数理解析研究所講究録
第 1799 巻 2012 年 195-202

2011 年度冬の LA シンポジウム [S11]

Pattern Formation by Fully Asynchronous Mobile Robots

Nao Fujinaga * Yukiko Yamauchi* Shuji Kijima* Masafumi Yamashita *

Abstruct

This paper considers a system R of anonymous mobile robots, each represented by a point in 2D Euclidean space. A robot, given an algorithm, repeats a "Look–Compute–Move" cycle, to observe the other robots' positions (in Look), to compute the next position by using the algorithm (in Compute), and to move toward the next position (in Move). The robots are anonymous in the sense that they do not have identifiers, and are controlled by the same algorithm. A basic and crucial assumption on the system is that they are not aware of the global coordinate system, and all the actions by robots are via their local coordinate systems, which may be inconsistent each other.

The problem of forming a given pattern by a set of mobile robots is called the pattern formation problem. Suzuki and Yamashita proposed *oblivious* algorithms with which robots' actions depend only on latest observation. They characterized, for any pattern, a necessary and sufficient condition on initial positions for the pattern formation both in oblivious and non-oblivious case, showing they are equivalent in Semi-Synchronous model [5].

This paper is concerned with pattern formation by fully-asynchronous (i.e., CORDA [3]) oblivious robots, and we present a pattern formation algorithm ψ_P for any pattern P, with which robots form P if initial position of the robots do not have a kind of symmetry.

1 Introduction

Autonomous mobile robot In autonomous mobile robots model, we consider a system consisting of anonymous mobile robots. A robot, given an algorithm, repeats a "Look–Compute–Move" cycle, to observe the other robots' positions (in Look phase), to compute the next position by using the algorithm (in Compute phase), and to move toward the next position (in Move phase). The robots are anonymous in the sense that they do not have identifiers (and are not identified just by their looks neither), and are controlled by the same algorithm. A basic and crucial assumption on the system is that they are not aware of the global coordinate system, and all the actions by robots are via their local coordinate systems, which may be inconsistent each other.

The problem of forming a given pattern by a set of mobile robots is called the pattern formation problem and has been studied extensively in various observation and synchronous models. Notably, Suzuki and Yamashita considered *oblivious* algorithms with which robots' action depend only on latest observation. They characterized for any pattern, a necessary and sufficient condition on initial positions for the pattern formation both in oblivious and non-oblivious case, showing they are equivalent with a mild synchronous assumption, i.e., semi-synchronous model (refer to section 2 for its detail).

Contribution This paper is concerned with the pattern formation by fully-asynchronous oblivious robots. We, for any pattern, give a sufficient condition on initial positions for the pattern formation. That is, any pattern is formable if initial position of robots does not have a kind of symmetry.

Organization This paper is organized as follows: In section 2, we introduce models of the robot system and summarize known results about the pattern formation problem. In section 3, we define the "clockwise matching", which plays key role in our pattern formation algorithm ψ_P. The definition of ψ_P and its correctness is presented in section 4.

*Department of Informatics, Graduate School of Information Science and Electrical Engineering, Kyushu University, Fukuoka 819-0395, Japan

2 Preliminaries

In the literatures, various communication and synchronous models are considered for autonomous mobile robots model, while the basic assumptions are shared. In this section, we briefly overview the various communication (i.e., observation) and synchronous models discussed in the literatures and summarize their results. After that, we provide our result.

Terminology Before, taking up the main subject, we develop terminologies we use throughout this paper. Let $\mathbb{N}$ denotes the set $\{0, 1, \dots\}$ of natural numbers, $\mathbb{R}$ denotes the set of real numbers, and $\mathbb{C}$ denotes the set of complex numbers with imaginary unit j. For $x+jy \in \mathbb{C}$, let $|x+jy| = \sqrt{x^2+y^2}$, i.e., for $p \in \mathbb{C}$, $|p|$ is the Euclidean norm of p. For $re^{j\theta} \in \mathbb{C}$, let $\arg(re^{j\theta}) = \theta$. We consider $\mathbb{C}$ to be ordered by: for $re^{j\theta}, r'e^{j\theta'} \in \mathbb{C}$,

- if $r < r'$ then $re^{j\theta} < r'e^{j\theta'}$, and
- if $r = r'$ and $\theta < \theta'$ then $re^{j\theta} < r'e^{j\theta'}$.

For $p, q \in \mathbb{C}$, let $\overline{pq} = \{p + t(q-p) : t \in \mathbb{R}, 0 \le t \le 1]\}$, $\overrightarrow{pq} = \{p + t(q-p) : t \in \mathbb{R}, 0 \le t\}$. Let

$$E_2^+ = \{f_{p,q} : p, q \in \mathbb{C}, |p| = 1\},$$
$$D_2^+ = \{f_{p,q} : p, q \in \mathbb{C}, |p| > 1\}$$

where $f_{p,q}(x) = px + q$. Let $\mathcal{P}_n = \{P : P \subset \mathbb{C}, |P| = n\}$. An element of $\mathcal{P}_n$ is called a *pattern*. A bijection (i.e., a matching) $M : A \to B$ is identified with the set $\{(a, f(a)) : a \in A\}$. For $A, B \in \mathcal{P}_n$, let $\mathcal{U}(A, B)$ denotes the set of all bijection from A to B, and for $M \in \mathcal{U}(A, B)$, let $d(M) = \sum_{(a,b)\in M} |a - b|$. For $A, B \in \mathcal{P}_n$, let $d(A, B) = \min\{d(M) : M \in \mathcal{U}(A, B)\}$. For $P \in \mathcal{P}_n$, let $c(P) = \sum P/n$ i.e., $c(P)$ is the center of P.

Autonomous mobile robot model In the model, we consider a system consisting of n robots $r_1, r_2, \dots, r_n$ in 2D Euclid space $\mathbb{C}$. Let $r_i(t)$ denotes the position of r_i at time t on the global coordinate system, (hence, $r_i(t) \in \mathbb{C}$,) and let $R(t) = \{r_1(t), r_2(t), \dots, r_n(t)\}$. We describe the *local coordinate system* of r_i at time t by some transformation $Z_{i,t}$ on $\mathbb{C}$. (See the next paragraph for the detail of local coordinate systems.)

Given an algorithm ψ, each robot r_i, repeats a "Look–Compute–Move" cycle:

Look: to observe the other robots' positions:

$$Z_{i,t}(R(t)) = \{Z_{i,t}(r_1(t)), Z_{i,t}(r_2(t)), \dots, Z_{i,t}(r_n(t))\}$$

of current time t via $Z_{i,t}$, and

Compute: to compute, by ψ, the next position:

$$q = \psi\left(Z_{i,t_0}(R(t_0)), Z_{i,t_1}(R(t_1)), \dots, Z_{i,t_m}(R(t_m))\right)$$

where $t_0 < t_1 < \cdots < t_m = t$ are the times when r_i performed Look so far, and

Move: to move directly toward the next position $Z_{i,t}^{-1}(q)$ of global coordinate system (which correspond to q of r_i's local coordinate system).

The system is *distributed* in the sence that 1) they are not aware of the global coordinate system, and all the observations by robots are via their local coordinate systems, which may be inconsistent each other, and 2) the timing of each Look, Compute and Move performed by robots may be inconsistent each other; the system is asynchronous. The detail of the observation and synchronous models are discussed in the following paragraphs. Moreover, the system is *anonymous* in the sense that they do not have identifiers (and are not identified just by their looks neither), and are controlled by the same algorithm.

Observation models Here, we introduce *local coordinate system*. We assume that a robot observes other robots' positions on its own local coordinate system. Let G be some group of transformation on 2D Euclid plane. A local coordinate system of r_i at time t is expressed by a transformation $Z_{i,t} \in G$ such that $Z_{i,t}(r_i(t)) = 0$; a robot r_i is located on the origin of its own local coordinate system. According to observation models, we consider a different group G as the set of possible local coordinate systems, and call it G-observation model. In G-observation model we always consider 1) a worst local coordinate system in G for each robot to observe other robots' positions, and 2) two patterns $P, P' \in \mathcal{P}_n$ to be similar if there exists $Z \in G$, such that $Z(P) = P'$, and write $P \simeq_G P'$, since there is no way the robots can distinguish two pattern P, P' such that $P \simeq_G P$.

Synchronous models As for the asynchrony, three kinds of synchronous models have been discussed in the literatures. A (fully) synchronous robot synchronously executes a Look–Compute–Move cycle; all robots simultaneously start and finish the Look, Compute and Move phases in each iteration, and they always reach their next positions (computed in their Compute phases) in their Move phases. We call this robot model F-synchronous [4].

An asynchronous (or CORDA [3]) robot, on the other hand, asynchronously executes a Look–Compute–Move cycle. Moreover, a Move phase may finish when a robot is still on the way to its next position. To give a lower bound on robots' mobility, we assume the existence of the minimum movable distance $\epsilon > 0$; every robots can move at least ϵ length in every Move phase. We call this robot model A-synchronous.

Finally, a semi-synchronous robot is the same as an asynchronous robot, except that Look and Move phases of two robots never overlap, or informally, no robots observe other robots moving. Furthermore, a semi-synchronous robot knows its own minimum movable distance. We call this robot model S-synchronous [4].

Note that in each model, adversarial fair schedular are assumed; we always consider a worst schedule for each model on condition that, in the schedule, every robot performs Look–Compute–Move cycle at least once for large enough time span.

In Figure 1, each of FSYNCH, SSYNCH and ASYNCH provides an instance of schedules of Look–Compute–Move by three robots, each representing F, S and A-synchronous model, respectively.

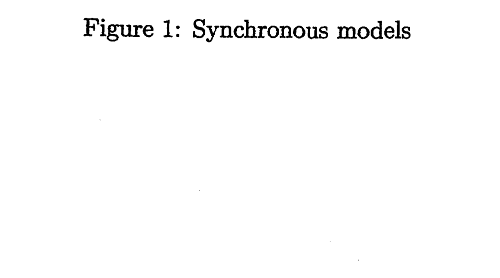

Figure 1: Synchronous models

Pattern formation problem The problem of forming a given pattern P comprising of n points by n mobile robots is called the pattern formation problem; we want to design a pattern formation algorithm ψ_P with which the robots form the given pattern P in finite time from any initial positions. Especially, we are focused on the oblivious algorithm; an algorithm is said to be oblivious if

$$\psi(P_0, P_1, \ldots, P_m) = \psi(P_m) , \tag{1}$$

i.e., robots' move only depend on the latest observation. The reason behind it is: oblivious algorithms for the formation of P tolerate any temporal crash failure (in which robots lost its memory and stop for finite duration), granting the system self-stabilizing property. However, such formation algorithms (even non-oblivious one) does not exist in general. The following Example provides such an instance.

Example Let us consider A-synchronous E_2^+-observation robots with right triangler initial position. Given any algorithm ψ, since our robots follow the same algorithm ψ, by letting the adversary always choose symmetric local coordinate systems as well as fully synchronous schedule, we can construct valid execution of ψ which never form a given target pattern, whenever the target pattern is not right triangle itself. □

This paper is concerned with pattern formation by oblivious A-synchronous E_2^+-observation robots. To be more precise, we define a set of possible executions $\mathrm{Ex}(\psi, \mathcal{I})$ of an algorithm $\psi : \mathcal{P}_n \to \mathbb{C}$ [1] from a set $\mathcal{I} \subseteq \mathcal{P}_n$ of initial positions in the model:

Definition 1. $\mathrm{Ex}(\psi, \mathcal{I})$ *is a set of* $R : \mathbb{N} \to \mathcal{P}_n$ *such that: for all* $i \in \{1, 2, \ldots, n\}$, *there exists infinite* t_k*s* $(0 = t_0 < t_1 < \ldots)$ *such that,*

1. *for all* $k \in \mathbb{N}$ *and* $t \in [t_k, t_{k+1})$, $r_i(t+1) - r_i(t) = a(d_{i,k} - r_i(t))$ *with some* $a \in [0, 1]$ *and,*
2. *there exists* $\epsilon > 0$ *such that* $|r_i(t_{k+1}) - r_i(t_k)| \geq \min\{|d_{i,k} - r_i(t_k)|, \epsilon\}$ *for all* $k \in \mathbb{N}$,

where $Z_{i,k} \in E_2^+$, $Z_{i,k}(r_i(t_k)) = 0$, $R(0) \in \mathcal{I}$ *and,*

$$d_{i,k} = \begin{cases} r_i(0) & k = 0 \\ Z_{i,k}^{-1} \circ \psi\left(Z_{i,k}(R(t_k))\right) & \textit{otherwise} . \end{cases} \tag{2}$$

[1] Only oblivious algorithms are considered

Furthermore, we define the formation of a pattern $P \in \mathcal{P}_n$ as follows:

Definition 2. *An algorithm ψ forms a pattern P from initial positions $\mathcal{I}$ in A-synchronous E_2^+-observation model, if for all $R \in \mathrm{Ex}(\psi, \mathcal{I})$, there exists t_0 such that $R(t) \simeq_{E_2^+} P$ for all $t > t_0$. P is formable from $\mathcal{I}$ by A-synchronous E_2^+-observation robots, if such algorithms exist.*

Since there is no algorithm ψ_P which forms given pattern P from all initial positions in general as we saw in Example, in this paper, we are concerned with a minimal assumption on initial position of robots for the formation of a pattern P. In order to characterize the symmetry, we define the *symmetricity* $\rho(P)$ of $P \in \mathcal{P}_n$, by:

$$\rho'(P) = \#\{Z \in E_2^+ : P = Z(P)\}$$

$$\rho(P) = \begin{cases} 1 & \rho'(P) > 1 \wedge \exists p \in P : p = c(P) \\ \rho'(P) & otherwise \end{cases}$$

and remark the following theorem.

Theorem 1. *A pattern P is formable from initial positions $\mathcal{I}$ by A-synchronous E_2^+-observation robots, only if $\rho(I)$ divides $\rho(P)$ for all $I \in \mathcal{I}$.*

2.1 Related works

Known results about the pattern formation problem are summarized as follows: Assume $n > 2$.

1. A pattern P is formable from initial positions $\mathcal{I}$ by D_2^+-observation F-synchronous robots (and hence by S and A-synchronous robots), only if $\rho(I)$ divides $\rho(P)$ for all $I \in \mathcal{I}$ [4, 5].

2. A pattern P is formable from initial positions $\mathcal{I}$ by D_2^+-observation A-synchronous non-oblivious robots (and hence for S and F-synchronous non-oblivious robots), if $\rho(I)$ divides $\rho(P)$ for all $I \in \mathcal{I}$ [2].

3. A pattern P is formable from initial positions $\mathcal{I}$ by D_2^+-observation S-synchronous oblivious robots (and hence for F-synchronous robots), if $\rho(I)$ divides $\rho(P)$ for all $I \in \mathcal{I}$ [5],

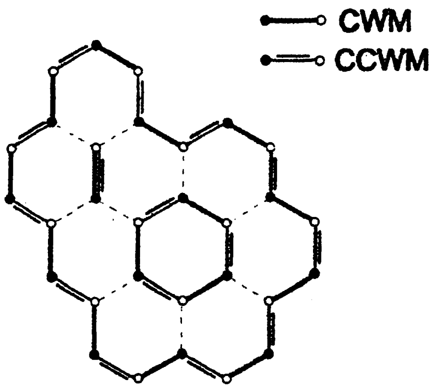

Figure 2: An example of the clockwise matching.

4. A pattern P is formable from initial positions $\mathcal{I}$ by D_2^+-observation A-synchronous oblivious robots, if $\rho(I) = 1$ for all $I \in \mathcal{I}$ [1]. [2]

2.2 Result

This paper is concerned with the pattern formation by A-synchronous E_2^+-observation robots. The contribution of this paper is summarized by the following theorems.

Theorem 2. *A pattern P ($|P| > 2$) is formable from initial positions $\mathcal{I}$ by oblivious A-synchronous E_2^+-observation robots if $\rho'(I) = 1$ for all $I \in \mathcal{I}$.*

The "clockwise matching" is defined in the following section, which plays key role in our pattern formation algorithm ψ_P. ψ_P and its correctness is discussed in section 4.

3 Clockwise matching

In this section, we define the "clockwise matching" CWM(A, B) of two patterns $A, B \in \mathcal{P}_n$, which plays a key role in our algorithm.

We consider a set $\mathcal{M}(A,B) = \arg\min_M\{d(M) : M \in \mathcal{U}(A,B), \forall(a,b),(c,d) \in M, \overline{ab} \not\subset \overline{cd}\}$ of matchings. Note that as an element of $\mathcal{M}(A,B)$, we do not allow matching whose edge includes

[2] The algorithm considered in [1] to obtain the result require to robots to move along some curves while robots in our model only moves directly to calculated point.

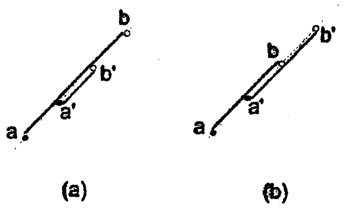

Figure 3:

its another edge as in Figure 3 (a), while allowing parallel edges as in Figure 3 (b). See that $\mathcal{M}(A,B) \neq \emptyset$, but not necessarily $|\mathcal{M}(A,B)| = 1$. The clockwise matching defines a canonical matching in $\mathcal{M}(A,B)$.

In section 3.1, a partial order ($\preceq$) is defined on $\mathcal{M} := \mathcal{M}(A,B)$. Intuitively, for $M, M' \in \mathcal{M}$, $M \preceq M'$ means that M is "closer to clockwise" than M'. Moreover, the following proposition (which will be shown in section 3.2) states there is the clockwise matching in $\mathcal{M}$.

Claim 1. *($\mathcal{M}, \preceq$) have the least element.*

Thus, the clockwise matching $\mathrm{CWM}(A,B)$ of two patterns $A, B \in \mathcal{L}_n$ is defined by:

$$\mathrm{CWM}(A,B) = \min \mathcal{M}(A,B) ,$$

where minimum is taken with respect to ($\preceq$).

3.1 Definition of ($\preceq$) on $\mathcal{M}$

Throughout this section, we consider a bipartite graph G with its vertex set $V = A \cup B$ ($A, B \in \mathcal{L}_n$) and edge set $E = \bigcup \mathcal{M}$. We draw vertices of A with black and vertices of B with white in Figures 2-. For a plane graph G, $P(G)$ expresses the set of all edges incident to G's exterior face.

Let us assume G to be a plane bipartite (by replacing each edge (a,b) with a line $\overline{ab}$). Then, as illustrated in Figure 4, for its two matchings $M, M' \in \mathcal{M}$, any edge $e \in M \oplus M'$ can be classified either as CW or CCW. Thus we define ($\preceq$) on $\mathcal{M}$ by:

Definition 3. $M \preceq M'$ $\overset{\text{def}}{\iff}$ *for all $e \in P(M \oplus M') \cap M$, e is clockwise.*

Indeed, we can regard G to be plane graph as follows:

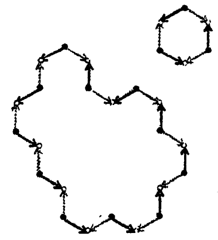

Figure 4: CW edges and CCW edges

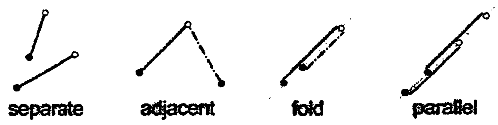

Figure 5: Possible relation between two edges of G

Lemma 1. *For any two edges $x = (a,b)$ and $y = (a',b')$ of G, either of the following holds: (See Figure 5 for illustration).*

- *(adjacent) x and y share exactly one end vertex v,*
  - *(simple) $\overline{x} \cap \overline{y} = \{v\}$.*
  - *(fold) not simple.*
- *(not adjacent) $a \neq a'$ and $b \neq b'$,*
  - *(separate) $\overline{x} \cap \overline{y} = \emptyset$.*
  - *(parallel) a, a', b, b' reside on one line in the order.*

Proof. By triangle inequality. □

We consider a cycle $C = e_1 e_2 \ldots e_m$ of G. Let's see, what kind of graph we can draw on the plane

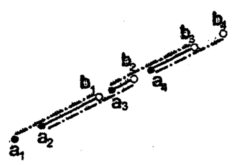

Figure 6: An example of a folded-path.

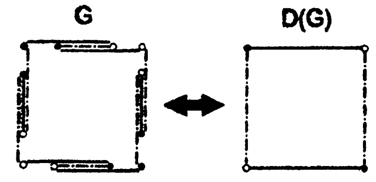

Figure 7: An example of G and $D(G)$.

as C. By Lemma 1, e_i and e_{i+1} is either (fold) or (simple). If it is (fold), the next edge e_{i+2} must be (parallel) with e_i. Then, for the next edge e_{i+3}, you can choose (simple) or (fold). However in case you choose (fold) you have to be careful not to draw the line too long and include e_i, and so on, and of course any of two edges could never cross each other.

With that observation, let us define the plane graph representation $D(G)$ of G as follow. We call an alternating path $a_1b_1 \ldots a_mb_m$ of G which satisfy $a_{i+1} \in \overline{a_ib_i}$ and $b_i \in \overline{a_{i+1}b_{i+1}}$ for all $i = 1 \ldots m-1$, a *folded-path*. Any edge is a folded-path with length 1. A *maximal folded-path* is a folded-path, which by extending the path with one more vertex, no longer holds above condition. $D(G)$ is a plane graph which is produced by replacing each maximal folded-path aPb of G with a line $\overline{ab}$. See that those two lines never intersect with each other except for end points and for any perfect matching of $D(G)$, there is a corresponding perfect matching of G since each aPb is an alternating path without branch from inner vertices. With this correspondence, we regard G to be a plane graph itself.

3.2 Proof of Claim 1

In this section, we prove Claim 1. First, we show that $\mathcal{M}$ is indeed partially ordered by $(\preceq)$.

Lemma 2. *$(\mathcal{M}, \preceq)$ is a partially ordered set i.e.,*

1. *$M \preceq M$ for all $M \in \mathcal{M}$,*
2. *$M \preceq M' \wedge M' \preceq M \Rightarrow M = M'$, for all $M, M' \in \mathcal{M}$, and*
3. *$L \preceq M \wedge M \preceq R \Rightarrow L \preceq R$, for all $L, M, R \in \mathcal{M}$.*

Proof. Reflexivity and antisymmetry is clear. We say transitivity. First we argue that for all $e \in P(L \cup M \cup R)$, $e \in L$ or $e \in R$. Let $e \in M \setminus (L \cup R)$, and consider the following cases.

- Case $e \in P(L \cup M)$ and $e \in P(M \cup R)$: By assumption, e is CW in $P(M \cup R)$ and CCW in $P(L \cup M)$. This contradict with $e \in P(L \cup M \cup R)$ and $e \in M \setminus (L \cup R)$.
- Case $e \notin P(L \cup M)$: e is in the inner face of $P(L \cup M)$, contradicting with the assumption.
- Case $e \notin P(M \cup R)$: Same as above.

Thus, $P(L \cup M \cup R)$ is consist of edges of L and R. Hence, if not $L \preceq R$, this contradict with $L \preceq M \preceq R$. □

Moreover, by the fact that there is a lower bound of any M and M' in $\mathcal{M}$, we obtain Claim 1.

4 Pattern formation

4.1 Algorithm ψ_F

To give constructive proof of Theorem 2, we consider the following algorithm ψ_P for any pattern P such that all points in P do not resides on single line. (When all points in P resides on single line, the formation can be easily accomplished, thus we omit the case.) The algorithm ψ_P, given input $X \in \mathcal{P}_n$, consider the set

$$C = \{\mathrm{CWM}(X, Z(P)) | Z \in E_2^+\}$$

and choose the matching M^* which minimize its cost (which is defined later) among all the candidate matchings in C, and according to the matching we let exactly one of robots move to its matched position. We guarantee that the same matching will be chosen even after the robot moved, by the fact that the cost of the matching decreased by the move is more than that of any other matchings.

Here, we consider the points of given pattern to have coordinates as well as IDs. That is, we consider our robots to share the coordinates $p_1, p_2, \ldots, p_n$ such that $\{p_i\} \simeq_{E_2^+} P$. (This can be done by letting $(p_1, p_2, \ldots, p_n) = \min\{P_z : Z \in E_2^+\}$ where P_z is a vector obtained by sorting the elements of $Z(P)$ in the increasing order, and the minimum is taken with respect to the lexicographical order.)

Using this labeled pattern, we define the cost $w(M)$ of $M \in C$ by $w(M) = (w_0, w_1, w_2, \ldots, w_n)$ where $w_0 = d(M)$, and $w_1, w_2, \ldots, w_n$ is the costs of elements of M in the increasing order when the cost $w(x, Z(f_i))$ of $(x, Z(f_i)) \in M$ is defined by $w(x, Z(f_i)) = (h_i, l_i, \theta_i, i)$ with

$$\begin{aligned} l_i &= |x - Z(f_i)|, \\ \theta_i &= \arg(f_i - Z^{-1}(x)), \\ h_i &= \# \left\{ (y, Z(f_j)) \in \eta : \overrightarrow{yZ(f_j)} \subset \overrightarrow{xZ(f_i)} \right\} . \end{aligned}$$

With these costs, let $M^* = \arg\min\{w(M) : M \in C\}$ and $(a^*, b^*) = \arg\min\{w(a, b) : (a, b) \in M^*, w(a, b) \neq 0\}$. Each of the above minimums is taken with respect to the lexicographic order. Finally, $\psi_P(X) = b^*$ when $a^* = 0$, and $\psi_P(X) = 0$, otherwise.

4.2 Correctness of ψ_F

We prove Theorem 2. Let $C(t)$ denotes the candidate matchings ψ_P calculates at time t, and M_0^* denotes the matching ψ_P calculates at time 0 (in the global coordinates system). M_0^* is unique since we are assuming $\rho'(R(0)) = 1$.

First, we show that for any time t, ψ_P calculates the matching $\{(r_i(t), M_0^*(r_i(0)))\} =: M_t^*$. By the definition of the algorithm, at time 0, there is exactly one robot (let it be r_i) which moves, unless $l_j(0) = 0$ for all j (in this case the formation is complete). Moreover, we can assume there is no other robot along its way blocking its movement; if there is such a robot r_j, $h_j < h_i$ which contradict with the fact that $(r_i(0), M_0^*(r_i(0)))$ was the minimum of M_0^*. Let assume, without loss of generality, that r_i moves from time 0 to t with ϵ length. Then for all $M_t \in C(t)$ $(M_t \neq M_t^*)$, $w(M_t^*) < w(M_t)$ (thus ψ_P again choose M_t^* at time t). This is because, by the move, the cost of M^* decreases from

$$((h_{i_1}^*, l_{i_1}^*, \theta_{i_1}^*, i_1), \ldots, (h_{i_n}^*, l_{i_n}^*, \theta_{i_n}^*, i_n)) := w(M_0^*)$$

to

$$((h_{i_1}^*, l_{i_1}'^* - \epsilon, \theta_{i_1}^*, i_1), \ldots, (h_{i_n}^*, l_{i_n}^*, \theta_{i_n}^*, i_n)) := w(M_t^*)$$

while cost of other matching M decreases from

$$\begin{aligned} ((h_{s_1}, l_{s_1}, \theta_{s_1}, s_1), \ldots, (h_{s_j}, l_{s_j}, \theta_{s_j}, s_j), \ldots, \\ (h_{s_n}, l_{s_n}, \theta_{s_n}, s_n)) := w(M_0) \end{aligned}$$

to

$$\begin{aligned} ((h_{s_1}, l_{s_1}, \theta_{s_1}, s_1), \ldots, (h_{s_j}, l_{s_j} - \delta, \theta_{s_j}, s_j), \ldots, \\ (h_{s_n}, l_{s_n}, \theta_{s_n}, s_n)) := w(M_t) \end{aligned}$$

where $\delta \leq \epsilon$ (remember that r_i moves toward $M_0^*(r_i(0))$ *directly*, hence $\delta \leq \epsilon$ by the triangle inequality). Adding to the fact that M_0^* was the minimum of $C(0)$, M_t^* is again the minimum of $C(t)$. Furthermore, obviously $(r_i(t), M_t^*(r_i(t)))$ is the minimum element of M_t^*. Thus ψ_P will choose r_i again unless $l_i(t) = 0$. When $l_i(t) = 0$, inductively applying the same argument, eventually, $l_j = 0$ for all j, completing the formation.

5 Discussion and conclusion

In this paper we showed that $\rho'(I) = 1$ for all $I \in \mathcal{I}$ is sufficient for a pattern P ($|P| > 2$) to be formable from initial positions $\mathcal{I}$ by oblivious A-synchronous E_2^+-observation robots.

Although we could not provide its proof, we conjecture that $\rho(I)$ dividing $\rho(P)$ for all $I \in \mathcal{I}$ is necessary and sufficient for a pattern P ($|P| > 2$) to be formable from initial positions $\mathcal{I}$ by oblivious A-synchronous E_2^+-observation robots (as in the case of S-synchronous D_2^+-observation robots).

Indeed, we conjecture that our algorithm ψ_P form P from initial positions $\mathcal{I}$ such that for all $I \in \mathcal{I}$, $\rho'(I)$ divides $\rho'(P)$. The problem is whether or not the matching ψ_P calculates is unique even when $\rho'(I) \geq 2$ (as long as $\rho'(I)$ divides $\rho'(P)$), which can be reduced to the following seemingly positive conjecture:

Conjecture 1. *Let $A, B \in \mathcal{P}_n$ be two pattern such that*

1. *$\rho'(A) > 2$,*
2. *$\rho'(A)$ divides $\rho'(B)$, and*
3. *all points in B do not resides on single lines.*

Let $B^* \in \arg\min_{B'}\{d(A, B') : B' \simeq_{E_2^+} B\}$. Then the center of B^* coincides with that of A.

References

[1] Y. Dieudonné, F. Petit, and V. Villain : Leader election problem versus pattern formation problem. *Proc. International Symposium*

on Distributed Computing 2010 (DISC 2010), Lecture Notes in Computer Science **6343**, pp. 267–281 (2010)

[2] H. Nagamochi, M. Yamashita, and T. Ibaraki: Distributed algorithms for cooperative controlling of anonymous mobile robots. *Technical Reports of IEICE,* COMP95-24, pp. 31–40 (1995) (in Japanese)

[3] G. Prencipe: Distributed coordination of a set of autonomous mobile robots. PhD Thesis, Universita di Pisa (2002)

[4] I. Suzuki and M. Yamashita: Distributed anonymous mobile robots: Formation of geometric patterns. *SIAM Journal on Computing,* **28**, pp. 1347–1363 (1999)

[5] M. Yamashita and I. Suzuki: Characterizing geometric patterns formable by oblivious anonymous mobile robots. *Theoretical Computer Science,* **411**, pp. 2433–2453 (2010)