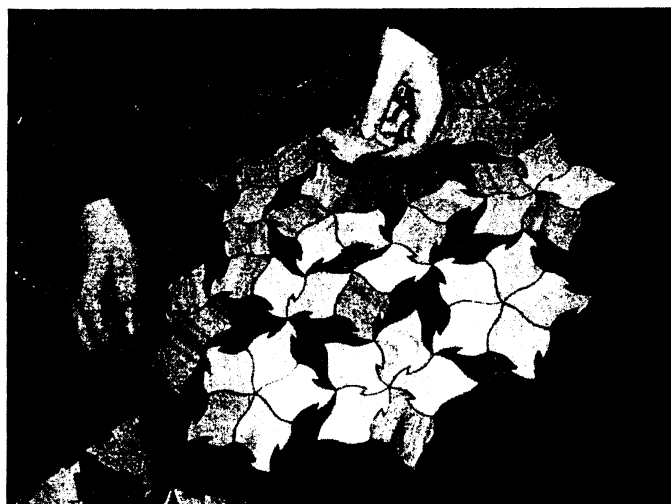


Aperiodic Tiles



Edmund Harriss, University
of Leicester
[www.mathematicians.org.uk/
eoh](http://www.mathematicians.org.uk/eoh)

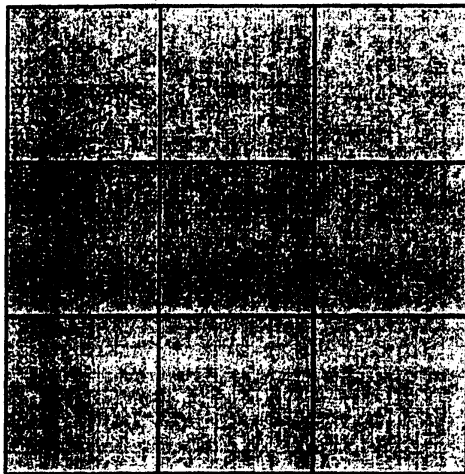
Mathematics and disaster relief: The hexayurt



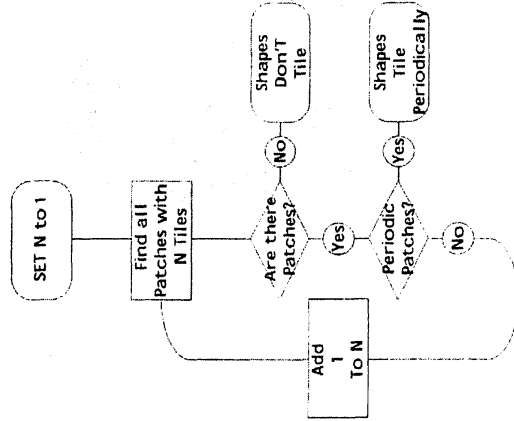
Vinay Gupta

www.hexayurt.com

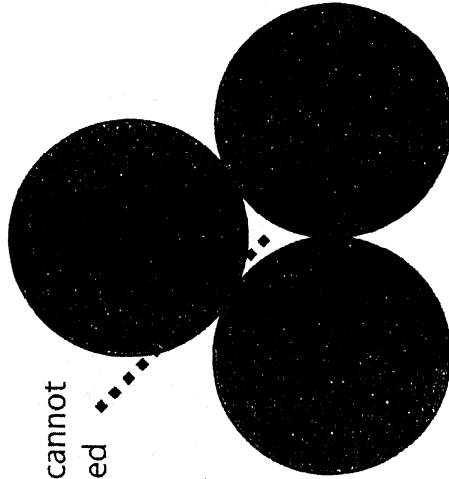
This can tile, periodically. What about another, a circle.



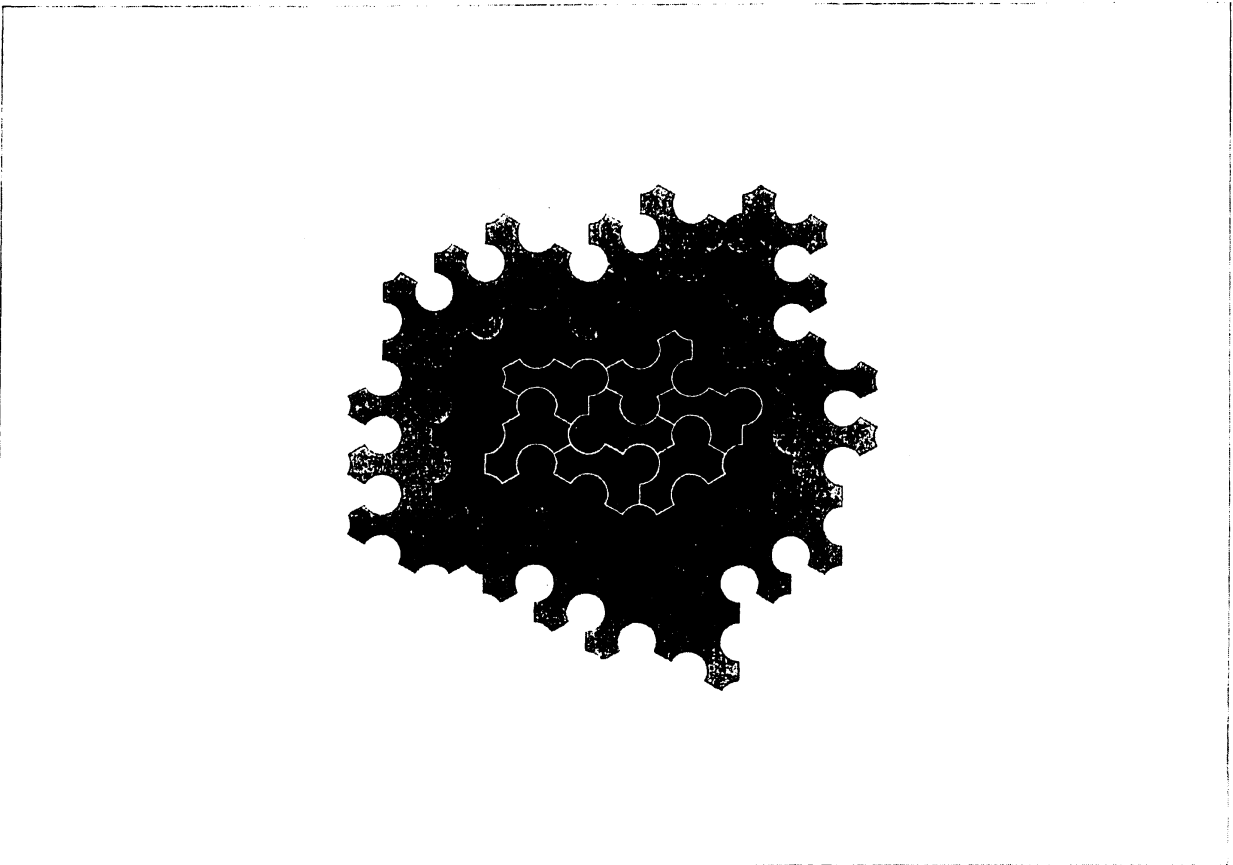
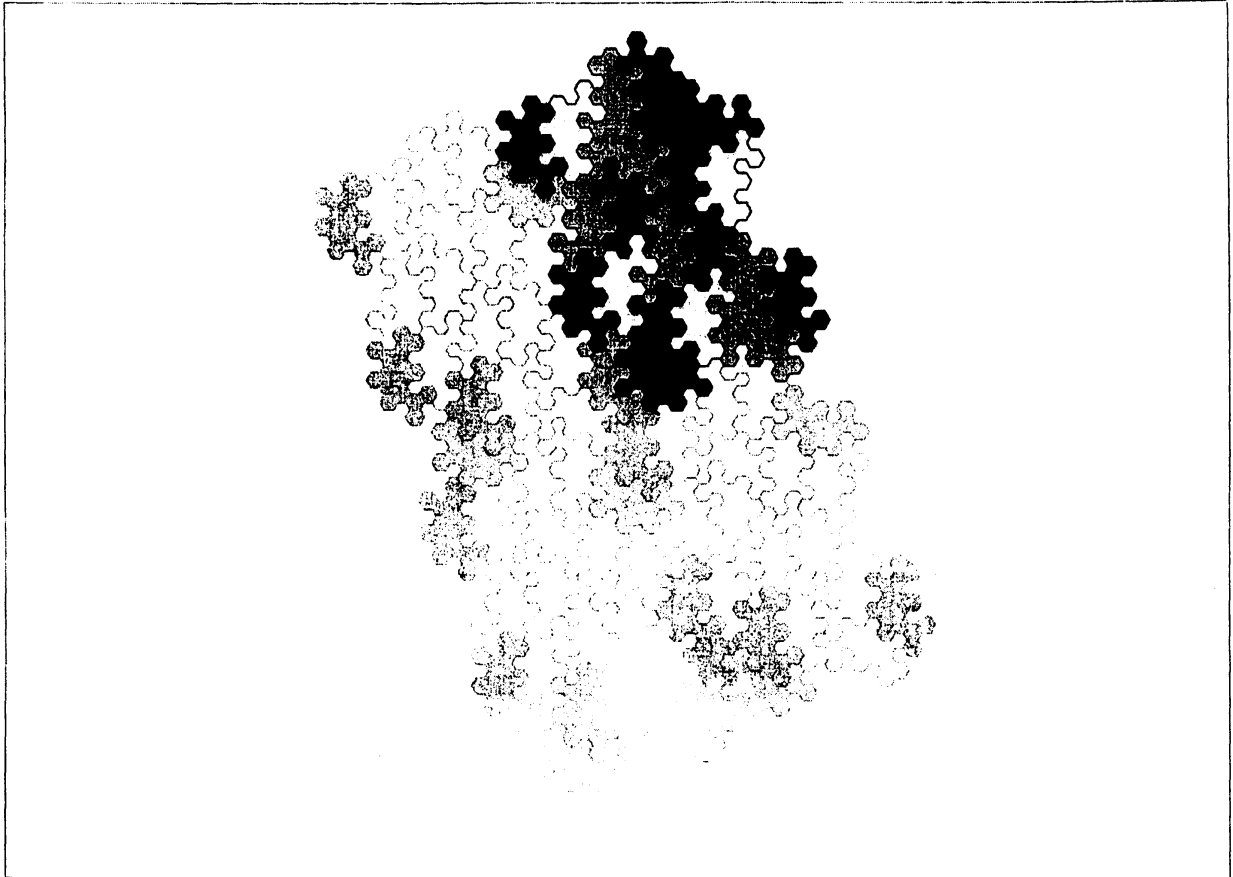
Consider what happens for the tiles we have...



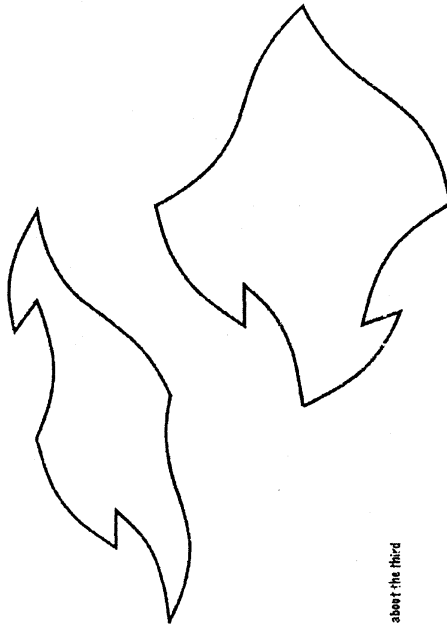
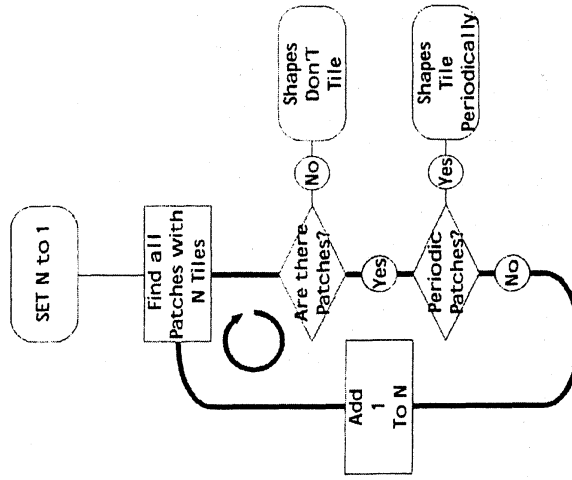
Hole that cannot be filled



This clearly cannot tile, but maybe we have a simple algorithm...



This will run around the loop forever.
 It gets worse. The question is undecidable
 Whatever algorithm you find, there is always
 a set of shapes that will cause it to run
 forever without giving an answer.



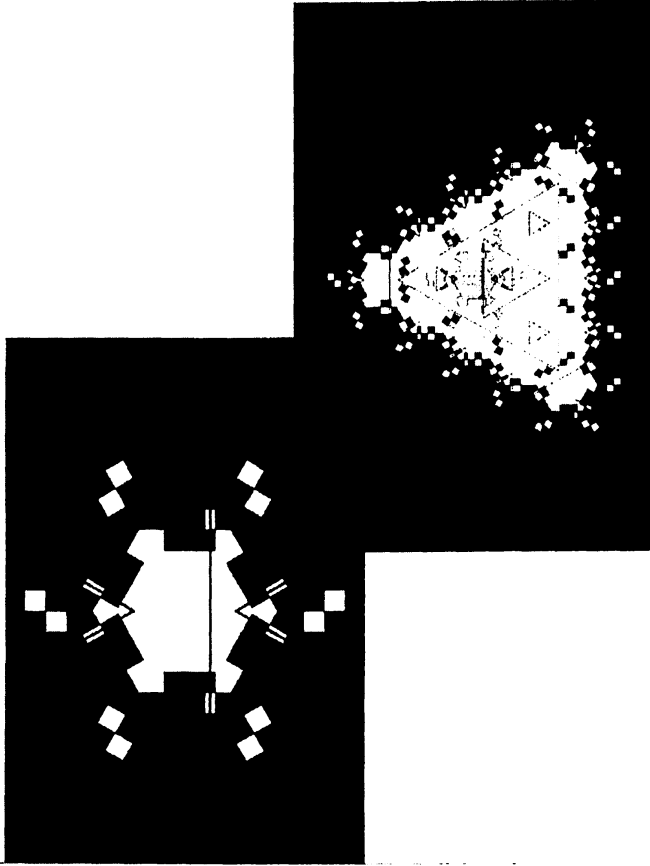
What about the third set...

Berger PhD Thesis
Simpler Proof: Robinson
Current state of the art:
5 Tiles Ollinger
This is a reminder of how deep
undecidability cuts into mathematics.
(of course it is also exciting simple
questions about tilings- will always yield
interesting new ideas.
Like Aperiodicity...

Robert Berger, *The undecidability of the
domino problem*,
Memoirs of the AMS 66, 1966

Raphael Robinson, *Undecidability and
nonperiodicity for tilings of the plane*,
Inventiones Mathematicae 12, 1971, pp.
177-209

Nicolas Ollinger: *Tiling the Plane with a Fixed
Number of Polyominoes*. Proceedings of LATA
2009, Lecture Notes in Computer Science
5457, Springer 2009, pp. 638-647.



Joshua Socolar and Joan Taylor,
An aperiodic hexagonal tile,
preprint: arXiv:1003.4279v1

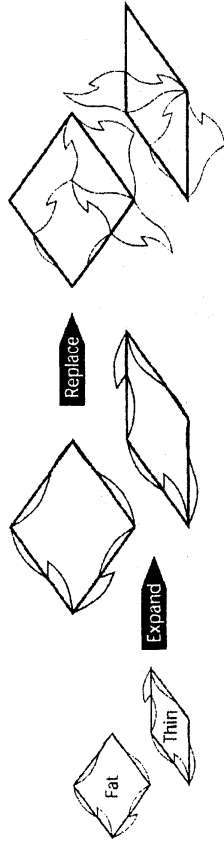
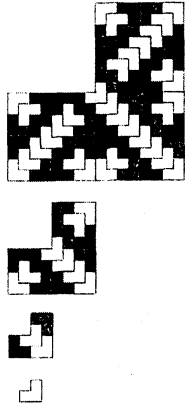
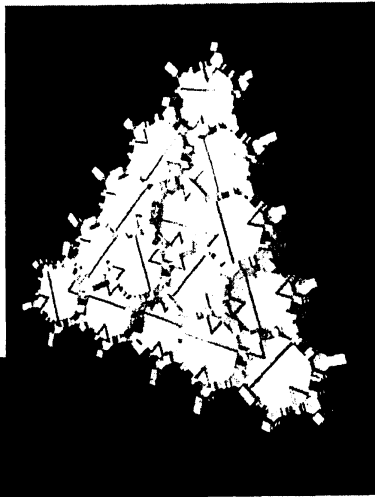
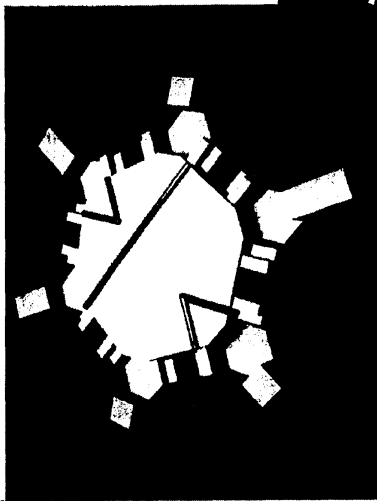
Joan Taylor,
Aperiodicity of a Functional Monotile,
preprint:
www.math.uni-bielefeld.de/stfb70/preprints/sfb10015.pdf

3d version, 1 periodic direction

Mentioned in New Scientist

How can you tell if these shapes tile at all?

The answer is a substitution rule.

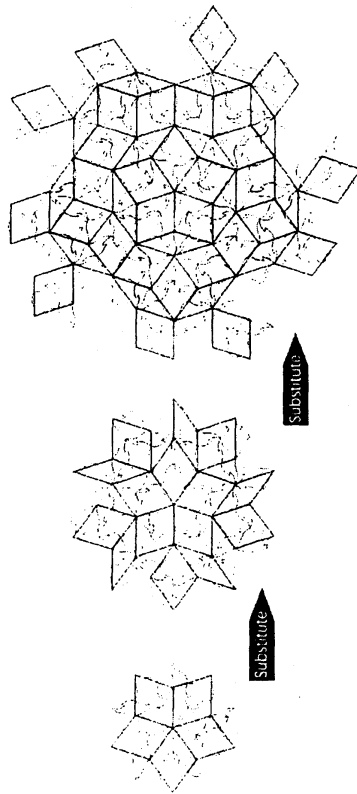
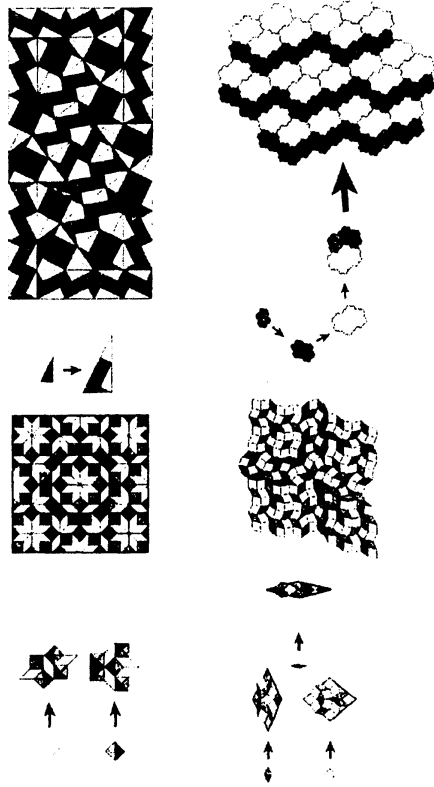


My work has been in creating and attempting to characterize substitution tilings.

The process of taking a substitution tiling

EG Penrose rhombs

and changing the edges to give a set of aperiodic tiles became known as matching rules...



A series of papers leading to a result that shows that we can get an aperiodic set of shapes from any substitution tiling.

Q. How?

This is an important result, but not well understood. So now.

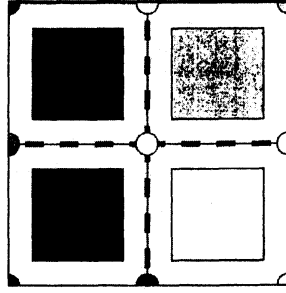
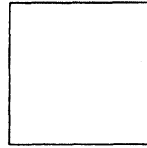
Raphael Robinson, *Undecidability and nonperiodicity for tilings of the plane*, *Inventiones Mathematicae* 12, 1971, pp. 177-209

Sharhar Mozes, *Tilings, substitution systems and dynamical systems generated by them*, *J. D'Analyse Math.* 53, 1989, pp.139-186

Chaim Goodman-Strauss, *Matching rules and substitution tilings*, *Annals of Mathematics* 147 No. 1, 1998, pp. 181-223

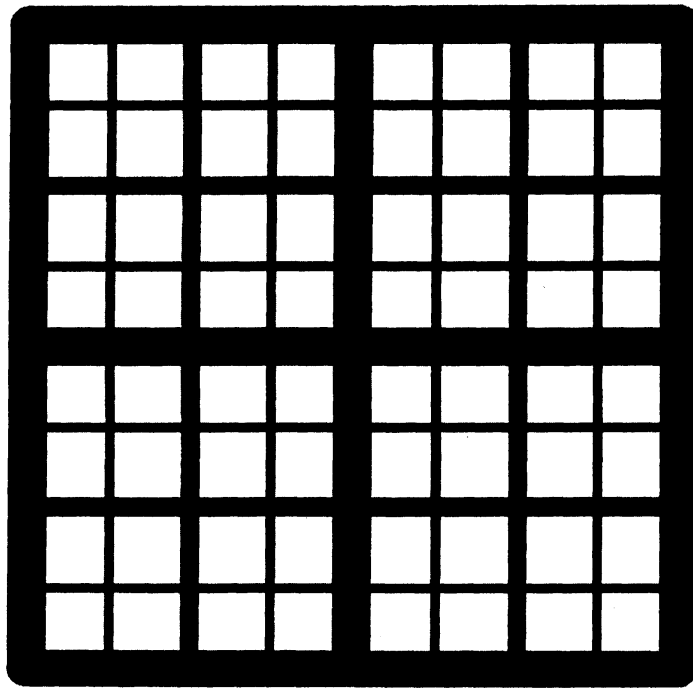
Start with the simplest possible substitution rule.

Label some features:
Edges, Vertices, Tiles



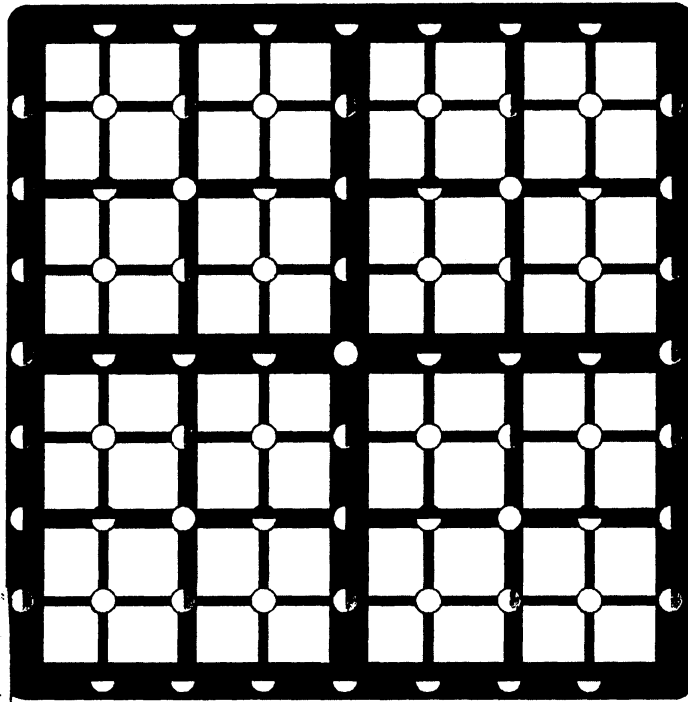
We can look at the hierarchy of the tiling.

Every edge ends up within a tile.

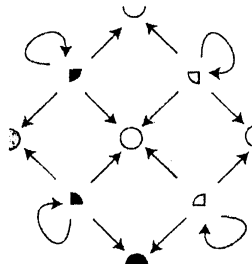
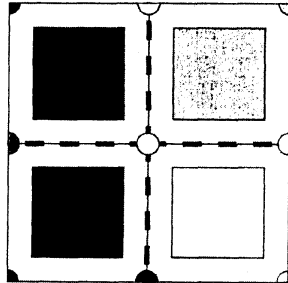


Some vertices will end up at the centre of a tile.

Others at the edge.



We can build a graph to show the possible routes that a vertex can take.



Every Tile knows:

Its tile type

The eventual type of its special vertex

Every Edge knows:

Its eventual type

What supertile it lies in:

The tile type

The eventual type of its special vertex

Every Vertex knows

Its eventual type

What edges join it

What supertile it lies in:

The tile type

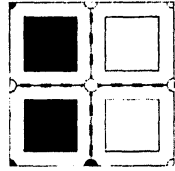
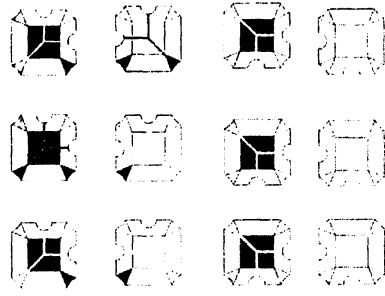
The eventual type of its special vertex

We want this information on the objects. The key is edges, they can grow transporting the information around the tiling

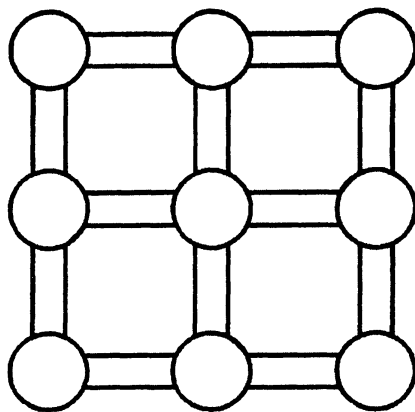
Now

We can start with the tiles. Each knows its type and the type of its special vertex.

The edges of the supertile will also need to know the type of the special vertex, so the information is passed up to the internal edges.

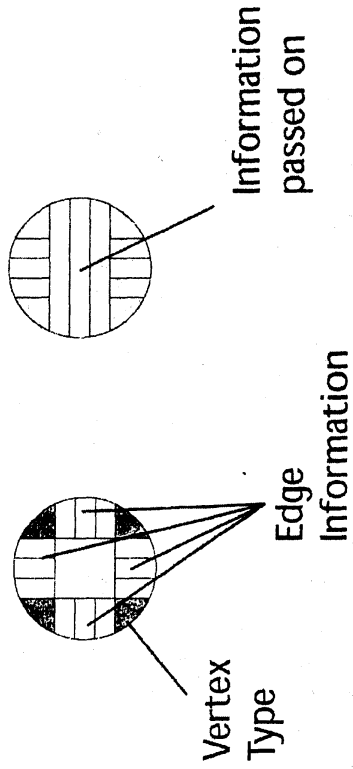
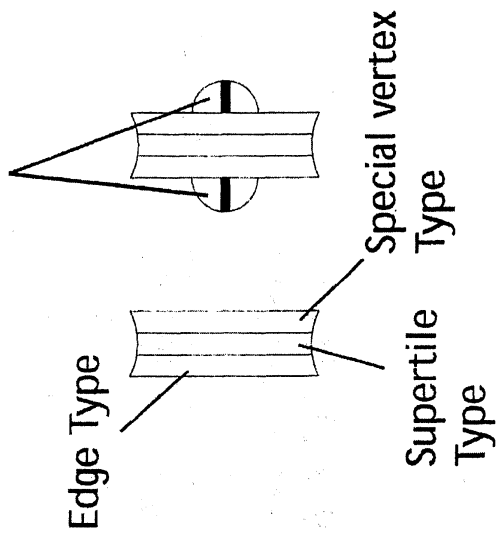


The tiles can be cut up to give the edges and vertices shape



Now look at edges. Each edge has three channels for the information it carries. There are also edges that plug into tiles.

Tile special vertex type



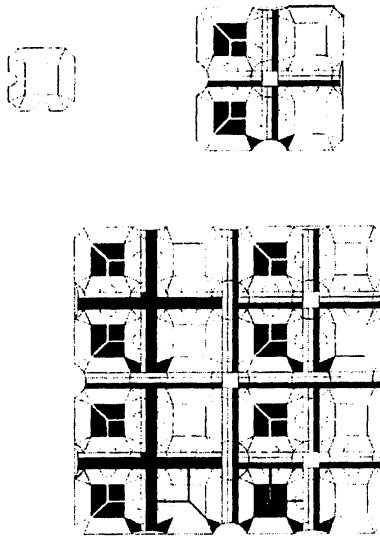
Lets build up a patch of tiling-

Note how the special vertex type is communicated up the hierarchy

Thus each element can have finite information so there are a finite number of tiles.

but...

there are quite a few choices so-



We end up with a lot of tiles!

The nice thing is that the information that travels round is explicit.

All the interactions are local, yet some information is forced to travel arbitrarily far. Something I at least find amazing.

