INTRODUCTION

In recent years, Smart Grid attracts attention as a new electric power system. The new system controls the flow of electric power from the both sides of supply and demand and enables us to use efficiently electric power with high quality. In addition, it is expected that the renewable energy such as photovoltaics (PV) leads to reduction of greenhouse gas. The Japanese government aims at a spread of photovoltaics [1] and started feed-in tariff. However, the photovoltaics will change due to a season, weather and time, so it has big influence on the other part of the electric power system. In order to reduce this instability, it has been proposed to introduce storage batteries into the grid[2, 3]. Then, it can stabilize the balance of electric power supply and demand by holding surplus power generated in photovoltaics at daytime and storing electric power with a low price at night.

This paper considers an optimal electric power management using photovoltaics and battery system. Assuming several management scenarios, we determine an optimal management plan for each scenario and discuss their costs. In section 2, we define the electric power management problem, and in section 3, we explain the mathematical model using a time-space network (TSN). Then we show the result of our analysis using practical data in section 4.
2 ELECTRIC POWER MANAGEMENT

We consider electric power management in a residential building using photovoltaic and battery system. Figure 1 shows the electric power network treated in this study. The demand of electric power in a residence differs at time, and the total supply from an external grid, PV and a battery must satisfy the demand at every time. PV generates electric energy at each time, and it can distribute the energy to three demands, i.e., the external grid, the residential demand and the battery.

In the current pricing system in Japan, the selling price is generally higher than the buying price. Therefore, everyone may consider to sell electricity by PV as much as possible, and to buy necessary electricity in a residence. This is an incentive to introduce PV, however, the essential purpose of introducing PV is local production for local consumption, not to save electricity cost for each consumer. We assume two types of regulation in the operation of delivering electricity from PV back into the external grid.

1. **Total amount type** : The delivered power should be no more than the power generated by PV. It is possible to sell the total electric energy of PV to the external grid.

2. **Surplus amount type** : The electric energy of PV is used preferentially for the residential demand. The delivered power should be no more than the surplus power.

For the battery, we have to make the regulations in charging resources and discharging timing. If there is no restriction on battery, that is, we can charge battery from external grid as well as PV, and discharge at any time, everyone may consider to charge the cheap electricity at night and to sell it during daytime by higher price, which is regardless the existence of PV. Here we consider two types of regulation in discharging from the battery:

1. **Normal type** : Discharging from the battery is not forbidden. It is always possible to discharge from the battery.

2. **Regulation type** : Discharging from the battery is forbidden while PV generates electricity.

Then it is possible to discharge only during night or a rainy day.

Figure 1: Electric Power Network in a Residence
Moreover, we assume two types of regulation in the source for charging the battery:

1. **PV and grid type**: The battery can charge from PV and the external grid.

2. **PV type**: The battery can charge from only PV.

There are several management scenarios by the combination of these constraints about PV and charging and discharging of the battery. Table 1 shows that electric power management scenarios considered in this study. Here, the scenario S1 is the most flexible management. It is possible to sell or to charge the electricity produced by PV, and also to charge the battery from grid. Since S1 has no restriction on electric power flow, S1 gives us the lowest cost of electricity usage, if we do not take into account of the cost of battery.

Comparing to S1, we consider some scenarios. S3 is added the restriction that the selling amount of electricity does not exceed the surplus. S5 is added the restriction that the discharging time is limited at night. S2 is added the restriction that the charging resource should be only PV. S4 is the current situation in Japan, where the battery is not connected to an external grid directly, and prohibited to sell to and to buy from the grid. We note that we add scenario S9 which removed the battery from the electric power network in order to verify the effect of a battery.

We also mention the unit price of electric power in Japan. When we buy electricity from the external supplier in the grid, the unit price is a fixed rate, which varies depending on the time and a day of the week. On the other hand, when we sell electricity back into the grid from PV, the unit price is fixed and is set to more expensive than buying price.

<table>
<thead>
<tr>
<th>Table 1: Electric Power Management Scenarios</th>
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</thead>
<tbody>
<tr>
<td>Discharging</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>Selling</td>
</tr>
<tr>
<td>Charging PV-Grid</td>
</tr>
<tr>
<td>Charging PV</td>
</tr>
<tr>
<td>No Battery</td>
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</tbody>
</table>

### 3 MATHEMATICAL MODEL

We explain a formulation for the electric power management using a time-space network (TSN) model. TSN extends the usual network in the direction of a time-axis. Figure 2 shows a TSN model for the electric power network in a residential building.

In this electric power network, we have to determine the amount of flows between the nodes at each time. Then, we formulate it as a mixed 0-1 integer programming problem (0-1MIP). In this formulation, we use a binary variable that denotes the existence of a flow. It enables us to describe constraints, such as the exclusion of charging and discharging operation of the battery, and the prohibition of the inflow and the outflow via the external grid at the same time. Furthermore, by using a continuous variable denotes the amount of a flow, we can describe the demand satisfied constraint. Under these constraints we consider minimizing the cost concerning electric power in a house. We show the notations of input data, decision variables and symbol of nodes as follows.
Figure 2: Time-Space Network Model for the Electric Power Network

Input data
- $p_t^b$ unit price in buying electricity at time $t$
- $p^s$ unit price in selling electricity
- $C$ capacity of the battery
- $g_{\text{max}}$ maximum electric power charged the battery
- $h_{\text{max}}$ maximum electric power discharged the battery
- $\varphi$ decay rate from AC to DC
- $\psi$ decay rate from DC to AC
- $T$ set of time periods for the management
- $E_t$ electric power generated by PV at time $t$
- $D_t$ demand in the residence at time $t$
- $w_0$ initial rate of electric energy on the battery at time 0

Decision variables
- $x_{i,j,t}$ a binary variable denoting the existence of a flow from node $i$ to node $j$ at time $t$
- $y_{i,j,t}$ a continuous variable denoting the amount of a flow from node $i$ to node $j$ at time $t$
- $w_t$ rate of electric energy on the battery at time $t$

Node symbols
- $p$ node of PV
- $c$ node of external grid
- $s$ node of battery
- $d$ node of demand

Formulation of mathematical model

Minimize

$$\sum_{t \in T} [(y_{c,d,t} + y_{c,s,t}) p_t^b - y_{p,c,t} p^s]$$

subject to

1. $y_{p,d,t} + y_{c,d,t} + \psi y_{s,d,t} = D_t$
2. $y_{p,c,t} + y_{p,d,t} + y_{p,s,t} = E_t$
3. $C w_t + \varphi (y_{p,s,t} + y_{c,s,t}) \leq C$
4. $C w_t - y_{s,d,t} \geq 0$
\[ \varphi(y_{p,s,t} + y_{c,s,t}) \leq g_{\text{max}} \]  

\[ x_{p,c,t} + x_{c,d,t} \leq 1, \]  

\[ x_{p,c,t} + x_{c,s,t} \leq 1 \]  

\[ x_{p,s,t} + x_{s,d,t} \leq 1, \]  

\[ x_{c,s,t} + x_{s,d,t} \leq 1 \]  

\[ \varphi(y_{p,s,t} + y_{c,s,t}) - y_{s,d,t} = C(w_{t+1} - w_t) \]  

\[ x_{s,d,t} \in \{0,1\}, \]  

\[ x_{p,c,t} \in \{0,1\}, \]  

\[ x_{p,d,t} \in \{0,1\}, \]  

\[ x_{p,s,t} \in \{0,1\} \]  

\[ 0 \leq w_t \leq 1 \]  

In this formulation, we consider all the constraints in \( t \in T \). The objective function (1) is the total price calculated by the amounts of electricity bought and sold via the external grid. Eq.(2) implies the residential demand should be satisfied, and eq.(3) is the distribution constraint about electric power generated by PV. Eqs.(4) and (5) show the capacity constraints of the battery with charging and discharging. Eq.(6) shows the limit of electric power in charging the battery. Eq.(7) shows the exclusion constraints of buying and selling electricity via the external grid. Eq.(8) shows the exclusion constraints of charging and discharging operations of the battery. Eq.(9) shows the conservation of electric energy of charging or discharging the battery. Eq.(10) defines binary variables for each electric flow. Eq.(11) defines the ranges for each electric flow. Eq.(12) shows the range of the charging rate of the battery.

This is a mathematical formulation for scenario S1, which is a basic model on this study and the other scenarios can be described by adding several constraints to this formulation. For example, in the scenario S3 and S4 we add the following constraint about the amount of delivered power from PV back into the external grid.

\[ \begin{cases} y_{p,c,t} = 0, & t \in T_1 = \{t \in T | E_t - D_t \leq 0\} \\ y_{p,c,t} \leq E_t - D_t, & t \in T_2 = \{t \in T | E_t - D_t > 0\} \end{cases} \]  

Eq.(13) shows the restriction on the delivered power considering the surplus of the electric power. Moreover, in the scenario S5–S8 we add the following constraint about the existence of the flow between the battery and the demand.

\[ x_{s,d,t} = 0, t \in T_3 = \{t \in T | E_t > 0\} \]  

Eq.(14) shows the prohibition of discharging from the battery while PV generating. Finally, we consider the constraint about the scenario S2, S4, S6 and S8.

\[ x_{c,s,t} = 0, t \in T \]  

Eq.(15) shows the prohibition of charging the battery from the external grid. Then the battery is supposed to charge from only PV.
4 CASE STUDY

We assume the management scenarios and evaluate each minimum cost. We note that the uncertainty about the amount of demand $D_t$ and that of PV output $E_t$ is not considered from the viewpoint of comparing the minimum costs among all the considered scenarios.

We get these data $D_t$ and $E_t$ from a real residential building for three different seasons, i.e., summer, winter and mid-term. Figure 3 shows changes for every one hour in the demand of electric energy and the supply by PV at summer.

![Figure 3: Changes in Demand of Electric Power and Supply by PV](image)

The management planning period of each season is 35 days. In each season, the unit price of buying electricity from the external grid is fixed with time and a day of the week. Table 2 shows the unit price of buying electricity.

<table>
<thead>
<tr>
<th>Day</th>
<th>Price [yen/kWh]</th>
<th>Time [h]</th>
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</thead>
<tbody>
<tr>
<td>weekday</td>
<td></td>
<td>0-7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7-10</td>
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<td>10-18</td>
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<td></td>
<td>18-24</td>
</tr>
</tbody>
</table>

Table 2: Unit Price of Buying Electricity

We have changed the unit price of selling electricity between 0 and 45 [yen/kWh] to see how each management scenario works and its minimum cost. We have used ILOG CPLEX 12.4 for solving the 0-1MIP.

We also note the other parameters about the management. The capacity of battery $C$ is 6 [kWh], and the maximum electric power of both charging and discharging the battery, $g_{\text{max}}$ and $h_{\text{max}}$, is 3 [kW], respectively. In charging and discharging the battery, electric power is decreased because of the conversion between AC and DC. The decay rate $\varphi$ from AC to DC is 0.9 and vice versa. Figures 4 and 5 show the comparison of the minimum cost over 35 days among the seven management scenarios in summer and winter.
From these figures, we obtained the following four findings:

1. The total cost decreases greatly by introducing a battery in case of a low unit price of selling electricity.
2. There is few effect of battery as the unit price goes up, when the source of charging a battery is only PV such as S2, S4, S6 and S8.
3. If there is no restriction such as S1, the reduction of total cost is remarkable in case of a high unit price of selling electricity.
4. Comparing summer and winter, the characteristics between scenarios is almost same in spite that the total cost has shifted.

In the next experimentation, we analyze the cost reduction effect of the management factors by using the analysis of variance (ANOVA). There are 4 factors, that is selling, charging, discharging and season, and they have each level as the table 3 shown.

<p>| Table 3: Management Factors and Their Levels |</p>
<table>
<thead>
<tr>
<th>Factor</th>
<th>Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Discharging</td>
<td>A1. Normal type</td>
</tr>
<tr>
<td>B. Selling</td>
<td>B1. Total amount type</td>
</tr>
<tr>
<td>C. Charging</td>
<td>C1. PV-Grid type</td>
</tr>
</tbody>
</table>

Here, we assume season's factor as an uncontrollable factor. Then we analyze the effect of the other 3 factors and their interactions by removing this factor. In this experiment, the unit price of selling is fixed at 42 yen per kWh and its price is same to current Japan.

Figure 6 show the result of ANOVA. For the factorial effect, there is a significant difference in all the factor. Especially, the factor of charging, that is factor C, shows the most significant effect of the three factors. Between PV-Grid type and PV type, there is a cost difference of 2,000 yen. Next, for the interaction effect, there is also a significant difference in all. Especially, the interaction of Discharging and Selling, that is Interaction A*B, is the most influential effect. At the level A2, that is the regulation type of discharging, there is no effect of Factor B, selling type.
5 CONCLUSION

In this study, we have considered an optimal electric power management in a residential building using PV and battery system. To get an optimal management plan, we formulate the problem as 0-1MIP through a TSN model. By solving it under several management scenarios, we show the comparison of each minimum cost. Finally, we analyze the cost reduction effect caused by the management factors by using ANOVA.

REFERENCES

