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Network generated by automaton and its fluctuation analysis

By

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Abstract. *A concept of one-direction relation is introduced and the networks which are generated by the automatons are constructed. Then we can obtain fluctuations with decreasing property. Moreover, fluctuations with power laws in molecular biology, music, language and sports can be simulated in this manner. Next we introduce analysis on a network and give its applications to the network of the one-direction relation automaton.*

Introduction

We have a lot of fluctuations with power law in various fields ([2,3,4]). The typical fluctuations can be observed in language, music and biology. Also we have such fluctuations in cosmology and social network. Recently we have also developed the theory of complex systems and discuss the small world, 6-degree relations. ([7]). Moreover, we have constructed network models, which are called Barbash-Albert model with the node distributions of power law([1]). At present we have not discussed the relationships between the phenomena and the theory. The theory on networks is developed independently from the analysis or description of the phenomena. The two contexts exist independently.

In this paper we shall try to make a bridge between these two fields and make analysis on the network. For this we introduce a method of automaton for the generation of networks whose distribution of nodes obeys power law. Our proposal is the introduction of automaton which is called "One-way direction relation". This concept plays a fundamental role in this paper. We make a network of the automaton. Then we see that the network has both distributions of nodes which show the power law and exponential power changing parameters there (see Figures ?? in section 2). Hence we may expect that fluctuations with power laws may be generated by these automatons. In the first three sections we generate networks in music, biology and language by the one-direction relation automatons systematically.

Here we want to notice that it is not easy to see whether the given fluctuations have power laws or not. Here we give two methods of analysis on the node distributions. The first one is arithmetic method which is based on the stick broken model.

We shall show that we can analyze our networks comparing them with the special distributions of stick broken models. We notice that the model was introduced by Mac-arther to describe the ecological system of big islands ([3]). We know the distribution of the broken sticks is exponential type which is very near to power law (see section 3). Hence we may assert the possibility of getting power law by this method. The second one is analytic method. We introduce a concept of evolution on network and try to develop analysis on the network. We introduce an evolution operator as follows: we have the equation:

$$\delta_h \psi = A\psi, \quad \delta_h = \frac{\psi_{n+1} - \psi_n}{h}, \, A\psi = A\psi_n,$$

where ψ_n are functions on each node of the network. We have the solution for a given initial value ψ_0 :

$$\psi_n = K_n \psi_0, \quad K_n = (1 + hA)^n .$$

We notice that we can obtain the exponential operator. Namely, putting $h = 1/n$, we

$$\lim_{n \to \infty} K_n = e^A$$

By this we discuss the node distribution in terms of the evolution theory of the network. Finally we suggest how to find a network where the following evolution operator of power law type $\delta_h \psi = c\psi^\gamma$, where c, γ are constants. Then we can apply Prof. Suyari's method for this network and we can obtain the theory of Tsallis entropy ([6]).

1. Automaton defined by one-way relation

In this section we introduce a concept of one-way relation and obtain its automatons.

Definition

(1) A system of nodes $\{P_n\}$ is called to satisfy one-way relation, when the following conditions are satisfied :For any pair $\{P_i, P_j\}$ of nodes, we have the following relation or no relation: $P_i \to P_j$. We denote the system is denoted by $\{P_i, \to\}$.

(2)For a system $\{P_i, \to\}$, we call a node P_k* start node, if there exists a P_l such that $P_{ki}* \to P_l$ but no P_r satisfying $P_r \to P_k*$.

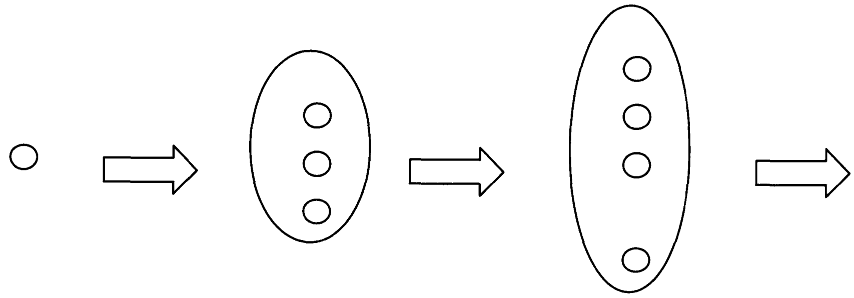
(3)For a system $\{P_i, \to\}$ a, we call a node P_j** the end node, if there exists a P_l such that $P_l \to P_i**$ but no P_r satisfying $P_k** \to P_r$.

We choose a system $\{P_i, \to\}$ and we generate the network which is generated by the relation, choosing the start node $P_\alpha*$. We choose a set of nodes $P_l^{(1)}$ satisfying $P_l^{(1)} \to P_{\alpha l}*$. We call the set the first generation: $P^{(1)}(P_\alpha^*) = \{P_l^{(1)} : P_{\alpha i}* \to P_l^{(1)}\}$. Repeating this process we can define the k-th generation:

$$P^{(k)}(P_\alpha^*) = \{P_l^{(k)} : P_r \to P_l^{(k)} (\exists P_r \in P^{(k-1)}(P_\alpha^*)\} :$$

Hence we obtain the network generated by the one way relation: $Q(P_\alpha^*) = \oplus_k P_\alpha^{(k)}$

The purpose of this paper is to show that the distribution of nodes has the decreasing character and fluctuations with power law may be realized by use of the network.



2. FLUCTUATIONS OF THE NETWORK OF ONE-WAY RELATION

In this section we give computer simulations of the fluctuations of the distributions of nodes for the network of the one-way relation. We give the simulations under the several different conditions. We consider the network of the node numbers N and of the relation numbers M. We denote the system by S(N,M).

(1) The distribution of a single start node

We give several computer simulations with a single start node.

(N,M)=(30,29)

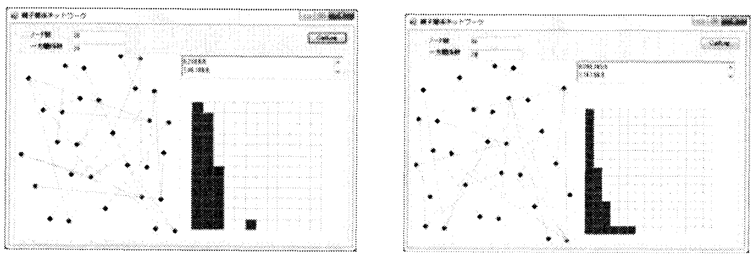


Figure 1

(2) The distribution of random start nodes

We give several computer simulations with random start nodes.

(N,M)=(30,100) (N,M)=(30,100) (N,M)=(30,100)

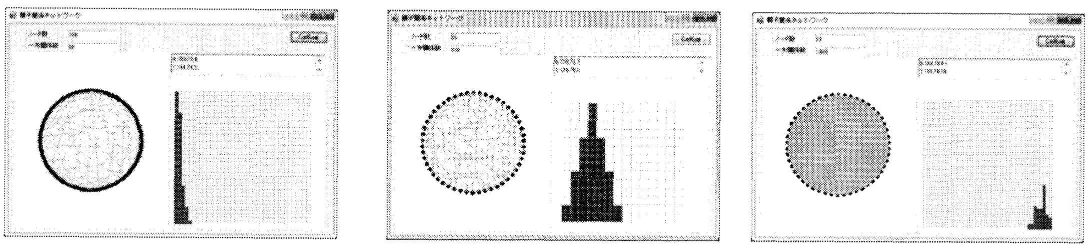


Figure 3

Hence we see that we have possibilities of realizing the both fluctuations with power law and the normal distributions in the same scheme. In the proceeding section we try to make fittings for the distributions in biology, language and music and others by this scheme.

3. FLUCTUATIONS IN BIOLOGY

In this section we observe many phenomena with power laws in molecular biology ([2]). The recent development in system biology tells us that we have a lot of phenomena whose networks show the power law in the node distributions. Here we shall show that we have big possibilities to realize the networks by use of the automaton generated by one-way relation.

(1)Transcription of protein production

We recall the transcription mechanism. We know that DNA constitutes by four kinds of elements A, T C and G. We have complementarity relation between these elements: $A \Leftrightarrow T \quad C \Leftrightarrow G$. Also we can produce m-RNA from DNA by Okazaki mechanism. The production of protein has a ternary construction method. Namely choosing three m-RNA, we can produce proteins following the codon table (Table 1):

	2								
1	U		C		A		G		3
U	UUU	Phe/F	UCU	Ser/S	UAU	Tyr/Y	UGU	Cys/C	U
	UUC	Phe/F	UCC	Ser/S	UAC	Tyr/Y	UGC	Cys/C	C
	UUA	Leu/L	UCA	Ser/S	[illegible]	[illegible]	[illegible]	[illegible]	A
	UUG	Leu/L	UCG	Ser/S	[illegible]	[illegible]	UGG	Trp/W	G
C	CUU	Leu/L	CCU	Pro/P	CAU	His/H	CGU	Arg/R	U
	CUC	Leu/L	CCC	Pro/P	CAC	His/H	CGC	Arg/R	C
	CUA	Leu/L	CCA	Pro/P	CAA	Gln/Q	CGA	Arg/R	A
	CUG	Leu/L	CCG	Pro/P	CAG	Gln/Q	CGG	Arg/R	G
A	AUU	Ile/I	ACU	Thr/T	AAU	Asn/N	AGU	Ser/S	U
	AUC	Ile/I	ACC	Thr/T	AAC	Asn/N	AGC	Ser/S	C
	AUA*	Ile/I	ACA	Thr/T	AAA	Lys/K	AGA	Arg/R	A
	AUG*	Met/M	ACG	Thr/T	AAG	Lys/K	AGG	Arg/R	G
G	GUU	Val/V	GCU	Ala/A	GAU	Asp/D	GGU	Gly/G	U
	GUC	Val/V	GCC	Ala/A	GAC	Asp/D	GGC	Gly/G	C
	GUA	Val/V	GCA	Ala/A	GAA	Glu/E	GGA	Gly/G	A
	GUG*	Val/V	GCG	Ala/A	GAG	Glu/E	GGG	Gly/G	G

Namely choosing three m-RNA, proteins can be produced following the codon table.Here we notice the following facts: (1)Three RNAs produce a protein. (2)20 kinds of proteins are produced (3)AUG is called start codon which gives the start of the production.(4) UAA, UGA, UAG are called the stop codons which stop the production

Table 1

We can introduce an automaton of the one way relation as the following transition table of the automaton. This is just special type of indefinite automaton. Hence we need not its explanation. Next we proceed to computer simulations. We make the computer simulations changing the numbers of stop codons. The first picture in Figure 6 is the case where the number of the stop codon is 1. The following figures are made following the numbers of stop codons. We can observe the decreasing property.

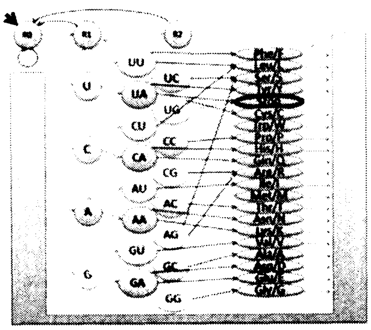
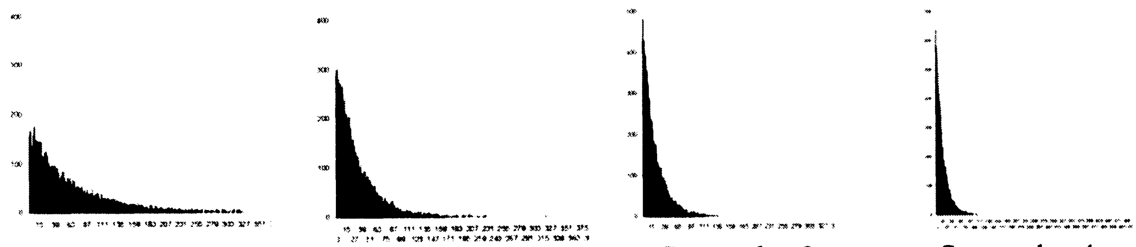


Figure 5



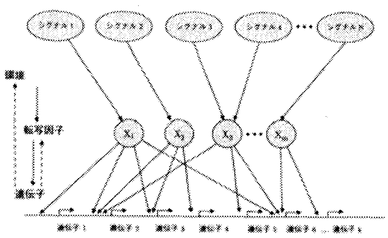
Stop codon 1, Stop codon 2 Stop codon 3 Stop codon 4

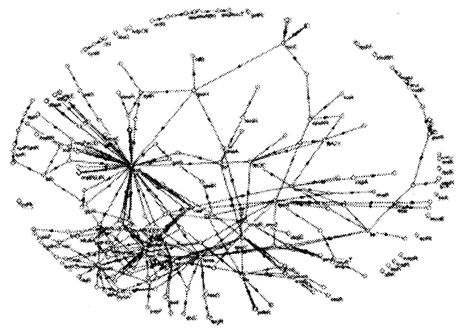
Figure 6

We can try to make the fitting of power law plot. Then we have the degree between -1.8 and -2.0 .

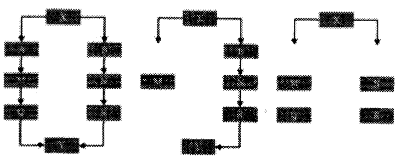
Next we proceed to other phenomena in the system of biology. Here we give examples without explanations and indicate the possibility of constructions of automaton of one-way relations ([2]).

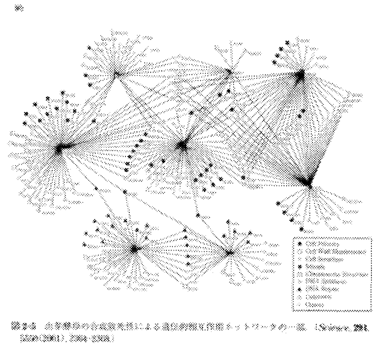
(2) Transcription mechanism



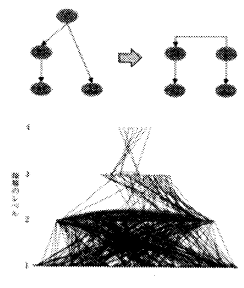


(2) Composite death mechanism





(3) Interrelationships in the protein productions



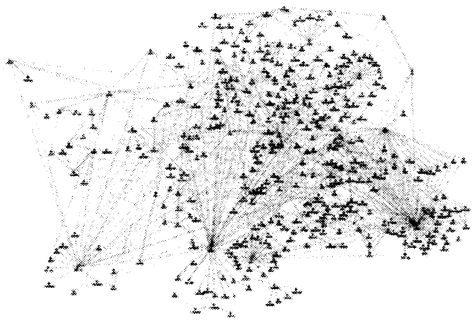


Figure 7

4. FLUCTUATIONS IN MUSIC

We can observe 1/f-fluctuation in music, especially in classic music. This fluctuation can be described as follows: We hear a music. Then we have the following Fourier expansion at any time: $\sum a_n f_n$.(see Figure 8 over). Considering the following function defined on integers: $f(n) = \log|a_n|$, we see that $f(n) \propto \frac{c}{n}$ which is called 1/f-fluctuation (see Figure 8 under).

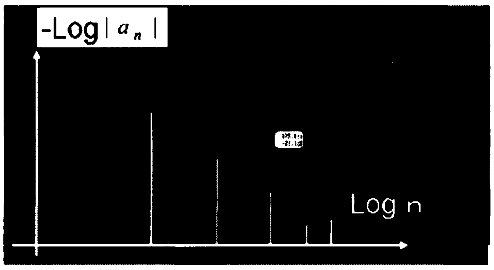


Figure 8

The 1/f-fluctuation phenomena is typical for classical music. But this is completely broken for contemporary music, especially for some "noisy music". In this paper we shall propose a method of automaton of the cadenza in classical music and we can show that we can realize the phenomena. At first we recall the cadenza in the classical music. Next we proceed to the constructions of automaton of one-way relation. The construction is given as follows:

(1) The start harmony triple is the class I harmony. (2) The end harmony triple is the class I. (3) Other process is given by the indeterminate automaton of cadenza .

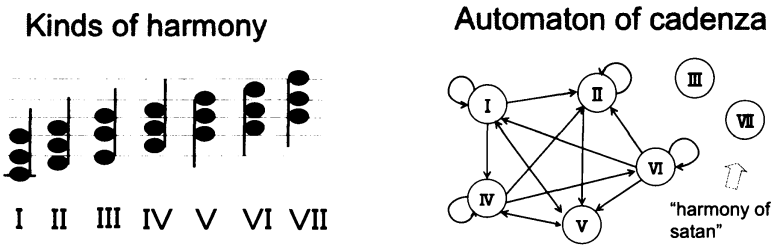
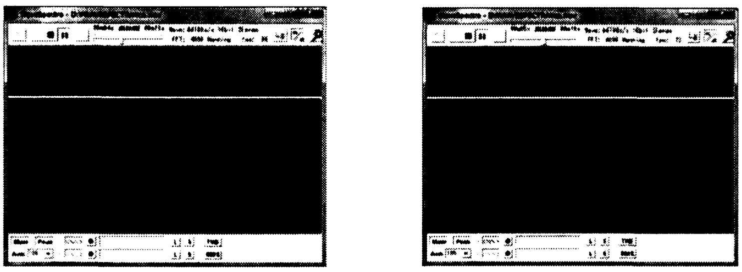


Figure 9

By this we can produce music and obtain the following fluctuation (Figure 10). We may say that the fluctuation can describe power law

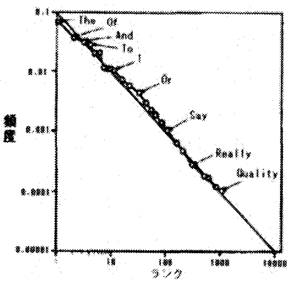


(The pictures are due K.Suzuki , K.Tsuchiya and K.Yano)
Figure 10

4. FLUCTUATIONS IN LANGAGE

The typical example of 1/f-fluctuation can be observed as the Zipf-law. Namely we choose a book written in English. Then we can consider the distribution of words with respect to the frequency in appearance. The ranking is stated as follows: (1) the ,

(2) of , (3) and ,….. Taking the logarsmic coordinate we have the Figure 11:



Although this phenomena is well known, we have no reasons of the distribution. We try to obtain by use of one-way automaton. We can discuss the generation of sentences producing sentences by use of a construction of simple grammar which is called PIGIN GRAMMAR

Figure 11

Here we shall introduce a simple grammar and obtain the distribution of the frequency of words. The most essential part of the grammar can be stated as follows:

Pigin words

(1) N-family $N_1, N_2, \ldots, N_M$ (Noun)

(2) V-family: $V_1, V_2, \ldots, V_N$ (Verb)

(3) A-family: $A_1, A_2, \ldots, A_R$ (Adjective)

Table of grammar

	N	V	A
N	×	○	×
V	○	○	○
A	○	×	×

Table 2

We give several examples of basic sentences:

Example1
$N \to V$ (I sleep)

Example2
$N \to V \to A$ (She is pretty)

Example 3
$N \to V \to N$ (I buy books)

Example 4
$N \to V \to N \to V \to N$
(I know, he bought books)

We produce sentences by the one-way relation as follows:

(1) Start word is N. (2) The end words are not deterministic. Namely words are end words when no more words can not be continued. (3) The production rules are given by the following grammar:○ implies the continuation possible and ×implies the continuation impossible. Then we have the following computer simulations on the distributions:

Computer simulation (I) Computer simulation (II)

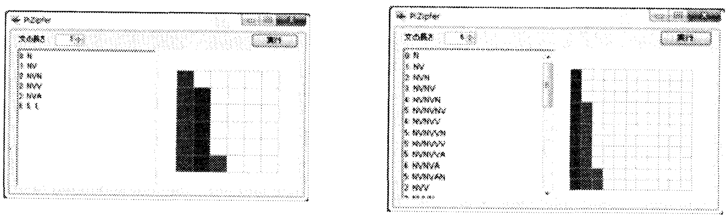


Figure 12

We see that we can obtain the decreasing distribution which does not against the Zipf law. We may try to obtain the Zipf's law making more realistic Pigin grammar.

5. FLUCTUATION FHENOMENA OF OTHER TYPES

We can observe many phenomena of power laws in several distributions. Here we give several examples of other types different from that of the one-way automaton:

(1) The fluctuations generated by both-way automaton

We give two examples: (1) Social network (2) Distribution of galaxies (Figure 13) :

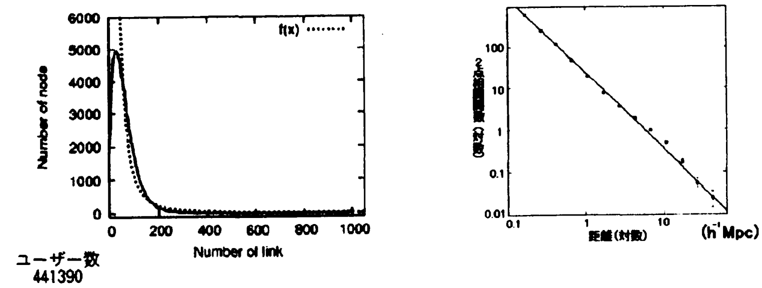


Figure 13

We notice that the both systems have different characters form that of one-way relations. For the case of the social network of MIXI, the network can be extended under the following conditions are satisfied: When one asks the other to have a contact, the other asked people should show it's acceptation. Under the both agreement, the net work can be extended. Hence we have to introduce the concept of both-way relation. Hence we put the following definition:

Definition

(1)A system of nodes $\{P_n\}$ is called to satisfy both-way relation, when the following conditions are satisfied:For any pair $\{P_i, P_j\}$ of nodes, we have the following one way relations or no relation: $P_i \to P_j$ and $P_j \to P_i$. We denote the system obtained by $\{P_i, \leftrightarrow\}$ (2)For a system $\{P_i, \leftrightarrow\}$ a, we call a node P_{ki} * the start node, if there exists a P_l such that $P_{ki}* \leftrightarrow P_l$ but no P_r satisfying $P_r \leftrightarrow P_k*$. (3)For a system $\{P_i, \leftrightarrow\}$ a, we call a node P_{ki} * the start node, if there exists a P_l such that $P_{ki}* \leftrightarrow P_l$ but no P_r satisfying $P_r \leftrightarrow P_k*$

Then we can construct the network system and we can obtain the fluctuation of decreasing properties. The details may be omitted.

As for the case of generation of galaxies, we may find the both-way direction because of the gravitational interactions introduce both-way relation:

$$f = G\frac{m_1 m_2}{r^2}$$

We want to make a comment on this network. In the theory of cosmology, we have still no understanding on the origin of the fluctuation of the galaxies. This is a serious problem on the origin of masses in this universe. Hence our method may have a possibility of the contribution to this problem.

(2) The fluctuations generated by stick broken model

We have another type of the system generation, which is well known as the stick

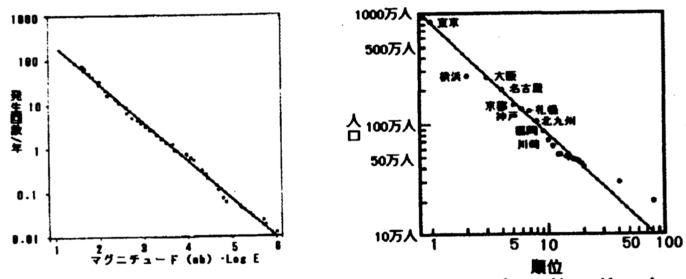


Figure 14

broken model. We give two examples: (1) The distribution of size of earth queak and (2) The ranking of the size of cities (in Japan).

The model to be fitted can be found in the so called stick broken model, which is initially introduced for the description of ecology in the big island. The distribution is described as the distribution of the pieces of a long stick which is known as the Hardy-Ramanjyan formula:

$$P(n) \approx \frac{\exp(An^{1/2})}{4n3^{1/2}} \qquad P_k(n) = \sum\nolimits_{n_1+\ldots+n_m=k} n_i P(n_1, n_2, \ldots, n_m)$$

Explicit treatment will be given in the forthcoming section, where $P(n)$ is the total number of possible decomposition of the length n and $P_k(n)$ is the numbers of pieces of length k. We give a computer simulation for the distribution of the pieces of broken sticks $P_k(14)$. This distribution is quite closed to the so called 1/f-fluctuation. Even though, we know that the distribution does not satisfy the power law.

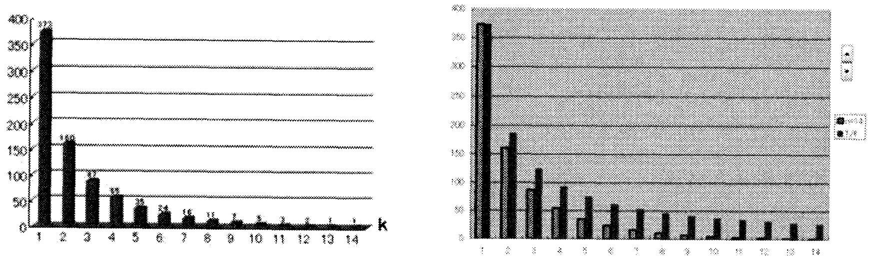
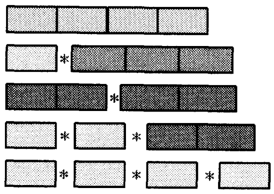


Figure 15

6. THE ANALYSIS ON THE NETWORK OF ONE-WAY RELATION (I)

In this section and next sections we give analysis on the network defined by automaton of an one-way relationship. Here we give numerical analysis on the distribution based on the stick broken model. In the next section we give differential and integral method.

We choose a stick with the length 4. Then we have the following distribution on the numbers of pieces of given length:



4
1 3
2 2
1 1 2
1 1 1 1

$$P(4) = 5$$
$$P_1(4) = 7, P_2(4) = 3$$
$$P_3(4) = 1, P_4(4) = 1$$

Figure 16

We can associate the network for this distribution:

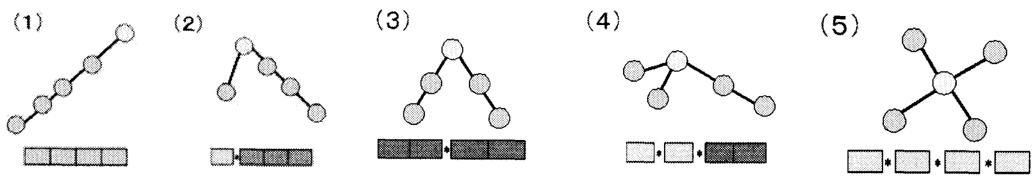


Figure 17

We can compare the node distributions of one-way relation automatons with that of stick broken model. We give an example of the demonstration for the evolution of Asian peoples by use of mitocondria ([2]).

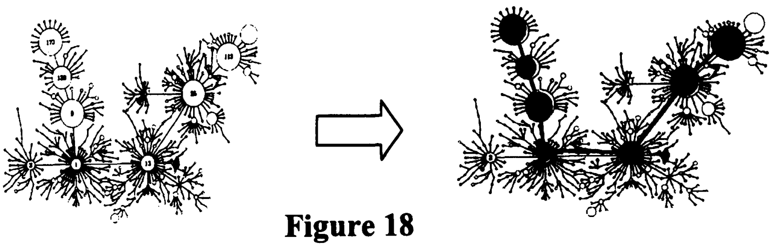


Figure 18

Hence we can analyze the distribution finding the part of stick broken model (Figure 18, right side) and deforming the model to the original distribution in the following steps:

(1) We see that there exists one to one correspondence between the counting of the distribution $P_k(n)$ of stick broken model and that of the distribution $Q_k(n)$ of nodes.
(2) Next we find the stick broken part of the network.
(3)Deformation of adding small nodes and realize them

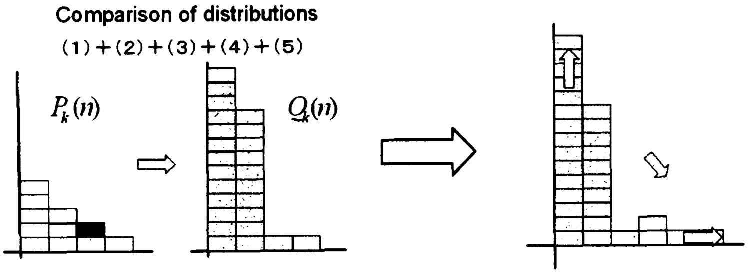


Figure 19

8. THE ANALYSIS ON THE NETWORK OF ONE-WAY RELATION (II)

In this section we give an analysis on an a network and give its application to the network of one-way automaton. At first we recall the evolution theory in mathematical physics. Then we introduce concepts of generating and annihilating operators for generation or dilating nodes of the network. Finally we proceed to evolution of the function space on the network and discuss the fluctuations .

(1) The diffusion (evolution) process

Here we recall the basic facts on diffusion process. The beginning is the heat equation: $\partial T/\partial t = a\partial^2 T/\partial x^2$. We can give the following solution for the initial value function $T(x,0) = f(x)$ by use of the heat kernel:

$$T(x,t) = \int_{-\infty}^{\infty} K(x,t) f(\xi) d\xi,\ K(x,t) = \frac{1}{2\sqrt{\pi a t}} \exp\left(\frac{(x-\xi)^2}{4at}\right)$$

The direct generalization of this formulation can be given as follows: $\partial u/\partial t = Au$, where A is an operator between some function space. The initial value problem with an initial function, we can give the solution under suitable conditions:

$$u(t) = (\exp A) f, \quad \exp A = \sum_{n=0}^{\infty} \frac{A^n}{n!}$$

(2) The discrete diffusion (evolution) process

We want to generalize the evolution process to a network. At first we introduce the sequence $\psi = \{\psi_n : n = 0,1,2,...\}$ of functions and define the discretized evolution equation by the following setting:

$$\frac{\partial}{\partial t}\psi \Rightarrow \frac{\psi_{n+1} - \psi_n}{h}, \quad A\psi \Rightarrow A\psi_n$$

Then we can define the equation:

$$\delta_h\psi = A\psi, \quad \delta_h = \frac{\psi_{n+1} - \psi_n}{h}, A\psi = A\psi_n$$

We can obtain the kernel operator and obtain the initial value problem as follows: For a given initial value ψ_0, we have the solution: $\psi_n = K_n\psi_0$, $K_n = (1 + hA)^n$. We notice that we can obtain the exponential operator by making continuation process: Putting $h = 1/n$, we have $\lim_{n\to\infty} K_n = e^A$.

(3) The generation of nodes and introduce a network

Next we introduce the evolution of a network of one-way relation. We begin with a description of generation and annihilation of nodes by operators. We choose a start node $P*$. For an arbitrary node P_j, we define the creation operator of P_j from $P*$ by

$$\hat{P}_j(P_k) = \begin{cases} P_j & when\ P_k = P* \\ \phi & (otherwise) \end{cases}$$

Also we introduce annihilation operator of point P_j as follows:

$$\hat{P}_j*(P_k) = \begin{cases} P* & when\ P_k = P_j \\ \phi & (otherwise) \end{cases}$$

For a pair of two points P_j and P_k, we define the transition operator $\hat{P}_{jk}$ as follows:

$$\hat{P}_{jk}(P_l) = \delta_{kl}P_j$$

In terms of these operators, we can describe a generation of network by use of creation and transition operators:

$$L^{(1)} = \{\hat{P}_j^{(1)}(P*) : (j = 1,2,...,N^{(1)})\}$$

$$L^{(2)} = \{\hat{P}_{kj}^{(2)}(P*) : (j = 1,2,...,N^{(1)}), (k = 1,2,...,N^{(2)}\}$$

....

$$L^{(k)} = \{\hat{P}_{kj}^{(k)}(P*) : (j = 1,2,...,N^{(k-1)}), (k = 1,2,...,N^{(k)}\}$$

Introducing the evolution operator by

$$\hat{A} = \sum \sum \hat{P}_{ij}^{(k)}$$

we can generate each node by the operator $\hat{A}$, which is denoted by

$$\delta\ P = \hat{A}P, \quad \delta\ = P_{n+1}P_n*, \hat{A}\psi = \hat{A}P_n$$

Introducing the exponential mapping by

$$\exp(\hat{A}) = \prod_{k=1}^{\infty} (1 + \sum \sum \hat{P}_{ij}^{(k)})$$

we can describe the total nodes by use of exponential mapping:

$$L = \exp \hat{A}(P^*),\ where\ L = \oplus L_j$$

(3) Discrete Suyari equation

Here we discuss the discrete version of Suyari equation. In order to describe the equation, we have to introduce the description of the network in the dual space. Choosing the characteristic function of each node, we can formulate the node distributions. Choosing $A = \psi^{-\gamma-1}$, we can generalize the equation to the discrete equation:

$$\delta_h \psi = \psi^{-\gamma}$$

Then for the initial value ψ^* , we have the solution:

$$\psi = \exp(\hat{A})\psi^* = \prod_{k=1}^{\infty} (1 + \sum \sum \hat{P}_{ij}^{(k)-\gamma-1})\,\psi^*$$

The details will be given in another paper.

(4) Distribution of nodes

Finally we shall describe the node distribution of the given network and find how to examine whether it obeys power law or not. We choose a network with a generator $\hat{A}$. Then introducing the generation operator A_N of the degrees of nodes at each node point, we obtain its difference equation. Then introducing the linear space which is generated by nodes, we can describe the node distribution N : $\delta N = A_N N$. We notice that the subspaces can be described as the eigen spaces of the Lapalcian operator on the network.

$$\Delta N(P) = n(P) N(P)$$

, where $n(P)$ is the degree of the node at P. Then we can discuss the fluctuation whether it obeys power law or exponential law or not by the discussions (2). The detail will be given in a forthcoming paper.

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