

数理解析研究所講究録 1797

RIMS 共同研究

Introductory Workshop on  
Feynman Path Integral and Microlocal Analysis

京都大学数理解析研究所

2012年6月

*RIMS Kôkyûroku 1797*

*Introductory Workshop on  
Feynman Path Integral and Microlocal Analysis*

*June 21 ~24, 2011*

*edited by Naoto Kumano-go, Susumu Yamazaki,*

*Byoung Soo Kim and Yasuo Chiba*

*June, 2012*

*Research Institute for Mathematical Sciences*

*Kyoto University, Kyoto, Japan*

This is a report of research done at the Research Institute for Mathematical Sciences, Kyoto University. The papers contained herein are in final form and will not be submitted for publication elsewhere.

## PREFACE

This volume contains a set of introductory articles which were contributed by the invited speakers of the RIMS joint research workshop: “Introductory Workshop on Feynman Path Integral and Microlocal Analysis”, which was held at Research Institute for Mathematical Sciences, Kyoto University in June 21–24, 2011. The aims of this workshop were to overview the present stage of Japanese and Korean research in Feynman path integral and microlocal analysis and to obtain prospects for the future research. In order to do so, all the speakers gave the introductory lectures about their research fields, which contributed to capturing a whole picture of the current progress in the field. The editors would like to thank all the speakers who submitted their papers to this volume and also all those participants who attended the workshop.

Naoto KUMANO-GO

Susumu YAMAZAKI

Byoung Soo KIM

Yasuo CHIBA

This joint research is supported by RIMS and JSPS KAKENHI(C)21540196 and (C)20540191. We would like to express our sincere gratitude to RIMS and, in particular, to the secretary staffs of RIMS.

RIMS Joint Research  
**Introductory Workshop on  
Feynman Path Integral and Microlocal Analysis**

Organizers: Naoto Kumano-go (Kogakuin University)  
Byoung Soo Kim (Seoul National University of Science and Technology)  
Susumu Yamazaki (Nihon University)  
Yasuo Chiba (Tokyo University of Technology)

Date: from June 21 (Tuesday) to June 24 (Friday), 2011

Venue: RIMS, Kyoto University, Room No. 111

The following program is tentative. We can get the present version of the program from the URL: <http://www.ns.kogakuin.ac.jp/~ft24343/workshop2011-6>

**June 21 (Tuesday)**

10:00~11:00 **Takashi Ichinose** (Kanazawa University)

Imaginary-time path integrals for three magnetic relativistic Schrödinger operators,  
Part 1

11:20~12:20 **Bong Jin Kim** (Daejin University)

A note on the integral transforms on function spaces, Part 1

14:00~15:00 **Takeyuki Hida** (Nagoya University)

White noise approach to Feynman path integrals, Part 1

15:20~16:20 **Kun Sik Ryu** (Hannam University)

Introduction to the analogue of Wiener measure space and its applications, Part 1

**June 22 (Wednesday)**

10:00~11:00 **Daisuke Fujiwara** (Gakushuin University)

Stationary phase method for oscillatory integrals over a space of large dimension,  
Part 1

11:20~12:20 **Bong Jin Kim** (Daejin University)

A note on the integral transforms on function spaces, Part 2

14:00~15:00 **Takeyuki Hida** (Nagoya University)

White noise approach to Feynman path integrals, Part 2

15:20~16:20 **Kun Sik Ryu** (Hannam University)

Introduction to the analogue of Wiener measure space and its applications, Part 2

**June 23 (Thursday)**

10:00~11:00 **Daisuke Fujiwara** (Gakushuin University)

Stationary phase method for oscillatory integrals over a space of large dimension,  
Part 2

11:20~12:20 **Dong Hyun Cho** (Kyonggi University)

A survey of an analogue of conditional analytic Feynman integrals on a function space,  
Part 1

14:00~15:00 **Byoung Soo Kim** (Seoul National University of Science and Technology)

Introduction to Feynman's operational calculi for noncommuting operators, Part 1

15:20~16:20 **Naoto Kumano-go** (Kogakuin University)

Phase space Feynman path integrals by time slicing approximation, Part 1

**June 24 (Friday)**

10:00~11:00 **Takashi Ichinose** (Kanazawa University)

Imaginary-time path integrals for three magnetic relativistic Schrödinger operators,  
Part 2

11:10~12:10 **Dong Hyun Cho** (Kyonggi University)

A survey of an analogue of conditional analytic Feynman integrals on a function space,  
Part 2

14:00~15:00 **Byoung Soo Kim** (Seoul National University of Science and Technology)

Introduction to Feynman's operational calculi for noncommuting operators, Part 2

15:20~16:20 **Naoto Kumano-go** (Kogakuin University)

Phase space Feynman path integrals by time slicing approximation, Part 2

This joint research is supported by RIMS and JSPS KAKENHI(C)21540196 and (C)20540191.

Introductory Workshop on Feynman Path Integral and Microlocal Analysis  
RIMS 共同研究報告集

2011年6月21日～6月24日

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