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Stationary problem to a One-dimensional Bipolar Hydrodynamic Model of Semiconductors

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Abstract
We consider a one-dimensional bipolar hydrodynamic model of semiconductors. In this paper, We study the stationary problem and devote to introducing of the modeling and a mathematical result.

1 Introduction

We use a personal computer, a flash memory and a cellular phone every day. In addition, a solar cell receives much attention in terms of energy-saving. Semiconductors play an important role in these machines. For example, LSI (Large Scale Integration) is used in the

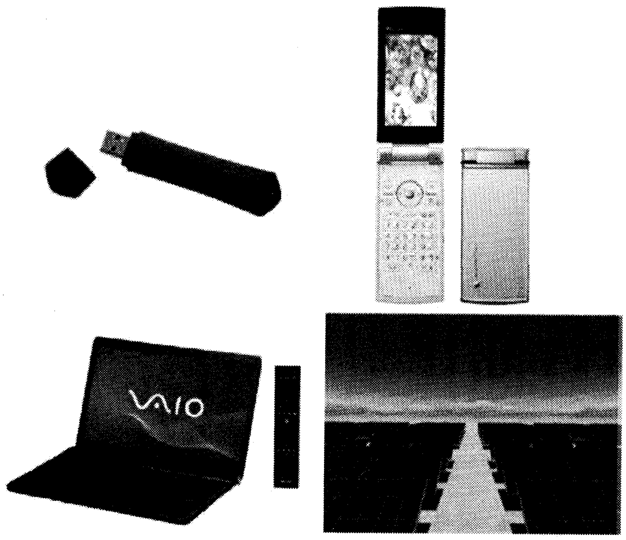

Figure 1: PC, flash memory, cellular phone and solar cell

personal computer and the flash memory. Moreover, LSI consists of diodes and transistors. Diodes and transistors consist of n-type semiconductors and p-type semiconductors.

N-type semiconductors are composed of group 14 elements (Si or Ge) and group 15 elements (P or As or Sb). Si has four electrons in the outermost shell. On the other hand, P has five electrons in the outermost shell. Therefore, if P units four silicones by covalent bound, one electron remains. Giving high temperature, this electron can move like a free electron.

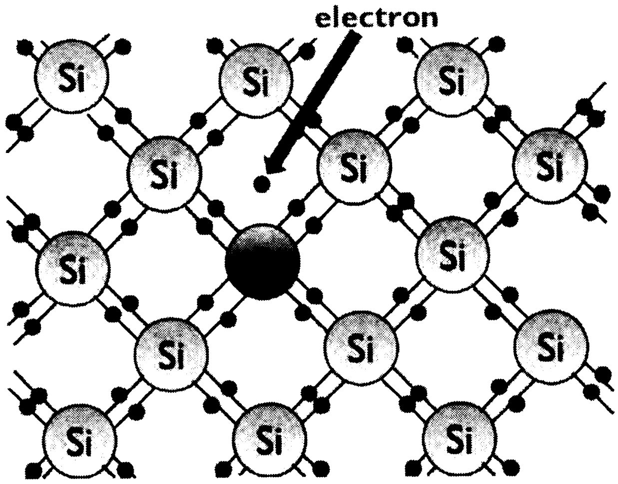

Figure 2: N-type semiconductor

P-type semiconductors are composed of group 14 elements (Si or Ge) and group 13 elements (B or Al or Ga or In). B has three electrons in the outermost shell. If B units four silicones by covalent bound, it is one electron short. Therefore, one electron hole arises. Then, giving high temperature, the nearby electron moves to the hole and another electron hole arises at the trace of the electron. As a results, the hole apparently moves.

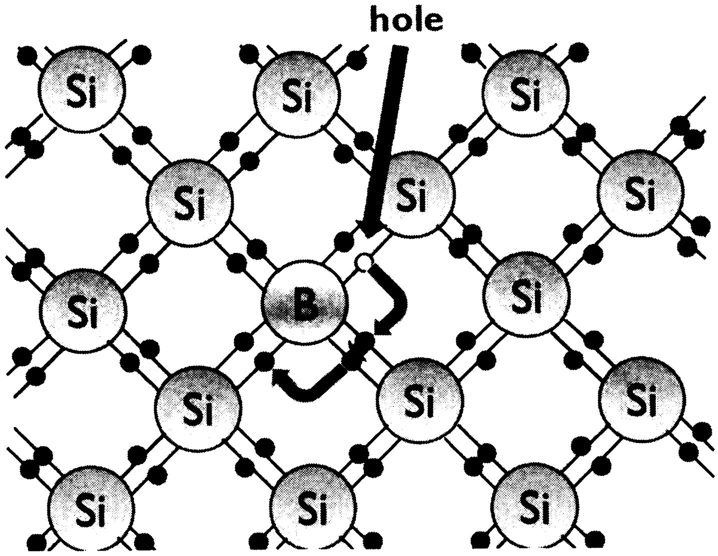

Figure 3: P-type semiconductor

2 MOS FET

In this section, we consider MOS FET, which is an abbreviation for metal oxide semiconductor field effect transistor. MOS FET is the mainstream of transistors. In fact, MOS FET is used in almost resent IC and analogue circuits in stead of diodes and transistors, because MOS FET has advantages. First, we can control the quantity of the current through MOS FET. Second, the electricity consumption of MOS FET is lower than other transistors.

We consider n-MOS FET in particular. N-MOS FET consists of the source, the gate, the drain and the body. There are n-type semiconductors under the source and the drain. The body consists of p-type semiconductors. The source, the gate and the drain consist of metal electrodes. These parts are made of silicone dioxide (SiO_2), where symbol p represents the region where much hole and little electron exist and symbol n+ represents the region where much more electron and little more hole exist.

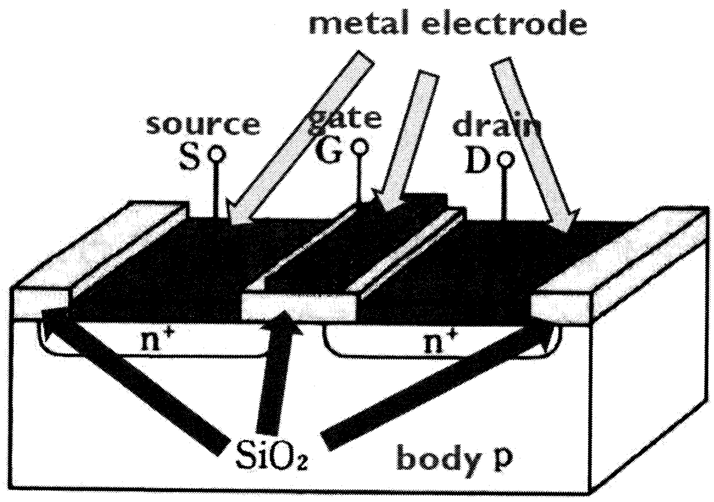

Figure 4: N-MOS FET

Now, we apply voltage between the source and the drain. Then much electron flows from the source to the drain (see Figure 5). Similarly little hole flows from the drain to the source. Next, we observe the property of the gate. By changing the voltage between the gate and the body, we can control the current flow between the source and the drain. For example, applying low voltage between the gate and the body, a small n-channel arises under the gate. Then little electron flows from the source to the drain. On the other hand, applying high voltage between the gate and the body, a large n-channel arises under the gate. Then, much electron flows from the source to the drain. Therefore, varying the voltage between the gate and the body modulates the conductivity of this layer.

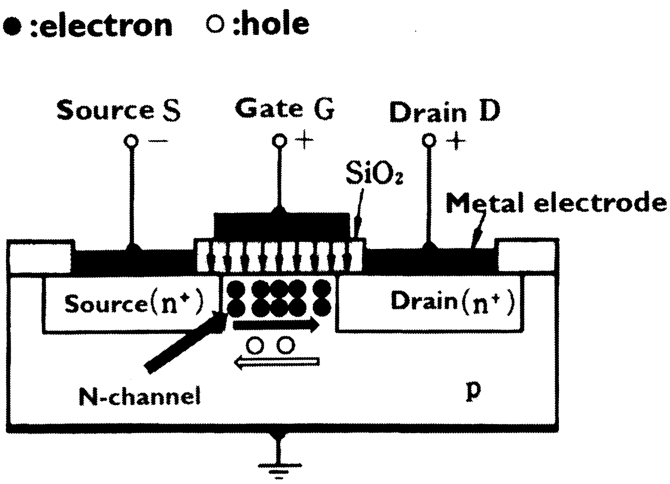

Figure 5: N-channel in n-MOS FET

3 Mathematical modeling of MOS FET

First, we introduce a hydrodynamic model of semiconductor. This equation represents the motion of the electron and the hole in a MOS FET. From now on, we denote this model by HD.

HD is derived from the Boltzmann equation by taking first three moments (see [B]). Moreover, letting some parameters of HD to zero, we can obtain the well-known drift-diffusion model. In addition to these models, there are many classical and quantum models for semiconductor (see [J, Chapter 1]). In this paper, we study the hydrodynamic model and treat with the electron and the hole as the fluid.

The drift-diffusion model can be treated easily. Therefore, the model is widely used. However, it is not suitable for the simulation of semiconductor with high bias. For this case, HD is more available. Moreover, HD is used for the simulation of the flash memory.

Next, we study the mathematical modeling of n-MOS FET. In particular, we consider the boundary condition and the position of the boundary data in MOS FET.

For real semiconductors, the electron (or hole) density of the source is the same as that of the drain, because the metal electrodes of the source and the drain are the same. Then we denote the electron density of the source and the drain by n_d (see Figure 6). Similarly, we denote the hole density of the source and the drain by h_d. Finally, we apply the voltage ϕ_r between the source and the gate.

If MOS FET whose depth is much longer than its width, we can regard MOS FET as a one-dimensional model. We then study the stationary problem of bipolar hydrodynamic model of semiconductors on the open interval $I = (0,1)$.

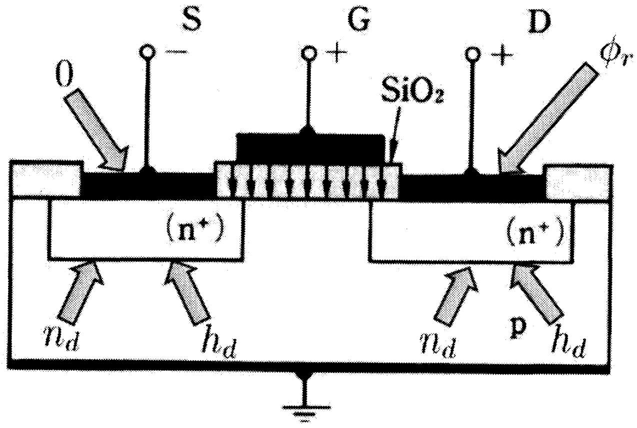

Figure 6: Boundary data in n-MOS FET

$$\begin{cases} j_x = 0, & \left(\dfrac{j^2}{n} + n\right)_x = n\phi_x - j, \\ k_x = 0, & \left(\dfrac{k^2}{h} + h\right)_x = -h\phi_x - k, \\ \phi_{xx} = n - h - D(x), \end{cases} \tag{3.1}$$

where h and n are the density of the electron and the hole respectively. j and k are the current density of the electron and the hole respectively. ϕ denotes the electrostatic potential. The doping profile $D \in C(\bar{I})$ is a known function, which represents the fixed background ion in semiconductors. The doping profile plays an important role in semiconductors, because it mainly determines the performance of semiconductors.

Then we consider the boundary problem (3.1) and the boundary conditions

$$\begin{aligned} &n|_{x=0} = n|_{x=1} = n_d\ (> 0), \quad h|_{x=0} = h|_{x=1} = h_d\ (> 0), \\ &\phi|_{x=0} = 0, \ \phi|_{x=1} = \phi_r\ (> 0). \end{aligned} \tag{3.2}$$

From the physical point of view, condition (3.2) represents Ohmic contacts (see [M], [S1] and [S2]).

4 Related results and the main theorem

We first introduce mathematical results for the unipolar hydrodynamics model (i.e., h =const. and $k = 0$ in (3.1).). The pioneer work in this field is Degond and Markowich [DM]. They investigated the existence and the uniqueness of stationary solutions. Subsequently, Nishibata and Suzuki [NS] proved the existence and the asymptotic stability of stationary solutions.

Next, we survey the bipolar case (3.1). Hattori and Zhu [HZ] discussed the stability of stationary solutions for the Cauchy problem. On the other hand, Li and Zhou [LZ] studied the existence and some limits of stationary solutions to a one-dimensional Dirichlet problem. Moreover, Gasser, Hsiao and Li [GHL] considered the asymptotic stability of classical solutions for the Cauchy problem. However, the doping profile is not considered in [LZ] and [GHL].

Now, in practical applications such as the simulation of n-MOS FET, the hydrodynamic model (3.1) is treated under the following conditions (see [M] and [S1]):

(C1) (3.1) is supplemented by the Dirichlet boundary conditions, such as (3.2).

(C2) The doping profile $D(x)$ has large derivative, that is, $D(x)$ is not flat.

In addition to the above papers, there are other mathematical papers for the bipolar case. Unfortunately, few results satisfy (C1) and (C2). In this paper, we shall consider a solution for the boundary condition (3.2) and an arbitrary doping profile.

From (3.1), we have

$$\begin{cases} j = \dfrac{\phi_r}{\displaystyle\int_0^1 \frac{1}{n}dx}, & \left(\dfrac{j^2}{2n^2} + \log n\right)_{xx} = n - h - D(x) + j\dfrac{n_x}{n^2}, \\ k = -\dfrac{\phi_r}{\displaystyle\int_0^1 \frac{1}{h}dx}, & \left(\dfrac{k^2}{2h^2} + \log h\right)_{xx} = h - n + D(x) + k\dfrac{h_x}{h^2}. \end{cases} \tag{4.1}$$

Then our main theorem is as follows.

Theorem 1 (Main Theorem[T]). *If ϕ_r is small enough, the boundary value problem* (3.2) *and* (4.1) *has a unique classical solution.*

The voltage of real semiconductors is small enough. Therefore, the assumption of the main theorem is adequate to the application. In addition, for the large voltage, it can be expected that the uniqueness of solutions does not hold generally (see [S1, Section 5.4]).

5 Outline of the proof

In this section, we introduce the outline of the proof. The detail can be found in [T]. The proof consist of two parts, existence and uniqueness.

The existence is proved as follows.

Step 1 We consider the semilinearized equation of (4.1).

Step 2 We consider the solution map T of the above semilinearized equation and prove that T maps a functional space X into itself by the maximum principle and the energy method.

Step 3 Applying the Schauder fixed point theorem to T , we obtain a fixed point, which is a solution of (4.1).

The uniqueness is proved as follows.

Step 4 We prove that any classical solution to the boundary problem (3.2) and (4.1) is uniformly bounded by the maximum principle.

Step 5 We prove the uniqueness by the energy method.

Throughout the above proof, the most difficult point is the uniformly bounded estimate of solutions in Step 2 and 4.

5.1 Bounded estimates of Step 2

In Step 2, we first assume that given functions $(n,h) \in C^1(\bar{I})$ satisfy

$$C_m\, e^{a(x^2-1)} \le n(x) \le C_M\, e^{a(1-x^2)}, \quad C_m\, e^{a(x^2-1)} \le h(x) \le C_M\, e^{a(1-x^2)}, \tag{5.1}$$

where

$$\begin{aligned} &D_M := \max_{x\in\bar{I}} |D(x)|, \; a := \max\{3, D_M\}, \; C_m := \min\{n_d, h_d\}, \\ &C_M := \max\{n_d, h_d\}. \end{aligned}$$

Next we prove the existence and the uniqueness of solutions to the following semilinearized equation of (4.1).

$$\begin{cases} \left(\dfrac{1}{n} - \dfrac{j^2}{n^3}\right) N_{xx} + \left(-\dfrac{1}{n^2} + \dfrac{3j^2}{n^4}\right)(N_x)^2 - \dfrac{j}{n^2} N_x \\ \qquad = b(N-n) + N - h - D(x), \\ \left(\dfrac{1}{h} - \dfrac{k^2}{h^3}\right) H_{xx} + \left(-\dfrac{1}{h^2} + \dfrac{3k^2}{h^4}\right)(H_x)^2 - \dfrac{k}{h^2} H_x \\ \qquad = b(H-h) + H - n + D(x), \\ N|_{x=0} = N|_{x=1} = n_d, \quad H|_{x=0} = H|_{x=1} = h_d, \end{cases}$$

where we define $j = \phi_r / \int_0^1 \frac{1}{n} dx$, $k = -\phi_r / \int_0^1 \frac{1}{h} dx$ and b is a constant which we choose large enough.

Then we prove the following proposition.

Proposition 2. *If* (n,h) *satisfies* (5.1), (N,H) *also satisfies*

$$C_m\, e^{a(x^2-1)} \le N(x) \le C_M\, e^{a(1-x^2)}, \quad C_m\, e^{a(x^2-1)} \le H(x) \le C_M\, e^{a(1-x^2)}.$$

In order to obtain the upper (resp. lower) bound, we apply the maximum principle to $\tilde{N} = e^{ax^2} N$ and $\tilde{H} = e^{ax^2} H$ (resp. $\tilde{N} = e^{-ax^2} N$ and $\tilde{H} = e^{-ax^2} H$).

Then we define the solution map $T : (n,h) \mapsto (N,H)$. From the above lemma and the energy method, we find that T maps a functional space X to itself and is compact.

5.2 Bounded estimates of Step 4

In step 4, we prove the following proposition.

Proposition 3. *If ϕ_r is small enough, classical solutions to the boundary value problem* (3.1)-(3.2) *satisfy*

$$C_m e^{a(x^2-1)} \leq n(x) \leq C_M e^{a(1-x^2)}, \quad C_m e^{a(x^2-1)} \leq h(x) \leq C_M e^{a(1-x^2)}.$$

This proposition is deduced from the following three lemmas.

Lemma 4. *If $n(x)$ and $h(x)$ are classical solutions to the boundary value problem* (3.1) *and* (3.2)*, $n(x)$ and $h(x)$ are positive.*

Lemma 5. *If ϕ_r is small enough, classical solutions to the boundary value problem* (3.1) *and* (3.2) *satisfy*

$$n(x) \leq C_M e^{a(1-x^2)}, \quad h(x) \leq C_M e^{a(1-x^2)}.$$

Lemma 6. *If ϕ_r is small enough, there exists a positive constant C such that classical solutions to the boundary value problem* (3.1) *and* (3.2) *satisfy*

$$C < n(x), \quad C < h(x), \tag{5.2}$$

where C depends only on n_d, h_d and D_M.

The proof of Lemma 5 is similar to that of Proposition 2. However, we cannot apply the similar method to Lemma 6. To solve this problem, we consider the Riemann invariants.

$$\frac{j}{n} + \log n, \quad \frac{j}{n} - \log n. \tag{5.3}$$

The lower and upper bound of (5.3) yields (5.2).

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