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Homeomorphism groups of non-compact manifolds with the Whitney topology

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1. Introduction

Homeomorphism groups of manifolds admits several natural topologies; the compact-open topology, the Whitney topology, the uniform topology and the direct limit topology. For compact manifolds, all of these topologies coincide and its properties are familiar to many researchers in various fields. However, when the manifold is non-compact, each of these topologies has its own nature crucially different from others and in some literatures these differences seem to be not recognized correctly.

In this article, we explain some basic properties of the Whitney topology of homeomorphism groups of non-compact manifolds based upon the recent joint works [1, 2, 3, 4, 16], and then compare them with those of the compact-open topology [14, 15, 17].

The main observation in this article can be summarized as follows: the Whitney topology corresponds to the countable box product and small box product of l_2, while the compact-open topology corresponds with the usual countable Tychonoff product and weak product of l_2. In Sections 3 and 4 these assertions are exhibited explicitly for the 1 and 2-dimensional cases. (In the dimension ≥ 3 our way is obstructed by the homeomorphism group conjecture for compact manifolds.) The preliminary section 1 is devoted to the basics on box products

2. Box products and small box products

The Whitney topology is closely related to the box products and small box products. First we recall some basic facts on the box products. The index set of non-negative integers is denoted by the symbol ω.

Definition 2.1. (1) The box product $\square_{n\in\omega} X_n$ of a sequence of topological spaces $(X_n)_{n\in\omega}$ is the product $\prod_{n\in\omega} X_n$ with the box topology. This topology is generated by the basic open subsets of the form $\prod_{n\in\omega} U_n$, where U_n is an open subset of X_n for each $n \in \omega$.

(2) The small box product $\boxdot_{n\in\omega}X_n$ of a sequence of pointed topological spaces $(X_n, *_n)_{n\in\omega}$ is the subspace of $\square_{n\in\omega}X_n$ consisting of the points of the form

$$(x_0,\ x_1,\ \ldots,\ x_k,\ *_{k+1},\ *_{k+2},\ \ldots).$$

The small box product $\boxdot_{n\in\omega}X_n$ can be written as the increasing union of the finite products (under the obvious identification):

$$\boxdot_{n\in\omega}X_n = \bigcup_{n\in\omega}\left(\prod_{i\leq n}X_i\right).$$

This implicitly shows that the small box products are closely related to the direct limits in some sense. To simplify the notations, we use the symbols

$$(\square,\boxdot)_n X_n := (\square_n X_n, \boxdot_n X_n) \quad \text{and} \quad (\square,\boxdot)^\omega X := (\square,\boxdot)_{n\in\omega}X$$

to denote the pairs of box and small box products.

Example 2.1. The basic example is the pair $(\square,\boxdot)^\omega l_2$ of countable box and small box products of l_2 (with the origin 0 as the distinguished point). The box topology is so fine that $\square^\omega l_2$ is neither locally connected nor normal. On the other hand, from the topological classification of LF spaces (= the direct limits of Fréchet spaces) due to P. Mankiewicz [11], it follows that $\boxdot^\omega l_2 \approx l_2 \times \mathbb{R}^\infty$, where $\mathbb{R}^\infty$ denotes the direct limit of the tower of Euclidean spaces

$$\mathbb{R}^1 \subset \mathbb{R}^2 \subset \mathbb{R}^3 \subset \cdots.$$

The small box product is not metrizable, even if each factor is metrizable. For the paracompactness, we have the following result [2, Proposition 2.2].

Proposition 2.1. The small box product $\boxdot_{n\in\omega}X_n$ is paracompact if the finite product $\prod_{i\leq n}X_i$ is paracompact for each $n\in\omega$.

As in the case of Tychonoff products, any sequence of continuous maps $f^n : X_n \to Y_n$ $(n\in\omega)$ induces the continuous map

$$\square_n f^n : \square_n X_n \to \square_n Y_n : \quad (x_n)_n \mapsto (f^n(x_n))_n.$$

Obviously, for any sequence of pointed continuous maps $f^n : (X_n, *_n) \to (Y_n, *_n)$ $(n\in\omega)$, the map $\square_n f^n$ restricts to the map $\boxdot_n f^n : \boxdot_n X_n \to \boxdot_n Y_n$ between the small box products. Here we need some care for homotopies. For any sequence of pointed homotopies $h^n_t : (X_n, *_n) \to (Y_n, *_n)$ $(n\in\omega)$, the small box product $\boxdot_n h^n_t : \boxdot_n X_n \to \boxdot_n Y_n$ determines a pointed homotopy, while the box product $\square_n h^n_t$ itself is not continuous in t. This remark is useful to deduce some (local) homotopical properties of small box products from the corresponding properties of the factors.

Next we consider the (small) box products of topological groups ([2, Section 2]). For any topological group G we choose the unit element e as the base point of G.

If $(G_n)_{n\in\omega}$ is a sequence of topological groups, then the box product $\square_n G_n$ forms a topological group under the coordinatewise multiplication and the small box product $\boxdot_n G_n$ becomes a subgroup of $\square_n G_n$.

Suppose G is a topological group with the unit element e and $(G_n)_{n\in\omega}$ is a sequence of subgroups such that

$$G_n \subset G_{n+1}\ (n \in \omega), \quad G = \bigcup\nolimits_n G_n.$$

In this case we can define the multiplication map

$$p : \boxdot_n G_n \to G \quad \text{by} \quad p(x_0, x_1, \ldots, x_k, e, e, \ldots) = x_0 x_1 \cdots x_k.$$

Proposition 2.2. The multiplication map $p : \boxdot_n G_n \to G$ has the following properties:

(1) The map p is a continuous surjection.

(2) If the map p is an open map (at the unit element $(e)_n$ of $\boxdot_n G_n$), then G is the direct limit of the sequence $G_0 \subset G_1 \subset G_2 \subset \cdots$ in the category of topological groups. (This is denoted by the symbol $G = \text{g-}\varinjlim_n G_n$.)

(3) If the map p has a local section at e, then the following holds :

  (i) If each G_n is locally contractible, then so is G.

  (ii) A subgroup H of G is homotopy dense in G if (a) $H \cap G_n$ is homotopy dense in G_n for each $n \in \omega$ and (b) G is paracompact.

Here, a subspace A of a topological space X is said to be homotopy dense in X if there exists a homotopy $\varphi_t : X \to X$ $(0 \le t \le 1)$ such that $\varphi_0 = \mathrm{id}_X$ and $\varphi_t(X) \subset A$ $(0 < t \le 1)$.

3. Homeomorphism groups of non-compact manifolds with the Whitney topology

In this section we explain topological properties of homeomorphism groups of non-compact manifolds with the Whitney topology.

3.1. Homeomorphism groups with the Whitney topology.

First we recall the general properties of homeomorphism groups with the Whitney topology. Suppose M is a connected n-manifold possibly with boundary. By $\mathcal{H}(M)$ we denote the group of homeomorphisms of M endowed with the Whitney topology. In this topology each $h \in \mathcal{H}(M)$ has the fundamental neighborhood system :

$$\mathcal{U}(h) = \{g \in \mathcal{H}(M) : (h, g) \prec \mathcal{U}\} \quad (\mathcal{U} \in \mathrm{cov}(M)),$$

where $\mathrm{cov}(M)$ is the collection of all open coverings of M and $(h,g) \prec \mathcal{U}$ means that each $x \in M$ admits $U \in \mathcal{U}$ with $h(x), g(x) \in U$. It is seen that $\mathcal{H}(M)$ is a topological group.

Let $\mathcal{H}(M)_0$ denote the connected component of id_M in $\mathcal{H}(M)$ and $\mathcal{H}_c(M)$ denote the subgroup of $\mathcal{H}(M)$ consisting of homeomorphisms with compact support. One can see that any compact subset $\mathcal{K}$ of $\mathcal{H}_c(M)$ has a common compact support (i.e., there exists a compact subset K of M with $\mathrm{supp}\, h \subset K$ for any $h \in \mathcal{K}$).

For a subset A of M we have the subgroup $\mathcal{H}(M,A) = \{h \in \mathcal{H}(M) : h|_A = \mathrm{id}_A\}$. The notations $\mathcal{H}(M,A)_0$ and $\mathcal{H}_c(M,A)$ are defined similarly.

When M is a PL manifold, we can consider the subgroup $\mathcal{H}^{\mathrm{PL}}(M)$ of $\mathcal{H}(M)$ consisting of PL-homeomorphisms of M.

3.2. Homeomorphism groups of compact manifolds.

When M is a compact connected n-manifold possibly with boundary, the Whitney topology of $\mathcal{H}(M)$ coincides with the compact-open topology and $\mathcal{H}(M)$ is separable, completely metrizable and locally contractible ([6], [7]).

Conjecture 3.1. $\mathcal{H}(M)$ is an l_2-manifold.

This conjecture is equivalent to the assertion that $\mathcal{H}(M)$ is an ANR ([13]), and it is known that it holds for $n = 1, 2$ ([10]) and is still unsolved for $n \geq 3$.

When M is a compact PL n-manifold, the group $\mathcal{H}^{\mathrm{PL}}(M)$ is an l_2^f-manifold and the inclusion $\mathcal{H}^{\mathrm{PL}}(M)_0 \subset \mathcal{H}(M)_0$ is a weak homotopy equivalencefor any dimension n. It is also known that $\mathcal{H}^{\mathrm{PL}}(M)$ is homotopy dense in $\mathcal{H}(M)$ for $n = 1, 2$ (and for $n \geq 3$ if the conjecture is solved affermatively). (cf. [8])

3.3. Homeomorphism groups of non-compact manifolds.

Suppose M is a non-compact connected n-manifold possibly with boundary.

Proposition 3.1. The group $\mathcal{H}_c(M)$ has the following properties [2] :

(1) $\mathcal{H}_c(M)$ is paracompact and locally contractible.

(2) $\mathcal{H}(M)_0$ is an open normal subgroup of $\mathcal{H}_c(M)$ and

$$\mathcal{H}(M)_0 = \{h \in \mathcal{H}(M) : \ h \text{ is isotopic to } \mathrm{id}_M \text{ by an isotopy with compact support}\}$$

(3) The mapping class group $\mathcal{M}_c(M) = \mathcal{H}_c(M)/\mathcal{H}(M)_0$ has the discrete quotient topology and $\mathcal{H}_c(M) \approx \mathcal{H}(M)_0 \times \mathcal{M}_c(M)$ as topological spaces.

(4) Suppose $(M_i)_{i\in\mathbb{N}}$ is a sequence of compact subsets of M such that

$$M_i \subset \mathrm{Int}_M M_{i+1} \quad (i \geq 1) \quad \text{and} \quad M = \cup_i M_i.$$

This induces the increasing sequence of subgroups $G_i = \mathcal{H}(M, M \setminus M_i)$ $(i \geq 1)$ of $\mathcal{H}_c(M)$ and determines the multiplication map $p : \boxdot_i G_i \to \mathcal{H}_c(M)$. Then, the map p has a local section and hence $\mathcal{H}_c(M) = \text{g-}\varinjlim_i \mathcal{H}(M, M \setminus M_i)$.

In comparison with the last statement (4) the next remark is important.

Remark 3.1. The group $\mathcal{H}_c(M)$ with the direct limit topology is not a topological group.

Next we consider the local/global topological type of the pair $(\mathcal{H}(M), \mathcal{H}_c(M))$. The 1-dimensional case was treated in [1].

Theorem 3.1. $(\mathcal{H}(\mathbb{R}), \mathcal{H}_c(\mathbb{R})) \approx (\square, \boxdot)^\omega l_2$.

Therefore, it is natural to take the pair $(\square, \boxdot)^\omega l_2$ as a local model of $(\mathcal{H}(M), \mathcal{H}_c(M))$ for any non-compact n-manifold M. In this article, we use the following definition of local homeomorphisms of pairs:

(1) $(X, A) \approx_\ell (Y, B)$ at $a \in A$ $\iff$ There exists an open neighborhood U of a in X and an open subset V of Y such that $(U, U \cap A) \approx (V, V \cap B)$.

(2) $(X, A) \approx_\ell (Y, B)$ $\iff$ For each $a \in A$ we have $(X, A) \approx_\ell (Y, B)$ at $a \in A$.

Conjecture 3.2. $(\mathcal{H}(M), \mathcal{H}_c(M)) \approx_\ell (\square, \boxdot)^\omega l_2$. In particular, $\mathcal{H}_c(M)$ is a paracompact $(l_2 \times \mathbb{R}^\infty)$-manifold.

Theorem 3.1. The conjecture holds for $n = 2$ ([2]).

For $n \geq 3$, the conjecture is still open. For the global topological type of $\mathcal{H}_c(M)$ we have the following conclusion [3]:

Theorem 3.2. For $n = 2$

(1) $\mathcal{H}(M)_0 \approx l_2 \times \mathbb{R}^\infty$;

(2) $\mathcal{H}_c(M) \approx \mathcal{H}(M)_0 \times \mathcal{M}_c(M), \qquad \mathcal{M}_c(M) \approx \begin{cases} \mathbb{N} & (M : \text{generic}) \\ 1 \text{ point} & (M : \text{exceptional}). \end{cases}$

Here, we say that M is exceptional if $M \approx X - K$, where

(i) X is an annulus, a disk or a Möbius band, and

(ii) K is a nonempty compact subset of one boundary circle of X.

As for PL-homeomorphism groups we have

Proposition 3.3. If M is a PL n-manifold $(n = 1, 2)$, then $\mathcal{H}_c^{\mathrm{PL}}(M)$ is homotopy dense in $\mathcal{H}_c(M)$.

3.4. **Sketch of proofs of the main theorems.**

In this subsection we give short comments on the proofs of the main theorems:

Theorem 3.1. $(\mathcal{H}(M), \mathcal{H}_c(M)) \approx_\ell (\Box, \boxdot)^\omega l_2$

Theorem 3.2. $\mathcal{H}(M)_0 \approx \boxdot^\omega l_2 \approx l_2 \times \mathbb{R}^\infty$

Suppose M is a non-compact connected 2-manifold possibly with boundary. Choose any sequence $(M_i)_{i\in\mathbb{N}}$ of compact 2-submanifolds of M such that $M_i \subset \mathrm{Int}_M M_{i+1}$ $(i \geq 1)$ and $M = \cup_i M_i$.

Sketch of Proof of Theorem 3.1.

Let $L_i := M_i - \mathrm{Int}_M M_{i-1}$ $(i \geq 1)$. Consider the space of embeddings

$$\mathcal{E}(L_i, M)^* = \{f|_{L_i} : f \in \mathcal{H}(M)\}$$

with the compact-open topology. Then we have the local homeomorphisms of pairs

$$(\mathcal{H}(M), \mathcal{H}_c(M)) \approx_\ell (\Box, \boxdot)_i \, \mathcal{E}(L_{2i}, M)^* \times (\Box, \boxdot)_i \, \mathcal{H}(M, M \setminus L_{2i-1}) \approx_\ell (\Box, \boxdot)^\omega l_2. \quad \Box$$

Sketch of Proof of Theorem 3.2.

[1] Argument in [BMSY, arXiv:0802.0337v1] (2007 - 2008).

It is based on the next key lemma:

Lemma 3.1. Suppose G is a topological group and $(G_i)_i$ is a sequence of closed subgroups of G such that $G_i \subset G_{i+1}$ $(i \geq 1)$ and $G = \cup_i G_i$. We assume that the following conditions are satisfied:

($*_1$) the multiplication map $q : \boxdot_i G_i \to G$, $q(x_1, \ldots, x_m, e, e, \cdots) = x_m \cdots x_1$ is an open map.

($*_2$) for each $i \geq 1$, the projection $\pi_i : G_i \to G_i/G_{i-1}$ has a section $s_i : G_i/G_{i-1} \to G_i$ (i.e., $\pi_i s_i = \mathrm{id}$).

Then, the sections $(s_i)_i$ induces the map $s = \boxdot_i s_i : \boxdot_i(G_i/G_{i-1}) \to \boxdot_i G_i$ and the composition $qs : \boxdot_i(G_i/G_{i-1}) \to G$ is a homeomorphism.

$$\begin{tikzcd} & \boxdot_i G_i \arrow[dr, "p"] & \\ \boxdot_i(G_i/G_{i-1}) \arrow[ur, "s = \boxdot_i s_i"] \arrow[rr, "ps", "\approx"'] & & G \end{tikzcd}$$

We can apply this lemma to the group $\mathcal{H}(M)_0$ and the subgroups $G_i = \mathcal{H}(M, M \setminus M_i)_0$ $(i \geq 1)$. It follows that

$$\mathcal{H}(M)_0 = \bigcup_i G_i \approx \boxdot_i(G_i/G_{i-1}) \approx \boxdot_i \ell_2.$$

[2] Argument in [3] (2009 - 2010).

T. Banakh - D. Repovš [5] obtained the following topological characterization of $l_2 \times \mathbb{R}^\infty$:

Theorem 3.2. Suppose X is a non-metrizable topological space. Then, $X \approx l_2 \times \mathbb{R}^\infty$ iff $X \approx \text{u-}\varinjlim X_n$ (the uniform direct limit) of a tower $(X_n)_{n \in \mathbb{N}}$ of metrizable uniform spaces such that each X_n satisfied the following conditions:

(i) X_n is uniformly locally equiconnected,

(ii) X_n is a uniform neighborhood retract in X_{n+1},

(iii) X_n has a uniform frill in X_{n+1}

(iv) X_n is contractible in X_{n+1} (v) X_n is an l_2-manifold

This has the following conclusion for topological groups [4].

Corollary 3.1. Suppose G is a non-metrizable topological group and $(G_i)_i$ is a sequence of closed subgroups of G such that $G_i \subset G_{i+1}$ $(i \geq 1)$ and $G = \cup_i G_i$. We assume that the following conditions are satisfied:

$(*_1)$ the multiplication map $p : \Box_i G_i \to G$ is an open map.

$(*_2)$ for each $i \geq 1$ (i) G_i^L is a uniform neighborhood retract of G_{i+1}^L and (ii) $G_i \approx l_2$.

Then $G \approx l_2 \times \mathbb{R}^\infty$.

Theorem 3.2 follows from this corollary. □

4. Homeomorphism groups of non-compact 2-manifolds with the compact-open topology

In this section, in comparison with the Whitney topology, we list some properties of the compact-open topology on homeomorphism groups of non-compact 2-manifolds [14, 15, 16].

Suppose M is a non-compact connected 2-manifold possibly with boundary. Below we say that M is exceptional if M is the plane, the open Möbius band, the open annulus or the half open annulus.

By $\mathcal{H}(M)^{co}$ we denote the group $\mathcal{H}(M)$ endowed with the compact-open topology. Let $\mathcal{H}(M)_0^{co}$ be the connected component of id_M in $\mathcal{H}(M)^{co}$ and set

$$\mathcal{H}_c(M)_1^* := \{h \in \mathcal{H}(M) : h \text{ is isotopic to } \mathrm{id}_M \text{ by an isotopy with compact support}\}.$$

Note that $\mathcal{H}_c(M)_1^* = \mathcal{H}(M)_0$ (with the Whitney Topology) as sets.

Theorem 4.1. [14, 15] (1) $\mathcal{H}(M)_0^{co} \approx \begin{cases} l_2 & (M : \text{generic}) \\ l_2 \times \mathbb{S}^1 & (M : \text{exceptional}) \end{cases}$

(2) $\mathcal{H}_c(M)_1^*$ is homotopy dense in $\mathcal{H}(M)_0^{co}$.

Corollary 4.1. [17] $(\mathcal{H}(M)_0^{co}, \mathcal{H}_c(M)_1^*) \approx \begin{cases} \left(\prod^\omega l_2, \sum^\omega l_2\right) & (M : \text{generic}) \\ \left(\prod^\omega l_2, \sum^\omega l_2\right) \times \mathbb{S}^1 & (M : \text{exceptional}) \end{cases}$

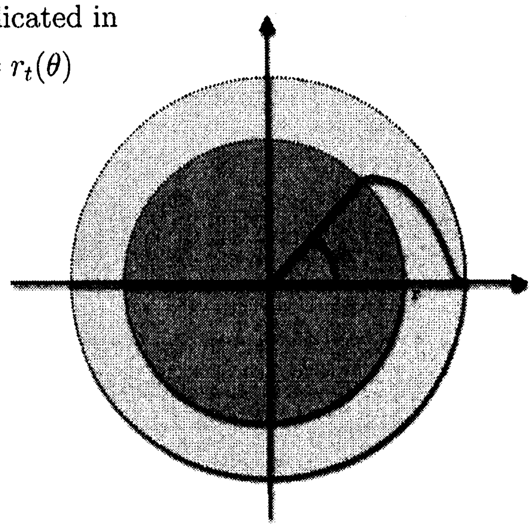
Example 4.1. $M = \mathbb{R}^2$ [15] :

(1) The homotopy equivalence $\mathcal{H}(\mathbb{R}^2)_0^{co} \simeq \mathbb{S}^1$ is induced by the loop of θ rotations $\varphi(\theta)$ $(\theta \in [0, 2\pi])$.

(2) $\mathcal{H}_c(\mathbb{R}^2)^{co}$ is homotopy dense in $\mathcal{H}(\mathbb{R}^2)_0^{co}$. Thus we have a homotopy of loops $\varphi_t(\theta)$ $(t \in [0,1])$ such that $\varphi_1(\theta) = \varphi(\theta)$ and $\varphi_t(\theta) \in \mathcal{D}_c(\mathbb{R}^2)$ $(t < 1)$. In fact, we can take $\varphi_t(\theta)$ as the loop of truncated θ rotations indicated in the picture, where the level of truncation $r = r_t(\theta)$ need to satisfy the next conditions :

(i) for any $t < 1$, $r \to \infty$ as $\theta \to 2\pi$.

(ii) $r \to \infty$ uniformly as $t \to 1$.



Example 4.2. $M =$ the open annulus:

Suppose h is the Dehn twist on M along the center circle of M. Then we have

(1) $h \in \mathcal{H}(M)_0 \setminus \mathcal{H}_c(M)_1^*$;

(2) There exists an isotopy $h_t \in \mathcal{H}(M)_0$ $(0 \le t \le 1)$ such that $h_0 = h$, $h_1 = \mathrm{id}_M$ and $h_t \in \mathcal{H}_c(M)_1^*$ $(0 < t \le 1)$.

The isotopy h_t is obtained by introducing the reverse Dehn twist from ∞.

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