

On the Igusa modular form of weight 10

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Abstract. This paper is related to the authors' talk at the RIMS conference 2011 on: *Automorphic forms, trace formulas and zeta functions* in Kyoto. The Igusa modular form of weight 10 is the unique Siegel modular form which is a Borcherds and a Saito-Kurokawa lift.

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1 Introduction

The Igusa modular form χ_{10} appeared first in the famous theorem of Jun-ichi Igusa about the generators of graded algebra of Siegel modular forms of even weight and degree 2 (see [Ig1]). The algebra is equal to

$$(1.1) \quad \mathbb{C}[E_4^2, E_6^2, \chi_{10}, E_{12}^2].$$

We normalized the Siegel type Eisenstein series E_k^2 of weight k such that the Fourier coefficient related to 0-dim cusp at infinity is one. The Igusa modular form χ_{10} is a cusp form of weight 10. Igusa introduced the form in terms of Eisenstein series ([Ig1], page 192).

$$\chi_{10} := -43867 \cdot 2^{-12} \cdot 3^{-5} \cdot 5^{-2} \cdot 7^{-1} \cdot 53^{-1} (E_4^2 E_6^2 - E_{10}^2).$$

It is known that χ_{10} is a Saito-Kurokawa lift ([Za]) and a Borcherds lift ([GN1], [GN2]).

The square root of this modular form is related to the denominator formula for a generalized Borcherds-Kac-Moody super algebra (Gritsenko, Nikulin). Moreover it is as a partition function of BPS dyons in the toroidally compactified heterotic string theory. To study a generalized Kac-Moody algebra one has to know the imaginary simple roots and the multiplicities of all positive roots. It is absolutely crucial that the underlying modular form has a degenerate Fourier expansion (Saito-Kurokawa lift) and an infinite product (Borcherds lift). We refer to ([CD], [CV]) for more details. The following theorem states that there are no other Siegel modular forms of degree 2 which are Borcherds and Saito-Kurokawa lifts .

Theorem *Let F be a Siegel modular form of degree 2. If F is a Borcherds lift and a Saito-Kurokawa lift, then F is proportional to the Igusa modular form.*

We note that the Borcherds lift is multiplicative and the Saito-Kurokawa lift additive.

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2 Siegel modular forms, Witt operator and Taylor expansions

For an introduction to the theory of Siegel modular forms we refer to Klingen's book ([Kl]). Let Γ_n be the Siegel modular group and \mathfrak{H}_n the upper half space of degree n :

$$\begin{aligned}\Gamma_n &:= \left\{ \gamma \in \mathrm{GL}_{2n}(\mathbb{Z}) \mid {}^t \gamma \begin{pmatrix} \mathbf{0}_n & \mathbf{1}_n \\ -\mathbf{1}_n & \mathbf{0}_n \end{pmatrix} \gamma = \begin{pmatrix} \mathbf{0}_n & \mathbf{1}_n \\ -\mathbf{1}_n & \mathbf{0}_n \end{pmatrix} \right\} \\ \mathfrak{H}_n &:= \{ Z \in \mathrm{M}_n(\mathbb{C}) \mid {}^t Z = Z, \mathrm{Im}(Z) > 0 \},\end{aligned}$$

where $\mathbf{0}_n$ (respectively $\mathbf{1}_n$) is the zero (respectively identity) matrix of degree n . Then we denote by $M_k(\Gamma_n)$ the space of Siegel modular forms of weight k on Γ_n and by $S_k(\Gamma_n)$ the subspace of cusp forms. In the case $n = 1$ we usually drop the index and for $n = 2$ which we are mainly interested in we often write (τ_1, z, τ_2) for a point

$$\begin{pmatrix} \tau_1 & z \\ z & \tau_2 \end{pmatrix} \in \mathfrak{H}_2.$$

Next we introduce two useful operators. Let $F \in M_k(\Gamma_2)$. Define

$$\begin{aligned}\Phi(F)(\tau) &:= \lim_{y \rightarrow \infty} F(\tau, 0, iy) \quad (\tau \in \mathfrak{H}_1), \\ \mathcal{W}(F)(\tau_1, \tau_2) &:= F(\tau_1, 0, \tau_2) \quad (\tau_1, \tau_2 \in \mathfrak{H}_1).\end{aligned}$$

Then $\Phi(F) \in M_k(\Gamma)$ and $\mathcal{W}(F) \in \mathrm{Sym}^2(M_k(\Gamma))$. The operator Φ (respectively \mathcal{W}) is called the *Siegel* (respectively *Witt*) operator. Then $S_k(\Gamma_2) = \{F \in M_k(\Gamma_2) \mid \Phi(F) = 0\}$.

Let f_1, f_2, \dots, f_d be a basis of newforms of S_k and $f_0 = e_k$. Here e_k denotes the elliptic Eisenstein series with constant term $a(0) = 1$.

Then we define

$$(2.1) \quad \mathrm{Sym}^2(M_k(\Gamma))^D := \left\{ \sum_{i=0}^d \alpha_i f_i \otimes f_i \mid \alpha_i \in \mathbb{C} \right\}.$$

By $\mathrm{Sym}^2(S_k(\Gamma))^D$ we denote the cuspidal part.

A Siegel modular form $F \in M_k(\Gamma_2)$ admits the Fourier expansion

$$F(\tau_1, z, \tau_2) = \sum_{n,r,m \in \mathbb{Z}} A_F(n, r, m) \mathbf{e}(n\tau_1 + rz + m\tau_2),$$

where we put $\mathbf{e}(z) = \exp(2\pi iz)$ for $z \in \mathbb{C}$. Note that $A_F(n, r, m) = 0$ unless $n, m, 4nm - r^2 \geq 0$. We also use the following shortcuts: $q := \mathbf{e}(\tau)$, $q_1 := \mathbf{e}(\tau_1)$, $\zeta := \mathbf{e}(z)$, $q_2 := \mathbf{e}(\tau_2)$. It is easy to see that:

$$(2.2) \quad \Phi(F)(\tau) = \sum_{n=0}^{\infty} A_F(n, 0, 0) q^n$$

$$(2.3) \quad \mathcal{W}(F)(\tau_1, \tau_2) = \sum_{n,m=0}^{\infty} \left(\sum_r A_F(n, r, m) \right) q_1^n q_2^m.$$

We define the order of the q -expansion of a modular form $F \in M_k(\Gamma_2)$ by

$$\text{ord}(F) := \min \{n \in \mathbb{N}_0 \mid A_F(n, r, m) \neq 0\}.$$

Remark 2.1. If $\text{ord}(F) \geq 2$, then $F \notin \text{Sym}^2(M_k(\Gamma))^D$.

Let k be even. Then $F \in M_k(\Gamma)$ has the Taylor expansion

$$(2.4) \quad F(\tau_1, z, \tau_2) = \sum_{l=0}^{\infty} \Psi_{2l}(\tau_1, \tau_2) z^{2l}.$$

It is clear that Ψ_0 is the image of the Witt operator and an element of $\text{Sym}^2(M_k(\Gamma))$. Moreover if Ψ_0 is identically zero then $\psi_2 \in \text{Sym}^2(S_{k+2}(\Gamma))$.

Finally let E_k^n denote the Siegel-type Eisenstein series on Γ_n , normalized by $\Phi^n(E_k^n) = 1$. Here Φ^n denotes the n -th iteration of the Φ operator. Let $E_k^n(f)$ denote the Klingen Eisenstein series attached to $f \in S_k(\Gamma), f \neq 0$. Note that $\Phi^{n-1}(E_k^n(f)) = f$. Let further $M_k^{2,0}$ be the 1-dim space generated by Siegel Eisenstein series of weight k and degree 2, let $M_k^{2,1}$ be the space generated by all Klingen type Eisenstein series of weight k and degree 2 and let $M_k^{2,2} = S_k(\Gamma_2)$. Then

$$(2.5) \quad M_k(\Gamma_2) = M_k^{2,0} \oplus M_k^{2,1} \oplus M_k^{2,2}.$$

The direct sum is related to the Petersson scalar product. Moreover this decomposition is respected by the Siegel Φ operator. Let $F \in M_k(\Gamma_2)$ with decomposition $F_0 + F_1 + F_2$. Then

$$(2.6) \quad \Phi(F) = \Phi(F_0) + \Phi(F_1) + \Phi(F_2)$$

$$(2.7) \quad = c_1 E_k + c_2 f \quad (c_1, c_2 \in \mathbb{C}, f \in S_k(\Gamma)).$$

3 Saito-Kurokawa lifts

One can find an overview in Zagier's Bourbaki article [Za]. Let k be an even integer. Then there exists an injective linear map

$$(3.1) \quad SKL : M_{2k-2}(\Gamma) \longrightarrow M_k(\Gamma_2),$$

where Hecke eigenforms f map to Hecke eigenforms $F = SKL(f)$. For a Hecke eigenform f , the spinor L-function $Z(SKL(f), s)$ is given by

$$Z(SKL(f), s) = \zeta(s - k + 1) \zeta(s - k + 2) L(f, s),$$

where $L(f, s)$ is the Hecke L-function of f and $\zeta(s)$ denotes the Riemann zeta function. We are interested in the image of the lifting, which is given by the so-called Maass Spezialschar:

$$(3.2) \quad M_k^{\text{Spez}} := \left\{ F \in M_k(\Gamma_2) \mid A_F(n, r, m) = \sum_{d \in \mathbb{N}, d \mid (n, r, m)} d^{k-1} A_F\left(\frac{nm}{d^2}, \frac{r}{d}, 1\right) \right\}.$$

Here (n, r, m) denotes the greatest common divisor of n, r, m (We put $1 := (0, 0, 0)$). To prove our main result we use the following properties of the Maass Spezialschar. If $F \in M_k^{\text{Spez}}$, then F is non-trivial iff Ψ_0^F or Ψ_2^F is not identically zero. Moreover

$$(3.3) \quad \Psi_0^F \in \text{Sym}^2(M_k(\Gamma))^D.$$

If Ψ_0^F is identically zero then

$$(3.4) \quad \Psi_2^F \in \text{Sym}^2(S_{k+2}(\Gamma))^D.$$

Remark 3.1. Let $F \in M_k(\Gamma_2)$ has the decomposition $F_0 + F_1 + F_2$ as described before. If F_1 is non-trivial, then F is not in the Spezialschar.

4 Borchers lifts

Roughly speaking a Borchers lift BL is a correspondence between modular forms of weight $1 - \frac{m}{2}$ on \mathfrak{H} with possible singularities at the cusps and certain meromorphic automorphic forms with possible character on symmetric domains of type IV related to orthogonal groups $O(2, m)$ ($m \in \mathbb{N}$) ([Bo1],[Bo2], [Bo3]). We note that

$$BL(f + g) = BL(f) \cdot BL(g).$$

Lifts to Siegel modular forms of degree 2 are related to the case $m = 3$, where the image is uniquely (up to a scalar) determined by the divisor

$$(4.1) \quad \text{div}(BL(f)) = \sum_{d \in \mathcal{D}} n_d H_d.$$

Here \mathcal{D} is the set of all positive integers congruent to 0 or 1. The sum is finite and $n_d \in \mathbb{Z}$. The H_d are the Humbert surfaces (see also the following subsection), for general m they are called Heegner divisor. The image could be an element of $M_k(\Gamma_2, \nu)$, a Siegel modular form with the unique non-trivial character ν on Γ_2 .

Remark 4.1. The coefficients of the principal part of the input function are related to the n_d . A priori it is not clear when the nontrivial character in the image occurs. Moreover even when not all coefficients in the principal part are non-negative, the image could be holomorphic.

4.1 Humbert surfaces

Let

$$Q := \begin{pmatrix} & & & 1 \\ & & 1 & \\ & -2 & & \\ 1 & & & \\ & 1 & & \end{pmatrix}.$$

Put $Q(X, Y) := {}^tXQY$ and $Q[X] := Q(X, X)$ for $X, Y \in \mathbb{C}^5$. For $Z = (\tau_1, z, \tau_2) \in \mathfrak{H}_2$ put $\tilde{Z} := {}^t(-\tau_1\tau_2 + z^2, \tau_1, z, \tau_2, 1) \in \mathbb{C}^5$. Note that $Q[\tilde{Z}] = 0$ and $Q(\tilde{Z}, \tilde{Z}) = 4 \det(\text{Im}(Z)) > 0$. There exists a homomorphism $\iota: \text{Sp}_2(\mathbb{R}) \rightarrow O(Q)_{\mathbb{R}}$ such that $\widetilde{g\langle Z \rangle} = j(g, Z)^{-1} \iota(g) \tilde{Z}$ for $g \in \text{Sp}_2(\mathbb{R})$ and $Z \in \mathfrak{H}_2$.

Let $L := \mathbb{Z}^5, L^* := Q^{-1}L$ and $L^*_{\text{prim}} := \{\lambda \in L^* \mid n^{-1}\lambda \notin L^* \text{ for any integer } n > 1\}$. For an integer $d \in \mathbb{Z}$, let

$$\mathcal{H}_d := \sum_{X \in \mathcal{L}_d} \left\{ Z \in \mathfrak{H}_2 \mid Q(X, \tilde{Z}) = 0 \right\},$$

where $\mathcal{L}_d := \{X \in L^*_{\text{prim}} \mid Q[X] = -d/2\}$. Note that $\mathcal{H}_d = \emptyset$ unless $d > 0$ and $d \equiv 0$ or $1 \pmod{4}$. Since L^*_d is $\iota(\Gamma_2)$ -invariant, \mathcal{H}_d is Γ_2 -invariant. Denote by H_d the image of \mathcal{H}_d in $\Gamma_2 \backslash \mathfrak{H}_2$ by the natural projection $\mathfrak{H}_2 \rightarrow \Gamma_2 \backslash \mathfrak{H}_2$. The divisor H_d of $\Gamma_2 \backslash \mathfrak{H}_2$ is called the *Humbert surface* of discriminant d . It is known that H_d is nonzero and irreducible if $d \equiv 0$ or $1 \pmod{4}$ (see [Ge2], page 212, Theorem 2.4; see also [GH], Section 3). Note that

$$\mathcal{H}_1 = \bigcup_{\gamma \in \Gamma_2} \gamma \{(\tau_1, 0, \tau_2) \mid \tau_1, \tau_2 \in \mathfrak{H}\}$$

$$\mathcal{H}_4 = \bigcup_{\gamma \in \Gamma_2} \gamma \{(\tau, z, \tau) \mid \tau \in \mathfrak{H}, z \in \mathbb{C}\}.$$

4.2 Properties of Borchers lifts and examples

Recently [HM] we found an explicit description of the Borchers lifts related to single Heegner divisors. As a by-product one can see that the character is only related to the divisors H_1 and H_4 .

Theorem 4.2.

- (i) For each positive integer d with $d \equiv 0$ or $1 \pmod{4}$, there exists an $F_d \in M_{k_d}(\Gamma_2, \nu^{\alpha_d})$ with $\alpha_d \in \{0, 1\}$ satisfying $\text{div}(F_d) = H_d$.
- (ii) We have $F_1 \in S_5(\Gamma_2, \nu), F_4 \in S_{30}(\Gamma_2, \nu)$ and $F_d \in M_{k_d}(\Gamma_2)$ if $d > 4$.
- (iii) A Borchers lift $F \in M_k(\Gamma_2, \nu^\alpha)$ ($\alpha \in \{0, 1\}$) is a constant multiple of $\prod_d F_d^{A(d)}$, where d runs over the positive integers with $d \equiv 0$ or $1 \pmod{4}$, and $A(d)$ is a nonnegative integer ($A(d) = 0$ except for a finite number of d) satisfying $A(1) + A(4) \equiv \alpha \pmod{2}$.

Here $S_k(\Gamma_2, \nu)$ denotes the cuspidal subspace of $M_k(\Gamma_2, \nu)$

It is well-known that $\dim S_{10}(\Gamma_2) = 1$ (see also [Kl]). Hence χ_{10} is proportional to F_1^2 .

Remark 4.3. The Borchers lifts in $M_k(\Gamma_2)$ with $k \leq 60$ are listed as follows:

Borcherds lift	weight	divisor
F_1^{2a} ($1 \leq a \leq 6$)	$10a$	$2aH_1$
$F_1^{2a+1}F_4$ ($1 \leq a \leq 2$)	$10a + 35$	$(2a + 1)H_1 + H_4$
$F_1^{2a}F_5$ ($1 \leq a \leq 3$)	$10a + 24$	$2aH_1 + H_5$
F_4^2	60	$2H_4$
F_5^2	48	$2H_5$
F_8	60	H_8

The table shows that every Borcherds lift of weight less than or equal to 60 is a monomial of F_1, F_4, F_5 and F_8 . We also see that there is no holomorphic Borcherds lift of weight 12.

Assume that $F \in M_k(\Gamma_2)$ is a Borcherds lift. Then $\Phi(F)$ is proportional to a power Δ^r of the modular discriminant Δ with $r \geq 0$.

5 Proof of the Theorem

In the following we give a sketch of the proof of the main theorem. The complete proof will appear elsewhere. Let $F \in M_k(\Gamma_2)$, $F \neq 0$. Let F be a Borcherds lift (BL) and Saito-Kurokawa lift (SKL). First of all we can assume that the weight is even (SKL). This implies that $k \geq 4$. The structure theorem (BL) leads to

$$(5.1) \quad F \sim \prod_{d \in \mathcal{D}} F_d^{n_d}.$$

The product is finite, $n_1 + n_4 \equiv 0 \pmod{2}$ and $n_d \in \mathbb{N}_0$. The symbol \sim indicates that two functions are equal up to a non-zero constant.

Remark 5.1. A refined analysis of the modular forms F_d shows that

$$\text{ord}(F_1) = \frac{1}{2}, \text{ord}(F_4) = \frac{3}{2}.$$

If $d \geq 5$ then $\text{ord}(F_d) \geq 2$ iff d is a square and $\text{ord}(F_d) = 0$ otherwise.

Since F is also a SKL we have $\text{ord}(F) \leq 1$. This leads to

$$(5.2) \quad F \sim F_1^\alpha \cdot \prod_{d \geq 5, d \text{ not a square}} F_d^{n_d} \quad (\alpha = 0, 2).$$

Put $G := F/F_1^\alpha$. Since G is a BL and not a cusp form we have [HM]

$$\Phi(G) \sim \Delta^r \quad \left(r = \frac{k - 5\alpha}{12} \in \mathbb{N}\right).$$

Then it is easy to see that

$$\mathcal{W}(G) \sim \Delta^r \otimes E_k + E_k \otimes \Delta^r + \text{cuspidal}.$$

This shows that, if $\alpha = 0$, then $G = F$ is not a SKL, a contradiction. Thus we have $\alpha = 2$. Finally the case $\alpha = 2$ remains. We show that $r \geq 1$ is not possible (then the theorem is proven).

Let in the following $F \sim F_1^2 \dot{G}$, with $\Phi(G) = \Delta^r$ ($r \geq 1$). Then Ψ_0^F is identically 0. Since F is a SKL and not identically zero, we can assume that $\Psi_2^F \neq 0$ and that

$$(5.3) \quad \Psi_2^F \in \text{Sym}^2(S_{k+2}(\Gamma))^D.$$

Since the second Taylor coefficient of F_1^2 is proportional to $\Delta \otimes \Delta$ we obtain

$$(5.4) \quad \Psi_2^F \sim (\Delta \otimes \Delta) \cdot \mathcal{W}(G).$$

On the other hand $\mathcal{W}(G)$ can be expressed in terms of the modular function j and the primitive modular polynomial. This can be directly proven by comparing the weights and the divisors on $\mathfrak{H} \times \mathfrak{H}$.

For $m \in \mathbb{Z}_{>0}$, let \mathcal{M}_m^* be the set of primitive matrices in $M_2(\mathbb{Z})$ of determinant m . As is well-known, there exists a polynomial Φ_m^* in $\mathbb{Z}[X, Y]$, called the primitive modular polynomial of degree m , such that

$$\prod_{M \in \text{SL}_2(\mathbb{Z}) \setminus \mathcal{M}_m^*} (X - j(M\langle\tau\rangle)) = \Phi_m^*(X, j(\tau)).$$

Here $\tau \mapsto M\langle\tau\rangle$ denotes the action on \mathfrak{H} . The degree of $\Phi_m^*(X, Y)$ in X is larger than m for $m > 1$. Then

$$(5.5) \quad \mathcal{W}(G)(\tau_1, \tau_2) \sim (\Delta^r(\tau_1) \otimes \Delta^r(\tau_2)) \prod_{n>0} \Phi_n^*(j(\tau_1), j(\tau_2))^{a(n)},$$

where $a(n) \in \mathbb{N}_0$. Hence we obtain

$$(5.6) \quad \Psi_2^F(\tau_1, \tau_2) \sim (\Delta^{r+1}(\tau_1) \otimes \Delta^{r+1}(\tau_2)) \prod_{n>0} \Phi_n^*(j(\tau_1), j(\tau_2))^{a(n)}.$$

Combining this property with (5.3) leads to a contradiction by employing well-known properties of the modular polynomial, multiplicative properties of the Fourier coefficients of primitive Hecke eigenforms and the explicit Fourier expansion of the Δ -function.

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