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ON A REDUCTION OF NON-COMMUTATIVE REIDEMEISTER TORSION FOR HOMOLOGY CYLINDERS

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1. Introduction

Let $\Sigma_{g,1}$ be a compact oriented surface of genus $g \geq 1$ with one boundary component.

Homology cylinders over a surface were first introduced by Goussarov [4] and Habiro [6] in their surgery theory of 3-manifolds developed for the study of finite-type invariants. In [3, 9] Garoufalidis and Levine introduced the homology cobordism group of homology cylinders, which can be seen as an enlargement of the mapping class group of the surface. We denote by $C_{g,1}$, $C_{g,1}^{irr}$ and $\mathcal{H}_{g,1}$ the monoid of homology cylinders over $\Sigma_{g,1}$, the submonoid consisting of irreducible ones as 3-manifold and the smooth homology cobordism group respectively. The Johnson filtrations $C_{g,1}[k]$, $\mathcal{H}_{g,1}[k]$ of $C_{g,1}$, $\mathcal{H}_{g,1}$ are defined as the kernels of the actions on $\pi_1\Sigma_{g,1}/(\pi_1\Sigma_{g,1})_k$, where the lower central series G_k of a group G is defined inductively by $G_1 := G$ and $G_{k+1} := [G_k, G]$.

Sakasai [15, 16] studied torsion invariants of homology cylinders with in general non-commutative coefficients and showed by the degrees of these invariants associated to elements of $H^1(\Sigma_{g,1})$ as a reduction that the submonoids $C_{g,1}^{irr} \cap C_{g,1}[k]$ for $k \geq 2$ and $\mathrm{Ker}(C_{g,1} \to \mathcal{H}_{g,1})$ have abelian quotients isomorphic to $(\mathbb{Z}_{\geq 0})^\infty$. Note that since the connected sum of a homology cylinder and a homology 3-sphere is another homology cylinder, it is reasonable to restrict our attention to $C_{g,1}^{irr}$ in considering "size" of $C_{g,1}[k]$. Morita [12] showed by using his "trace maps" defined in [11] that the abelianization of $\mathcal{H}_{g,1}[2]$ has infinite rank. Goda and Sakasai [5] showed by using sutured Floer homology theory that $C_{g,1}^{irr}$ has an abelian quotient isomorphic to $(\mathbb{Z}_{\geq 0})^\infty$. Cha, Friedl and Kim [1] showed by using abelian torsion invariants that the abelianization of $\mathcal{H}_{g,1}$ contains a direct summand isomorphic to $(\mathbb{Z}/2)^\infty$, and that the abelianization of $\mathcal{H}_{g,1}[2]$ contains a direct summand isomorphic to $(\mathbb{Z}/2)^\infty$ and one isomorphic to $\mathbb{Z}^\infty$ if $g \geq 2$.

The aim of this note is to present another reduction of non-commutative torsion invariants introduced in [8] and to give another approach to Sakasai's result for $C_{g,1}^{irr} \cap C_{g,1}[k]$. More precisely, we consider the coefficients of the maximum order terms of torsion invariants associated to bi-orders of $\pi_1\Sigma_{g,1}/(\pi_1\Sigma_{g,1})_k$ and use them to prove that the group completion of $C_{g,1}^{irr} \cap C_{g,1}[k]$ has an abelian group quotient of infinite rank for $k \geq 2$. In [8] we can find an analogous work on submonoids of $C_{g,1}^{irr}$ associated to solvable quotients of $\pi_1\Sigma_{g,1}$.

In this note all homology groups and cohomology groups are with respect to integral coefficients unless specifically noted.

2. Homology cylinders

First we recall the definitions of homology cylinders and their homology cobordisms. See [7], [17] for more details on homology cylinders.

To simplify notation we often write Σ, π instead of $\Sigma_{g,1}, \pi_1\Sigma_{g,1}$, respectively. We take a base point for π in $\partial\Sigma$.

Definition 2.1. A *homology cylinder* $(M, i_\pm)$ over Σ is defined to be a compact oriented 3-manifold M together with embeddings $i_+, i_-\colon \Sigma \to \partial M$ satisfying the following:
(i) i_+ is orientation preserving and i_- is orientation reversing,
(ii) $\partial M = i_+(\Sigma) \cup i_-(\Sigma)$ and $i_+(\Sigma) \cap i_-(\Sigma) = i_+(\partial M) = i_-(\partial M)$,
(iii) $i_+|_{\partial\Sigma} = i_-|_{\partial\Sigma}$,
(iv) $(i_+)_*, (i_-)_*\colon H_*(\Sigma) \to H_*(M)$ are isomorphisms.
Two homology cylinders $(M, i_\pm), (N, j_\pm)$ are called isomorphic if there exists an orientation preserving homeomorphism $f\colon M \to N$ satisfying $j_\pm = f \circ i_\pm$. We denote by $C_{g,1}$ the set of all isomorphism classes of homology cylinders over $\Sigma_{g,1}$.

A product operation on $C_{g,1}$ is given by stacking:

$$(M, i_\pm) \cdot (N, j_\pm) := (M \cup_{i_- \circ (j_+)^{-1}} N, i_+, j_-),$$

which turns $C_{g,1}$ into a monoid. The unit is given by the standard cylinder $(\Sigma \times [0,1], id \times 1, id \times 0)$.

As pointed out in [5, Proposition 2. 4] there is an epimorphism $F\colon C_{g,1} \to \theta^3$ as follows, where θ^3 is the monoid of homology 3-spheres with the connected sum operation. For $(M, i_\pm) \in C_{g,1}$, we can write $M = M' \sharp M''$, where M' is the prime factor of M containing ∂M. Then $F(M, i_\pm) := M''$. Therefore it is reasonable to consider the submonoid $C_{g,1}^{irr}$ consisting of all homology cylinders whose underlying 3-manifolds are irreducible.

Definition 2.2. Two homology cylinders $(M, i_\pm) \sim_m (N, j_\pm)$ are said to be *homology cobordant* if there exists a compact oriented smooth 4-manifold such that:
(i) $\partial W = M \cup_{i_+ \circ j_+^{-1}, i_- \circ j_-^{-1}} (-N)$,
(ii) $H_*(M) \to H_*(W), H_*(N) \to H_*(W)$ are isomorphisms.
We denote by $\mathcal{H}_{g,1}$ the quotient set of $C_{g,1}$ with respect to the equivalence relation of homology cobordism.

The monoid structure of $C_{g,1}$ naturally induces a group structure of $\mathcal{H}_{g,1}$. The inverse of $[M, i_\pm] \in \mathcal{H}_{g,1}$ is given by $[-M, i_\mp]$.

We set $N_k := \pi/\pi_k$. For $(M, i_\pm) \in C_{g,1}, (i_\pm)_*\colon N_k \to \pi_1 M/(\pi_1 M)_k$ are isomorphisms according to Stallings' theorem [18]. We define a homomorphism $\varphi_k\colon C_{g,1} \to \operatorname{Aut} N_k$ by $\varphi_k(M, i_\pm) := (i_+)_*^{-1} \circ (i_-)_*$. By abuse of notation we also denote by φ_k the naturally induced homomorphism $\mathcal{H}_{g,1} \to \operatorname{Aut} N_k$.

Definition 2.3. The *Johnson filtrations* of $C_{g,1}$ and $\mathcal{H}_{g,1}$ are the sequences

$$\cdots \subset C_{g,1}[k] \subset \cdots \subset C_{g,1}[2] \subset C_{g,1}[1] = C_{g,1},$$
$$\cdots \subset \mathcal{H}_{g,1}[k] \subset \cdots \subset \mathcal{H}_{g,1}[2] \subset \mathcal{H}_{g,1}[1] = \mathcal{H}_{g,1}$$

respectively, where $C_{g,1}[k]$ and $\mathcal{H}_{g,1}[k]$ are the kernels of φ_k.

3. A reduction of the torsion homomorphism

Next we review torsion invariants of homology cylinders and introduce another reduction of the group where these torsion invariants are defined, using a bi-order of the nilpotent quotient $N_k := \pi/\pi_k$.

Let $\mathbb{K}$ be a skew field. We write write $\mathbb{K}_{ab}^\times$ for the abelianization of the unit group $\mathbb{K}^\times$. For a finite CW-pair (X, Y) and a homomorphism $\rho\colon \mathbb{Z}[\pi_1 X] \to \mathbb{K}$ such that the twisted homology group $H_*^\rho(X, Y; \mathbb{K})$ associated to ρ vanishes, the Reidemeister torsion $\tau_\rho(X, Y) \in \mathbb{K}_{ab}^\times / \pm\rho(\pi_1 X)$ associated to ρ is defined. See [10] and [19] for more details.

We set $A_k := \pi_{k-1}/\pi_k$. Since A_k is torsion-free for all k, N_k has a finite filtration of normal subgroups such that all successive quotient are torsion-free abelian groups. It is known that for such a group G (called poly-torsion-free-abelian), $\mathbb{Q}[G]$ is a right (and left) Ore domain; namely $\mathbb{Q}[G]$ embeds in its classical right ring of quotients $\mathbb{Q}(G) := \mathbb{Q}[G](\mathbb{Q}[G]\setminus 0)^{-1}$ [13]. For $(M, i_\pm) \in \mathcal{C}_{g,1}$, we denote by ρ_k the composition of the homomorphisms

$$\mathbb{Z}[\pi_1 M] \to \mathbb{Z}[\pi_1 M/(\pi_1 M)_k] \xrightarrow{(i_+)_*^{-1}} \mathbb{Z}[N_k] \to \mathbb{Q}(N_k).$$

See [2, Proposition 2. 10] for a proof of the following lemma.

Lemma 3.1. *For $(M, i_\pm) \in \mathcal{C}_{g,1}$, $H_*^{\rho_k}(M, i_+(\Sigma); \mathbb{Q}(N_k)) = 0$.*

Definition 3.2. We define a map $\tau_k \colon \mathcal{C}_{g,1} \to \mathbb{Q}(N_k)^\times_{ab}/\pm N_k$ by

$$\tau_k(M, i_\pm) := \tau_{\rho_k}(M, i_+(\Sigma)).$$

The following proposition is a version of [1, Proposition 3. 5] and [8, Corollary 3.10]. See also [16, Proposition 6. 6] for a related result. The proof is almost same as that of [8, Corollary 3.10], and so we omit the proof.

Proposition 3.3. *The map $\tau_k \rtimes \varphi_k \colon \mathcal{C}_{g,1} \to (\mathbb{Q}(N_k)^\times_{ab}/\pm N_k) \rtimes \mathrm{Aut}(N_k)$ is a homomorphism.*

Corollary 3.4. *The map $\tau_k \colon \mathcal{C}_{g,1}[k] \to \mathbb{Q}(N_k)^\times_{ab}/\pm N_k$ is a homomorphism.*

We denote by $\bar{\cdot} \colon \mathbb{Z}[N_k] \to \mathbb{Z}[N_k]$ the involution defined by $\bar{\gamma} = \gamma^{-1}$ for $\gamma \in N_k$ and naturally extend it to $\mathbb{Q}(N_k)$. We set

$$D_k := \{\pm\gamma \cdot q \cdot \bar{q} \in \mathbb{Q}(N_k)^\times_{ab}\ ;\ \gamma \in N_k, q \in \mathbb{Q}(\Gamma_m)^\times_{ab}\}.$$

The following theorem is also a version of [1, Theorem 3. 10] and [8, Corollary 3.13]. See them for the proof.

Theorem 3.5. *The map $\tau_k \rtimes \varphi_k \colon \mathcal{H}_{g,1} \to (\mathbb{Q}(N_k)^\times_{ab}/D_k) \rtimes \mathrm{Aut}(N_k)$ is a homomorphism.*

Corollary 3.6. *The map $\tau_k \colon \mathcal{H}_{g,1}[k] \to \mathbb{Q}(N_k)^\times_{ab}/D_k$ is a homomorphism.*

A *bi-order* $\leq$ of a group G is a total order of G satisfying that if $x \leq y$, then $axb \leq ayb$ for all $a, b, x, y \in G$. A group G is called *bi-orderable* if G admits a bi-order. It is well-known that every finitely generated torsion-free nilpotent group is residually p for any prime p. Rhemtulla [14] showed that a group which is residually p for infinitely many p is bi-orderable. Together with the fact that N_k is torsion-free, we see that N_k is a bi-orderable.

In the following we fix a bi-order of N_{k-1}. We define a map $c \colon \mathbb{Z}[N_k]\setminus 0 \to \mathbb{Q}(A_k)^\times/\pm A_k$ by

$$c\left(\sum_{\delta\in N_{k-1}}\ \sum_{\gamma\in N_k, [\gamma]=\delta} a_\gamma \gamma\right) = \left[\left(\sum_{\gamma\in N_k, [\gamma]=\delta_{max}} a_\gamma\gamma\right)\gamma_0^{-1}\right],$$

where $\delta_{\max} \in N_{k-1}$ is the maximum with respect to the fixed bi-order such that for some $\gamma \in N_k$ with $[\gamma] = \delta_{max}$, $a_\gamma \neq 0$, and $\gamma_0 \in N_k$ is an element with $[\gamma_0] = \delta_{max}$. The proof of the following lemma is straightforward.

Lemma 3.7. *The map $c \colon \mathbb{Z}[N_k]\setminus 0 \to \mathbb{Q}(A_k)^\times/\pm A_k$ does not depend on the choice of γ_0 and is a monoid homomorphism.*

By the lemma we have a group homomorphism $\mathbb{Q}(N_k)^{\times}_{ab}/\pm N_k \to \mathbb{Q}(A_k)^{\times}/\pm A_k$ which maps $f\cdot g^{-1}$ to $c(f)\cdot c(g)^{-1}$ for $f,g \in \mathbb{Z}[N_k]\setminus 0$. By abuse of notation, we use the same letter c for the homomorphism. Since there is a natural section $\mathbb{Q}(A_k)^{\times}/\pm A_k \to \mathbb{Q}(N_k)^{\times}_{ab}/\pm N_k$ of d, $\mathbb{Q}(A_k)^{\times}/\pm A_k$ can be seen as a direct summand of $\mathbb{Q}(N_k)^{\times}_{ab}/\pm N_k$.

For irreducible $p,q \in \mathbb{Z}[A_k]\setminus 0$, we write $p \sim q$ if there exists $a \in A_k$ such that $p = \pm a\cdot q$. Since $\mathbb{Z}[A_k]$ is a unique factorization domain, every $x \in \mathbb{Q}(A_k)^{\times}/\pm A_k$ can be written as $x = \prod_{[p]}[p]^{e_{[p]}}$, where $e_{[p]}$ is a uniquely determined integer. We have an isomorphism $e\colon \mathbb{Q}(A_k)^{\times}/\pm A_k \to \oplus_{[p]}\mathbb{Z}$ defined by $e(x) = \sum_{[p]} e_{[p]}$. Thus we obtain a homomorphism $e\circ c\circ \tau_k\colon C_{g,1}[k] \to \oplus_{[p]}\mathbb{Z} = \mathbb{Z}^{\infty}$.

4. Construction and computation

In this section we systematically construct the images of $e\circ c\circ\tau_k\colon C_{g,1}[k] \to \mathbb{Z}^{\infty}$.

For nontrivial $\gamma \in \pi$ and a tame knot $K \subset S^3$, we construct a homology cylinder $M(\gamma,K)$ as follows. Let $* \in \Sigma$ be the base point for π. We choose a smooth path $f\colon [0,1] \to \Sigma$ representing γ such that $f^{-1}(*) = \{0,1\}$, and define $\tilde{f}\colon [0,1] \to \Sigma\times[0,1]$, $h\colon [0,1] \to \Sigma\times[0,1]$ by $\tilde{f}(t) = (f(t),t)$ and $h(t) = (*, 1-t)$. After pushed into the interior, $\tilde{f}\cdot h$ determines a tame knot $J \subset \mathrm{Int}(\Sigma\times[0,1])$. Let E_J be the complement of an open tubular neighborhood Z of J. We take a framing of J so that a meridian of J represents the conjugacy class of the generator of the kernel of $\pi_1\partial Z \to H_1(\Sigma\times[0,1])$ compatible with the orientation of J and that a longitude of J represents the conjugacy class of the image of γ by $(i_-)_*\colon \pi \to \pi_1 E_J$. Let E_K be the exterior of K. Now $M(\gamma,K)$ is the result of attaching $-E_K$ to E_J along the boundaries so that a longitude and a meridian of K correspond to a meridian and a longitude of J respectively.

Lemma 4.1. *For all nontrivial $\gamma \in \pi$ and all knots $K \subset S^3$, $M(\gamma,K) \in C^{irr}_{g,1} \cap (\cap_k C_{g,1}[k])$.*

Proof. If K is a trivial knot, then $M(\gamma,K)$ is the unit of $C_{g,1}$ for all nontrivial $\gamma \in \pi$, and there is nothing to prove. In the following we assume that K is nontrivial.

Since E_J and E_K are both irreducible and ∂Z and ∂E_K are both incompressible, $M(\gamma,K)$ is also irreducible.

Extending a degree 1 map $(E_K,\partial E_K) \to (Z,\partial Z)$ by the identity map on E_J, we have $f\colon M(\gamma,K) \to \Sigma\times[0,1]$. The following commutative diagram of isomorphisms shows that $M(\gamma,K) \in C_{g,1}[k]$ for all k:

$$\begin{tikzcd}
 & \pi_1 M(\gamma,K)/(\pi_1 M(\gamma,K))_k \arrow[d, "f_*"] & \\
N_k \arrow[ur, "(i_+)_*"] \arrow[dr, "(i_-)_*"'] & & N_k \arrow[ul, "(i_+)_*"'] \arrow[dl, "(i_-)_*"] \\
 & \pi_1(\Sigma\times[0,1])/(\pi_1(\Sigma\times[0,1]))_k &
\end{tikzcd}$$

□

Proposition 4.2. *Let $\gamma \in \pi_k\setminus 1$. Then $\tau_{k+1}(M(\gamma,K)) = [\Delta_K(\gamma)]$ for all K.*

Proof. We have the following short exact sequences of twisted chain complexes:

$$0 \to C^{\rho_k}_*(\partial E_K) \to C^{\rho_k}_*(E_J, i_+(\Sigma)) \oplus C^{\rho_k}_*(E_K) \to C^{\rho_k}_*(M(\gamma,K), i_+(\Sigma)) \to 0,$$

$$0 \to C^{\rho_k}_*(\partial Z) \to C^{\rho_k}_*(E_J, i_+(\Sigma)) \oplus C^{\rho_k}_*(Z) \to C^{\rho_k}_*(\Sigma\times[0,1], i_+(\Sigma)) \to 0,$$

where all the coefficients are understood to be $\mathbb{Q}(N_k)$. It is easily checked that

$$H_*^{\rho_k}(\partial E_K; \mathbb{Q}(N_k)) = H_*^{\rho_k}(E_K; \mathbb{Q}(N_k)) = H_*^{\rho_k}(\partial Z; \mathbb{Q}(N_k)) = H_*^{\rho_k}(Z; \mathbb{Q}(N_k)) = 0.$$

Therefore by the homology long exact sequences

$$H_*^{\rho_k}(E_J, i_+(\Sigma); \mathbb{Q}(N_k)) = 0.$$

Considering multiplicativity of Reidemeister torsion in the above exact sequences we obtain

$$\tau_{\rho_k}(E_J, i_+(\Sigma)) \cdot \tau_{\rho_k}(E_K) = \tau_{\rho_k}(\partial E_K) \cdot \tau_{\rho_k}(M(\gamma, K), i_+(\Sigma)),$$
$$\tau_{\rho_k}(E_J, i_+(\Sigma)) \cdot \tau_{\rho_k}(Z) = \tau_{\rho_k}(\partial Z) \cdot \tau_{\rho_k}(M(id), i_+(\Sigma)).$$

Here

$$\tau_{\rho_k}(E_K) = [\Delta_K(\gamma)(\gamma - 1)^{-1}],$$
$$\tau_{\rho_k}(Z) = [(\gamma - 1)^{-1}],$$
$$\tau_{\rho_k}(\partial E_K) = \tau_{\rho_k}(\partial Z) = \tau_{\rho_k}(\Sigma \times [0, 1], i_+(\Sigma)) = 1,$$

which are easy to check. Now these equations give the desired formula. □

Recall that for every monoid S, there exists a monoid homomorphism $g: S \to \mathcal{U}(S)$ to a group $\mathcal{U}(S)$ satisfying the following: For every monoid homomorphism $f: S \to G$ to a group G, there exists a unique group homomorphism $f': \mathcal{U}(S) \to G$ such that $f = f' \circ g$. By the universality $\mathcal{U}(S)$ is uniquely determined up to isomorphisms. Finally, using the homomorphism $e \circ c \circ \tau_k: C_{g,1}[k] \to \mathbb{Z}^\infty$, we give another proof of the following theorem which is a direct corollary of Sakasai's.

Theorem 4.3 ([16, Corollary 6.16]). *The group $\mathcal{U}(C_{g,1}^{irr} \cap C_{g,1}[k])$ has an abelian group quotient of infinite rank for $k \geq 2$.*

Proof. Let $\gamma \in \pi_{k-1} \setminus \pi_k$ and let $K \subset S^3$ be a tame knot. By Lemma 4.1 we see $M(\gamma, K) \in C_{g,1}^{irr} \cap C_{g,1}[k]$. By Proposition 4.2 we have

$$c \circ \tau_k(M(\gamma, K)) = [\Delta_K(\gamma)].$$

Since it is well-known that for any $p \in \mathbb{Z}[t, t^{-1}]$ with $p(t^{-1}) = p(t)$ and $p(1) = 1$, there exists a knot $K \subset S^3$ such that $\Delta_K = p$, the image of $e \circ c \circ \tau_k: C_{g,1}^{irr} \cap C_{g,1}[k] \to \oplus_{[p]}\mathbb{Z}$ contains a submonoid isomorphic to $\mathbb{Z}_{\geq 0}^\infty$. Therefore the image of the induced map $\mathcal{U}(C_{g,1}^{irr} \cap C_{g,1}[k]) \to \mathbb{Z}^\infty$ is a free abelian group of infinite rank, which proves the theorem. □

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References

[1] J. Cha, S. Friedl and T. Kim, *The cobordism group of homology cylinders*, arXiv:0909.5580.

[2] T. D. Cochran, K. E. Orr, P. Teichner, *Knot concordance, Whitney towers and L^2-signatures*, Ann. of Math. (2) **157** (2003) 433–519.

[3] S. Garoufalidis and J. Levine, *Tree-level invariants of three-manifolds, Massey products and the Johnson homomorphism*, Graphs and patterns in mathematics and theoretical physics, Proc. Sympos. Pure Math. **73**, Amer. Math. Soc., Providence, RI (2005) 173–203.

[4] M. Goussarov, *Finite type invariants and n-equivalence of 3-manifolds*, C. R. Acad. Sci. Paris Ser. I Math. **329** (1999) 517–522.
[5] H. Goda and T. Sakasai, *Abelian quotients of monoids of homology cylinders*, Geom. Dedicata **151** (2011) 387–396.
[6] K. Habiro, *Claspers and finite type invariants of links*, Geom. Topol. **4** (2000) 1–83.
[7] K. Habiro and G. Massuyeau, *From mapping class groups to monoids of homology cobordisms: a survey*, arXiv:1003.2512.
[8] T. Kitayama, *Homology cylinders of higher-order*, arXiv:1108.5903.
[9] J. Levine, *Homology cylinders: an enlargement of the mapping class group*, Algebr. Geom. Topol. **1** (2001) 243–270.
[10] J. Milnor, *Whitehead torsion*, Bull. Amer. Math. Soc. **72** (1966) 358–426.
[11] S. Morita, *Abelian quotients of subgroups of the mapping class group of surfaces*, Duke Math. J. **70** (1993) 699–726.
[12] S. Morita, *Symplectic automorphism groups of nilpotent quotients of fundamental groups of surfaces*, Groups of diffeomorphisms, Adv. Stud. Pure Math. **52**, Math. Soc. Japan, Tokyo (2008) 443–468.
[13] D. Passman, *The algebraic structure of group rings*, Wiley, New York (1977).
[14] A. H. Rhemtulla, *Residually F_p-groups, for many primes p, are orderable*. Proc. Amer. Math. Soc. **41** (1973) 31–33.
[15] T. Sakasai, *Higher-order Alexander invariants for homology cobordisms of a surface*, Intelligence of low dimensional topology 2006, Ser. Knots Everything **40**, World Sci. Publ., Hackensack, NJ (2007) 271–278.
[16] T. Sakasai, *The Magnus representation and higher-order Alexander invariants for homology cobordisms of surfaces*, Algebr. Geom. Topol. **8** (2008) 803–848.
[17] T. Sakasai, *A survey of Magnus representations for mapping class groups and homology cobordisms of surfaces*, arXiv:1005.5501.
[18] J. Stallings, *Homology and central series of groups*, J. Algebra **2** (1965) 170–181.
[19] V. Turaev, *Introduction to combinatorial torsions*, Lectures in Mathematics, ETH Zürich (2001).

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