as follows:

Lemma 3.1 ([1]). *Let D be a diagram of a knot and g a state cycle. Then g is a cycle of $C^0_{Lee}(D)$ i.e. $d^0(g) = 0$.*

In general, the homology class of a cycle of $C^0_{Lee}(D)$ has many representatives. Let $f_2(D)$ be the state cycle such that $f_2(D)(l) = 2$ for any strongly negative Seifert circle l. Then we obtain the following:

Theorem 3.2 ([1]). *Let D be a non-negative diagram of a knot K. Then $[f_o(D)] = [f_2(D)]$.*

We give an example which illustrates Theorem 3.2.

Example 3.3. Let D be the standard pretzel diagram of $P(3, -3, -3)$. Figure 2 illustrates D, its Seifert circles and strongly negative Seifert circles. Let $g \in C^{-1}_{Lee}(D)$ be the enhanced state as in Figure 3. Then $f_o(D) - d^{-1}(g)$ is also a state cycle as in Figure 3. Let $h \in C^{-1}_{Lee}(D)$ be the enhanced state as in Figure 4. Then $f_2(D) = f_o(D) - d^{-1}(g) - d^{-1}(h)$ as in Figure 4. Therefore three homology classes in Figure 5 are the same.

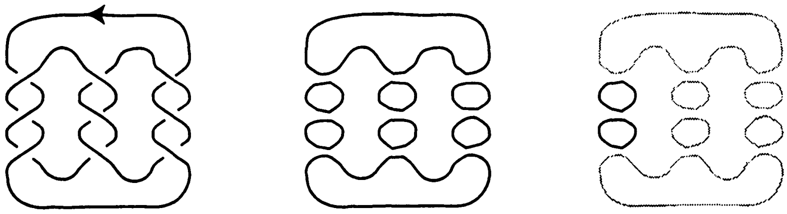

Figure 2: The standard pretzel diagram of $P(3,-3,-3)$, its Seifert circles and strongly negative Seifert circles

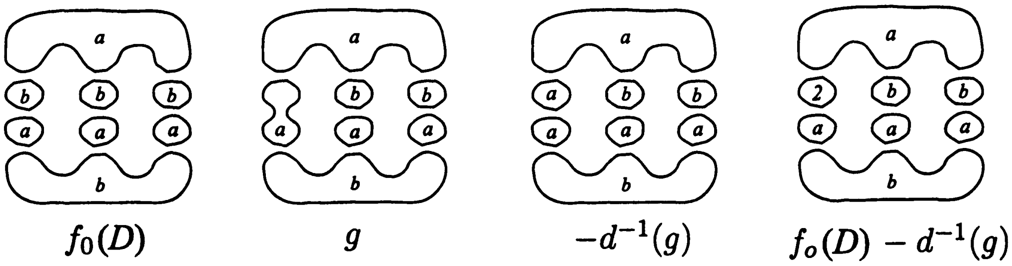

Figure 3: Some enhanced states

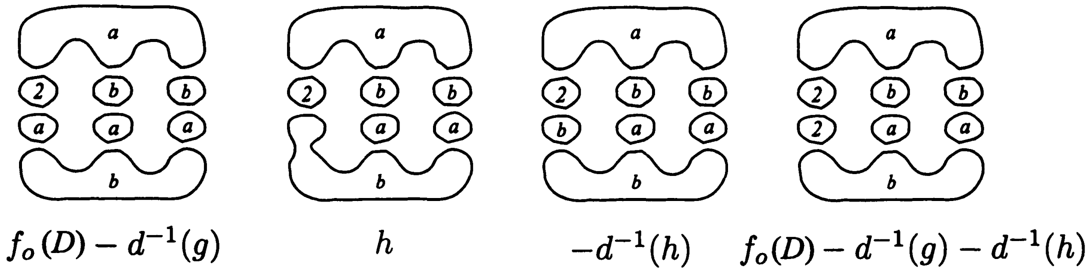

Figure 4: Some enhanced states

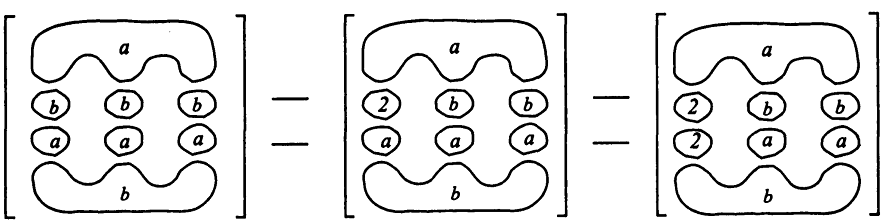

Figure 5: Three representatives of $[f_o(D)]$

4 The Rasmussen invariant of a knot and the sharper slice-Bennequin inequality

In this section, we recall the definition of the Rasmussen invariant of a knot and the sharper slice-Bennequin inequality for the Rasmussen invariant of a knot.

We define a grading p on V by setting $p(1) = 1$ and $p(x) = -1$ and extend it to $V^{\otimes n}$ by $p(v_1 \otimes v_2 \otimes, \cdots, v_n) = p(v_1) + p(v_2) + \cdots + p(v_n)$. Next we define a filtration grading q on $C^i_{Lee}(D)$ by $q(v) = p(v) + i + \omega(D)$, where v is a monomial of $C^i_{Lee}(D)$ and $\omega(D)$ is the writhe of D, and extend it to $C^i_{Lee}(D)$ by $\min\{q(v_j)\}$ where $v = \sum v_j \in C^*_{Lee}(D)$ and v_j is a monomial. Let

$$\mathcal{F}^i C^*_{Lee}(D) = \{v \in C^*_{Lee}(D) \setminus \{0\} | q(v) \geq i\} \cup \{0\}.$$

Then $\{\mathcal{F}^i C^*_{Lee}(D)\}$ is a filtration of $C^*_{Lee}(D)$. Rasmussen showed the following.

Lemma 4.1 ([19]). *Let D and D' be diagrams of a knot. Then $C^*_{Lee}(D)$ and $C^*_{Lee}(D')$ are filtered chain homotopic.*

We also denote by q the filtration grading on $H^*_{Lee}(K)$ which is induced by the filtration grading q on $C^*_{Lee}(D)$. Let

$$q_{\max}(K) = \max\{q(x) | x \in H^*_{Lee}(K), x \neq 0\},$$
$$q_{\min}(K) = \min\{q(x) | x \in H^*_{Lee}(K), x \neq 0\}.$$

The *Rasmussen invariant* of a knot K, $s(K)$, is define to be $\dfrac{q_{\max}(K) - q_{\min}(K)}{2}$. By Lemma 4.1, $s(K)$ does not depend on the choice of diagrams of K.

Lemma 4.2 ([19]). *Let K be a knot and D a diagram of K. Then*
(1) $q_{\min}(K) = q([f_o(D)]) = q([f_{\bar{o}}(D)])$.
(2) $q_{\max}(K) - q_{\min}(K) = 2$.

Note that $s(K)$ is equal to $q\big([f_o(D)]\big) + 1$ by Lemma 4.2. The following theorem is the sharper slice-Bennequin inequality for the Rasmussen invariant, which was first proved by Kawamura [10]. Note that the state cycle in Theorem 3.2 implies the sharper slice-Bennequin inequality for the Rasmussen invariant.

Theorem 4.3 ([1] and [10]). *Let D be a non-negative diagram of a K. Then*

$$w(D) - O(D) + 2O_<(D) + 1 \leq s(K),$$

where $O_<(D)$ is the number of strongly negative circles of D.

5 Kawamura-Lobb's inequality for the Rasmussen invariant

In this section, we recall Kawamura-Lobb's inequality for the Rasmussen invariant, which is stronger than the sharper slice-Bennequin inequality, and that the equality holds for homogeneous knots.

Let $O_+(D)$[1] and $O_-(D)$ be the numbers of connected components of the diagrams which is obtained from D by smoothing all negative and positive crossings of D, respectively. Kawamura [11] and Lobb [17] independently obtained a stronger estimation for the Rasmussen invariant as follows:

[1]In [11] and [17], it was denoted by $l_0(D)$ and #components($T^+(D)$), respectively.

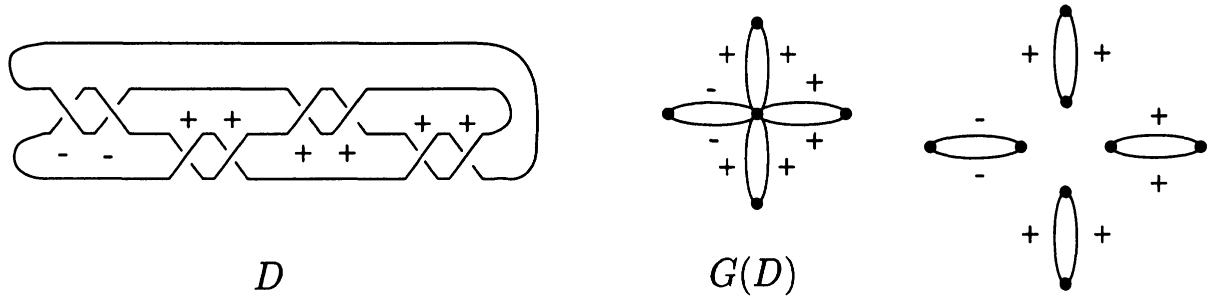

Figure 6: A non-alternating and non-positive diagram D and the graph $G(D)$

Theorem 5.1 ([11] and [17]). *Let D be a diagram of a knot K. Then*

$$w(D) - O(D) + 2O_{+}(D) - 1 \leq s(K),$$

where $\omega(D)$ denotes the writhe of D (i.e. the number of positive crossings of D minus the number of negative crossings of D) and $O(D)$ denotes the number of the Seifert circles of D.

Cromwell [5] introduced the notion of homogeneity for knots to generalize results on alternating knots. The notion of homogeneity is also defined for signed graphs and diagrams: A signed graph is *homogeneous* if each block has the same signs, and a diagram D of a knot is *homogeneous* if its Seifert graph, denoted by $G(D)$, is homogeneous (for more details, see [2]). A knot K is *homogeneous* if K has a homogeneous diagram. In [2], we determined the Rasmussen invariant of a homogeneous knot as follows:

Theorem 5.2 ([2]). *Let D be a homogeneous diagram of a knot K. Then*

$$s(K) = w(D) - O(D) + 2O_{+}(D) - 1.$$

6 A criteria on homogeneous knots

In this section, we consider some homogeneous knots and give a new criteria on homogeneous knots.

Cromwell [5] showed that alternating diagrams and positive diagrams are homogeneous. There are many homogeneous diagrams which are neither alternating nor positive. The following is a such example.

Example 6.1. Let D be the diagram as in Figure 6. Then $G(D)$ is homogeneous (see Figure 6). Therefore D is a homogeneous diagram which is neither alternating nor positive.

The class of homogeneous knots includes alternating knots and positive knots. Another example of a homogeneous knot is the closure of a homogeneous braid, a notion which was introduced by Stallings [21]. Let B_n be the braid group on n strands with generators $\sigma_1, \sigma_2, \cdots, \sigma_{n-1}$. A braid $\beta = \sigma_{i_1}^{\epsilon_1}\sigma_{i_2}^{\epsilon_2}\cdots\sigma_{i_k}^{\epsilon_k}, \epsilon_j = \pm 1$ $(j = 1, \cdots, k)$ is *homogeneous* if

(1) every σ_j occurs at least once,

(2) for each j, the exponents of all occurrences of σ_j are the same.

K	homogeneous?	homogeneous braid ?	K	homogeneous?	homogeneous braid ?
8_{19}	YES	YES	10_{140}	NO	-
8_{20}	NO	-	10_{141}	NO	-
8_{21}	NO	-	10_{142}	YES	NO
9_{42}	NO	-	10_{143}	NO	-
9_{43}	YES	YES	10_{144}	?	NO
9_{44}	NO	-	10_{145}	NO	-
9_{45}	NO	-	10_{146}	NO	-
9_{46}	NO	-	10_{147}	NO	-
9_{47}	YES	YES	10_{148}	NO	-
9_{48}	NO	-	10_{149}	NO	-
9_{49}	YES	NO	10_{150}	NO	-
10_{124}	YES	YES	10_{151}	?	?
10_{125}	NO	-	10_{152}	YES	YES
10_{126}	NO	-	10_{153}	NO	-
10_{127}	NO	-	10_{154}	YES	?
10_{128}	YES	NO	10_{155}	NO	-
10_{129}	NO	-	10_{156}	YES	?
10_{130}	NO	-	10_{157}	NO	-
10_{131}	NO	-	10_{158}	?	?
10_{132}	NO	-	10_{159}	NO	-
10_{133}	NO	-	10_{160}	?	?
10_{134}	YES	NO	10_{161}	YES	?
10_{135}	NO	-	10_{162}	NO	-
10_{136}	NO	-	10_{163}	NO	-
10_{137}	NO	-	10_{164}	NO	-
10_{138}	YES	YES	10_{165}	?	NO
10_{139}	YES	YES			

Table 1:

For example, the braid $\sigma_1\sigma_2^{-1}\sigma_1\sigma_2^{-1}$ is homogeneous, however, the braid $\sigma_1^2\sigma_2\sigma_1\sigma_2^{-1}$ is not homogeneous.

Lemma 6.2 ([5]). *Let β be a braid whose closure is a knot. Then β is homogeneous if and only if the knot diagram of the closure of β is homogeneous.*

A *homogeneous braid knot* is the closure of a homogeneous braid. By the above lemma, a homogeneous braid knot is homogeneous. The knot 9_{43} is a homogeneous braid knot which is not neither alternating nor positive.

Stallings [21] proved that a homogeneous braid knot is fibered. Notice that there exist homogeneous knots which are not homogeneous braid knots since some homogeneous knots are not fibered (for example, 5_2). The knot 9_{49} is a homogeneous knot which are neither homogeneous braid, alternating, nor positive. We give tables of non-alternating homogeneous knots and homogeneous braid knots up to 10 crossings, respectively. The table of non-alternating homogeneous knots up to 10 crossings is due to Cromwell [5].

In [2], we gave some characterizations of a positive knot. One of them is the following:

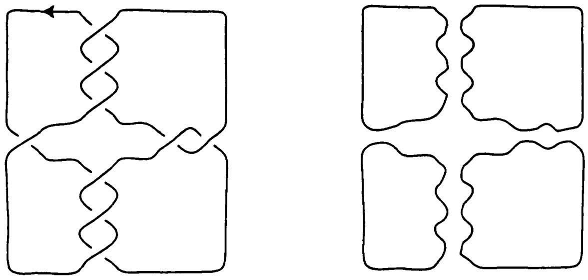

Figure 7: A diagram of 8_{19} and its Seifert circles

Theorem 6.3 (Theorem 1.3 in [2]). *A knot K is positive if and only if K homogeneous and $s(K)/2 = g_*(K) = g(K)$, where $g_*(K)$ is the 4-ball genus of K and $g(K)$ is the genus of K.*

As a corollary, we obtain the following:

Corollary 6.4. *If K is not positive and $s(K) = 2g(K)$, then K is not homogeneous.*

This corollary gives us a new method to show that some knots are not homogeneous. The following is such an example.

Example 6.5. Let K be the knot 10_{145}. Then K is not positive and $s(K) = 2g(K) = 4$ (see [4]). Therefore K is not homogeneous.

7 Non state cycles which represent canonical classes

In section 3, we described state cycles which give the sharper slice-Bennequin inequality for the Rasmussen invariant. In this section, we consider cycles which give Kawamura-Lobb' inequality for the Rasmussen invariant, which is stronger than the sharper slice-Bennequin inequality.

First we give some examples of knot diagrams such that Kawamura-Lobb' inequality gives a stronger estimation than the sharper slice-Bennequin inequality.

Example 7.1. Let D be the diagram of 8_{19} as in Figure 7. Then $\omega(D) = 3, O(D) = 4, O_<(D) = 0$ and $O_+(D) = 2$. Therefore the sharper slice-Bennequin inequality implies that $0 = 3 - 4 + 0 + 1 \leq s(8_{19})$ and Kawamura-Lobb' inequality implies that $2 = 3 - 4 + 2 + 1 \leq s(8_{19})$. Note that $s(8_{19}) = 2$ (see [4]).

Example 7.2. Let D be the alternating diagram as in Figure 8 and K the knot which is represented by D. Then $\omega(D) = -3, O(D) = 6, O_<(D) = 2$ and $O_+(D) = 4$. Therefore the sharper slice-Bennequin inequality implies that $-4 = -3 - 6 + 4 + 1 \leq s(K)$ and Kawamura-Lobb' inequality implies that $-2 = -3 - 6 + 6 + 1 \leq s(K)$. Since K is the connected sum of two figure-eight knots and the trefoil knot, $s(K) = -2$ (see [19]).

Next we give some cycles which give Kawamura-Lobb' inequality for the Rasmussen invariant.

Example 7.3. Let D be the diagram of 8_{19} as in Figure 7. Let $g \in C^{-1}_{Lee}(D)$ be the enhanced state as in Figure 9. Then $f_o(D) - d^{-1}(g)$ is not a state cycle as in Figure 10. Note that the cycle $f_o(D) - d^{-1}(g)$ implies that $2 = 3 - 4 + 2 + 1 \leq s(8_{19})$, which is Kawamura-Lobb' inequality.

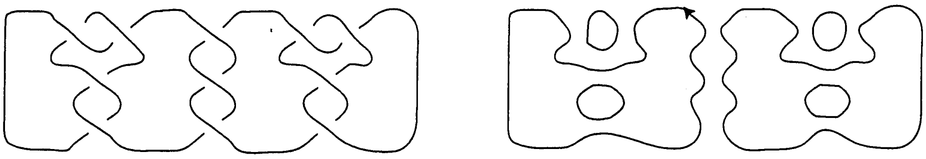

Figure 8: An alternating diagram and its Seifert circles

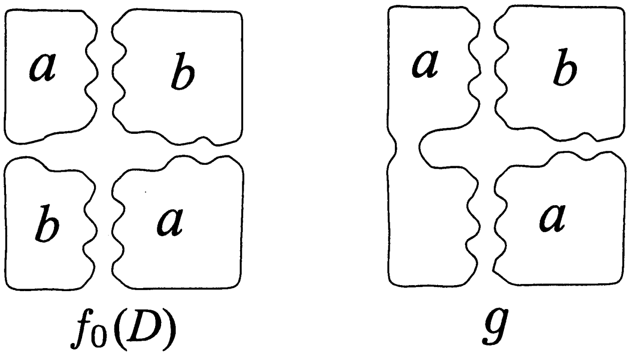

Figure 9: The state cycle $f_o(D)$ and the enhanced state g

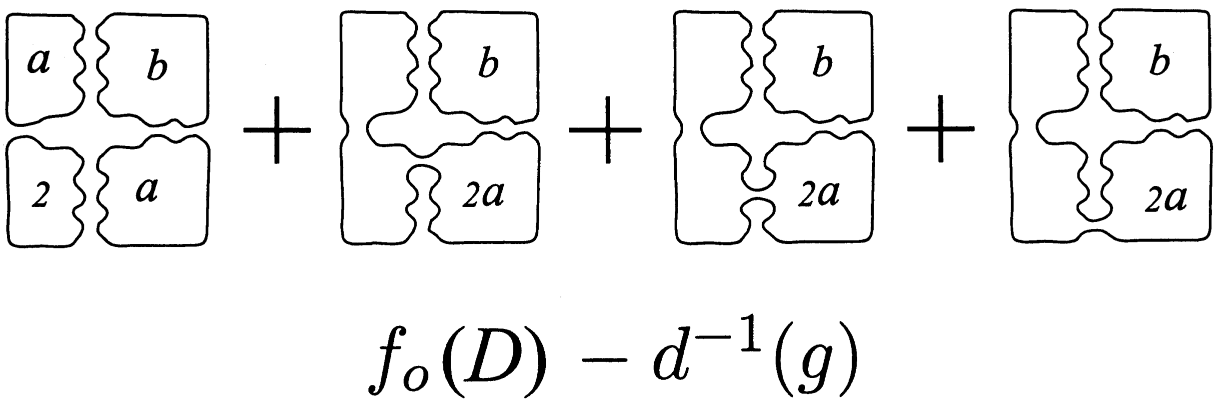

Figure 10: A representative of $f_o(D)$

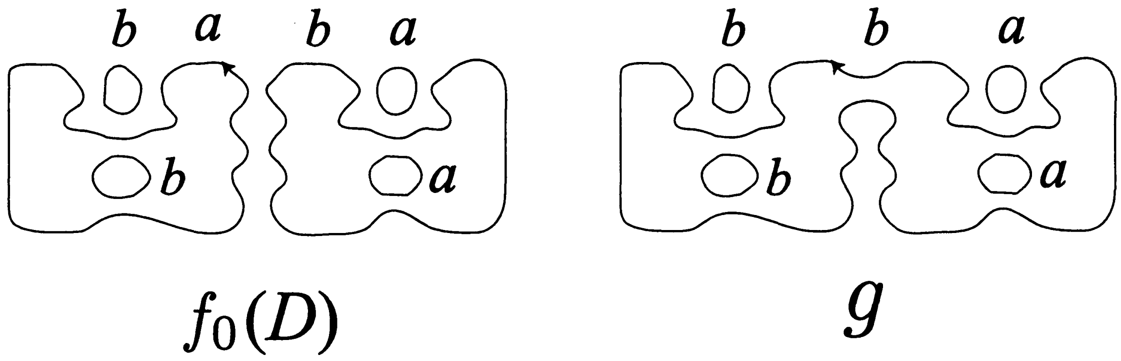

Figure 11: The state cycle $f_o(D)$ and the enhanced state g

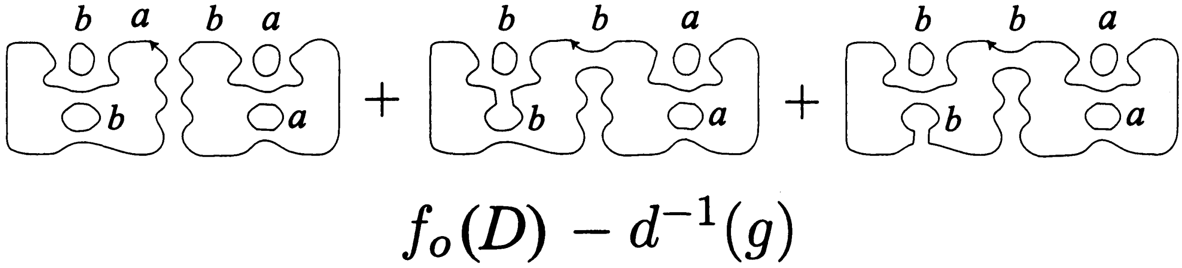

Figure 12: A representative of $f_o(D)$

Example 7.4. Let D be the alternating diagram as in Figure 8 and K the knot which is represented by D. Let $g \in C^{-1}_{Lee}(D)$ be the enhanced state as in Figure 11. Then $f_o(D) - d^{-1}(g)$ is not a state cycle as in Figure 10. Note that the cycle $f_o(D) - d^{-1}(g)$ implies that $-2 = -3 - 6 + 6 + 1 \leq s(K)$, which is Kawamura-Lobb' inequality.

Problem 7.5. *Let D be a knot diagram. Find an explicit presentation of a cycle $f(D)$ such that $[f_o(D)] = [f(D)]$ and $q(f(D)) = w(D) - O(D) + 2O_+(D) - 2$.*

In general, even for an alternating diagram D, we do not know an explicit presentation of a cycle $f(D)$ such that $[f_o(D)] = [f(D)]$ and $q(f(D)) = w(D) - O(D) + 2O_+(D) - 2$.

8 Non homogeneous knots

There are many non homogeneous knots. One of them is the pretzel knot of type $(p, -q, -r)$ for odd integer $|p| \geq 3, |q| \geq 3, |r| \geq 3$ (see [5]). Another example is the untwisted Whitehead double of a knot (see [5]). For diagrams of these knots, it seem to be more hard to describe cycles which determine the Rasmussen invariant.

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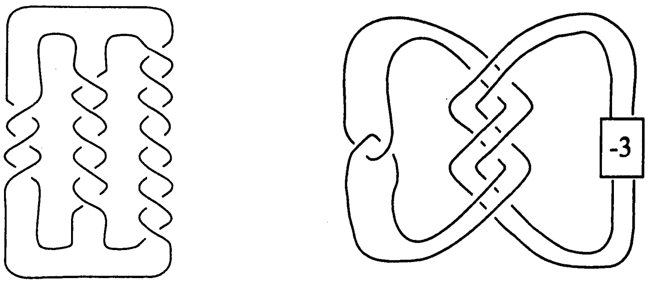

Figure 13: P(3,-5,-7) and the untwisted Whitehead double of the trefoil knot

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