数理解析研究所講究録
第 1883 巻 2014 年 13-20

Asymptotic stability of stationary solutions for the non-isentropic Euler-Maxwell system

Yoshihiro Ueda[1] and Shuichi Kawashima[2]

[1] Faculty of Maritime Sciences, Kobe University
Kobe 658-0022, Japan

[2] Faculty of Mathematics, Kyushu University
Fukuoka 819-0395, Japan

1 Introduction

In this note we consider the following non-isentropic Euler–Maxwell system in $\mathbb{R}^3$:

$$\begin{cases} n_t + \operatorname{div}(nu) = 0, \\ (nu)_t + \operatorname{div}(nu \otimes u) + \nabla p(n,\theta) = -n(E + u \times B) - \dfrac{1}{\tau_1} nu, \\ \Big\{ n\Big(c_V\theta + \dfrac{1}{2}|u|^2\Big)\Big\}_t + \operatorname{div}\Big\{ nu\Big(c_V\theta + \dfrac{1}{2}|u|^2\Big) + p(n,\theta)u \Big\} \\ \qquad\qquad = -nu \cdot (E + u \times B) - \dfrac{c_V}{\tau_2} n(\theta - \theta_\infty), \\ E_t - \operatorname{rot} B = nu, \\ B_t + \operatorname{rot} E = 0, \end{cases} \tag{1.1}$$

$$\operatorname{div} E = b(x) - n, \tag{1.2}$$

$$\operatorname{div} B = 0, \tag{1.3}$$

with the initial data

$$(n, u, \theta, E, B)(0, x) = (n_0, u_0, \theta_0, E_0, B_0)(x). \tag{1.4}$$

Here the mass density $n > 0$, the velocity $u \in \mathbb{R}^3$, the absolute temperature $\theta > 0$, the electric field $E \in \mathbb{R}^3$, and the magnetic induction $B \in \mathbb{R}^3$ are unknown functions of $t > 0$ and $x \in \mathbb{R}^3$, $b > 0$ is a given function of $x \in \mathbb{R}^3$, and θ_∞ is a positive constant. Relaxation parameters τ_1 and τ_2 are positive constants. Due to the Boyle–Charles law, the pressure p is explicitly given as a function of the density and the absolute temperature:

$$p = p(n, \theta) := Kn\theta,$$

where $K > 0$ is the gas constant. On the other hand, c_V is a positive constant which means the specific heat at constant volume. Due to Mayler's relation for the ideal gas, the specific heat c_V is expressed in terms of the gas constant K and the adiabatic constant $\gamma > 1$ as

$$c_V = \frac{K}{\gamma - 1}.$$

For example, when $K = 1$ and $\gamma = 5/3$, the specific heat becomes $c_V = 3/2$.

The non-isentropic Euler-Maxwell system (1.1), (1.2), (1.3) describes the dynamics of non-isentropic compressible electrons in plasma physics under the interaction of the magnetic and electric fields via the Lorentz force (see [1]).

In the authors' papers [4, 5, 6], the barotropic type Euler-Maxwell system was analyzed about its basic properties. More precisely, we showed that the Euler-Maxwell system is a symmetrizable hyperbolic system and has the weaker dissipative structure than the standard one characterized in [2, 3, 7]. Moreover, we observed that this dissipative structure is of the regularity-loss type in the sense that the regularity-loss occurs in the dissipation part of the energy estimates. This weaker dissipation causes some additional difficulties in establishing a global existence result and especially in obtaining the time asymptotic decay of solutions.

Based on the above known results and analysis, our purpose in this note is to show the asymptotic stability of stationary solutions for the the non-isentropic Euler-Maxwell system (1.1), (1.2), (1.3) in the whole space $\mathbb{R}^3$. We will prove the global existence of smooth solutions and its asymptotic convergence toward the stationary solution, that is, $(n, u, \theta, E, B)(t, x) \to (\tilde{n}(x), 0, \theta_\infty, \tilde{E}(x), B_\infty)$ as $t \to \infty$, where $(\tilde{n}(x), 0, \theta_\infty, \tilde{E}(x), B_\infty)$ is the stationary solution for the system (1.1), (1.2), (1.3). The key to the proof of our main result is to show the uniform a priori estimates by applying the energy method which makes use of the strict convexity of the physical energy together with the dissipative structure of the system (1.1), (1.2), (1.3).

2 Stationary problem

In this section, we consider the existence of the stationary solution for (1.1), (1.2), (1.3). We look for stationary solutions in the form $(\tilde{n}(x), 0, \theta_\infty, \tilde{E}(x), B_\infty)$, where $\tilde{n}(x) > 0$, $\tilde{E}(x) \in \mathbb{R}^3$, $\theta_\infty > 0$ is the constant in (1.1) and $B_\infty \in \mathbb{R}^3$ is a given constant. Then $\tilde{n}$ and $\tilde{E}$ satisfies the following stationary problem:

$$\begin{cases} \nabla p(\tilde{n}, \theta_\infty) = -\tilde{n}\tilde{E}, \\ \mathrm{div}\tilde{E} = b(x) - \tilde{n}, \\ \mathrm{rot}\,\tilde{E} = 0. \end{cases} \tag{2.1}$$

In order to solve this problem, it is convenient to introduce the electric potential $\psi \in \mathbb{R}$ such that

$$-\nabla\psi = \tilde{E}. \tag{2.2}$$

Then the stationary problem (2.1) is rewritten as

$$\begin{cases} \nabla(K\theta_\infty \log \tilde{n} - \psi) = 0, \\ -\Delta\psi = b(x) - \tilde{n}. \end{cases} \tag{2.3}$$

Now we consider the case where $b(x) = n_\infty + \bar{b}(x)$. Here n_∞ is an arbitrary positive constant and $\bar{b}(x) \to 0$ as $|x| \to \infty$. In this case, we look for solutions in the form $\tilde{n}(x) = n_\infty + \bar{n}(x)$. Then the problem (2.3) is reduced to

$$\begin{cases} \nabla(K\theta_\infty \log(1 + \bar{n}/n_\infty) - \psi) = 0, \\ -\Delta\psi = \bar{b}(x) - \bar{n}. \end{cases} \tag{2.4}$$

It follows from the first equation that $K\theta_\infty \log(1 + \bar{n}/n_\infty) - \psi = 0$, so that we have

$$\bar{n}(x) = n_\infty\{e^{\psi(x)/(K\theta_\infty)} - 1\}. \tag{2.5}$$

Here we have assumed that $(\bar{n}, \psi)(x) \to 0$ as $|x| \to \infty$. Substituting (2.5) into the second equation of (2.4), we obtain

$$-\Delta\psi + \frac{n_\infty}{K\theta_\infty}\psi = \bar{b}(x) - P(\psi), \tag{2.6}$$

where we define that

$$P(\psi) := n_\infty\Big\{e^{\psi/(K\theta_\infty)} - 1 - \frac{1}{K\theta_\infty}\psi\Big\}.$$

We note that $P(\psi) = O(\psi^2)$ for $|\psi| \ll 1$. Then, by applying the usual fixed point theorem, we can show the following existence results of a unique solution ψ for the reduced stationary problem (2.6).

Theorem 2.1. *Let $s \geq 0$ and $n_\infty > 0$. Suppose that $b - n_\infty \in H^s$. Then there exists a positive constant ε such that if $\|b - n_\infty\|_{H^s} \leq \varepsilon$, then the reduced stationary problem* (2.6) *has a unique solution $\psi \in H^{s+2}$ satisfying*

$$\|\psi\|_{H^{s+2}} \leq C\|b - n_\infty\|_{H^s}, \tag{2.7}$$

where C is a positive constant.

Corollary 2.2. *Under the same assumptions as in Theorem 2.1, the stationary problem* (2.1) *has a unique solution $(\tilde{n}, \tilde{E})$ such that $\tilde{n} - n_\infty \in H^{s+2}$, $\tilde{E} \in H^{s+1}$ and*

$$\|\tilde{n} - n_\infty\|_{H^{s+2}} + \|\tilde{E}\|_{H^{s+1}} \leq C\|b - n_\infty\|_{H^s},$$

where C is a positive constant. Also, $(\tilde{n}, 0, \theta_\infty, \tilde{E}, B_\infty)$ becomes a stationary solution to our original system (1.1), (1.2), (1.3).

By using Theorem 2.1 with the equations (2.2) and (2.5), we can derive Corollary 2.2 immediately. So we omit the proof of Corollary 2.2.

Proof of Theorem 2.1. For the reduced stationary problem (2.6), we apply the fixed point theorem and then show the existence of solution ψ. We define the mapping Ψ by

$$\Psi(\psi) := \Big(-\Delta + \frac{n_\infty}{K\theta_\infty}\Big)^{-1}(\bar{b}(x) - P(\psi)). \tag{2.8}$$

Then our solution ψ to the equation (2.6) can be obtained as a fixed point of the mapping Ψ, that is, $\psi = \Psi(\psi)$.

Now we consider the above mapping Ψ in the space H^2 and prove Theorem 2.1 for $s = 0$; the proof for $s \geq 1$ is similar and is omitted here. We take a closed convex subset S_σ of H^2 as $S_\sigma := \{\psi \in H^2;\ \|\psi\|_{H^2} \leq \sigma\}$, where $\sigma \in (0, 1]$ is a number which will be determined later. Here we note that $\|\psi\|_{L^\infty} \leq C_0\sigma$ for $\psi \in S_\sigma$, where C_0 is the constant appearing in the the following Gagliardo–Nirenberg inequality in $\mathbb{R}^3$:

$$\|v\|_{L^\infty} \leq C_0\|v\|_{L^2}^{1/4}\|\partial_x^2 v\|_{L^2}^{3/4}.$$

First we show that Ψ is a mapping of S_σ into itself if $\sigma \in (0, 1]$ is chosen suitably small. To this end, we let $\psi \in S_\sigma$ and put $\zeta = \Psi(\psi)$. Then we have from (2.8) that

$$-\Delta\zeta + \frac{n_\infty}{K\theta_\infty}\zeta = \bar{b}(x) - P(\psi). \tag{2.9}$$

Here we observe that $P(\psi) = O(\psi^2)$ for $|\psi| \to 0$ and that $\|\psi\|_{L^\infty} \leq C_0\sigma$. This yields $\|P(\psi)\|_{L^2} \leq C\|\psi\|_{L^2}^2 \leq C\sigma^2$ for $\sigma \in (0, 1]$. Therefore, applying the standard elliptic estimate to (2.9), we obtain

$$\|\zeta\|_{H^2} \leq C\|\bar{b} - P(\psi)\|_{L^2} \leq C_1(\|\bar{b}\|_{L^2} + \sigma^2) \tag{2.10}$$

for $\sigma \in (0, 1]$, where $C_1 \geq 1$ is a constant independent of σ. We choose σ such that

$$\sigma = 2C_1\|\bar{b}\|_{L^2} \tag{2.11}$$

and assume that $\|\bar{b}\|_{L^2}$ is so small that $\|\bar{b}\|_{L^2} \leq 1/(4C_1^2)$; note that we have $\sigma \in (0, 1]$ in this case. For this choice of σ and $\|\bar{b}\|_{L^2}$, the estimate (2.10) gives $\|\zeta\|_{H^2} \leq \sigma$. This shows that Ψ is a mapping of S_σ into itself.

Next we show the contraction property for the mapping Ψ. To this end, we let $\psi_j \in S_\sigma$ and put $\zeta_j = \Psi(\psi_j)$, where $j = 1, 2$. Then the difference $\zeta_1 - \zeta_2$ satisfies the equation

$$-\Delta(\zeta_1 - \zeta_2) + \frac{n_\infty}{K\theta_\infty}(\zeta_1 - \zeta_2) = -(P(\psi_1) - P(\psi_2)). \tag{2.12}$$

Since $P(\psi_1) - P(\psi_2) = O(|\psi_1| + |\psi_2|)(\psi_1 - \psi_2)$ for $|\psi_1| + |\psi_2| \to 0$, we see that

$$\|P(\psi_1) - P(\psi_2)\|_{L^2} \leq C(\|\psi_1\|_{L^2} + \|\psi_2\|_{L^2})\|\psi_1 - \psi_2\|_{L^2} \leq C\sigma\|\psi_1 - \psi_2\|_{L^2}.$$

Therefore, applying the standard elliptic estimate to (2.12), we find that

$$\|\zeta_1 - \zeta_2\|_{H^2} \leq C\|P(\psi_1) - P(\psi_2)\|_{L^2} \leq C_2\sigma\|\psi_1 - \psi_2\|_{L^2}, \tag{2.13}$$

where C_2 is a positive constant independent of σ. For our choice of σ in (2.11), we additionally assume that $\|\bar{b}\|_{L^2} \leq 1/(4C_1C_2)$. Then we have from (2.13) that $\|\zeta_1 - \zeta_2\|_{H^2} \leq \|\psi_1 - \psi_2\|_{L^2}/2$, which shows that Ψ is a contraction mapping from S_σ into itself. Consequently, Ψ has a fixed point ψ in S_σ with σ in (2.11). This fixed point ψ satisfies the equation (2.6) and verifies the estimate (2.7) for $s = 0$. Therefore this ψ is our desired solution to (2.6) for $s = 0$. Thus the proof of Theorem 2.1 for $s = 0$ is complete. □

3 Global existence

In this section, we give a statement of our main result. Before stating our result, we review some basic property of the system (1.1), (1.2), (1.3). We see that the equations (1.2) and (1.3) hold for any $t > 0$ if they hold initially. In fact, any solution to the initial value problem (1.1), (1.4) with the initial data verifying

$$\operatorname{div} E_0 = b(x) - n_0, \qquad \operatorname{div} B_0 = 0 \tag{3.1}$$

satisfies (1.2) and (1.3) for $t > 0$.

To state our theorem, we introduce

$$w = (n, u, \theta, E, B)^T, \quad \tilde{w} = (\tilde{n}, 0, \theta_\infty, \tilde{E}, B_\infty)^T, \quad w_0 = (n_0, u_0, \theta_0, E_0, B_0)^T$$

which are regarded as column vectors in $\mathbb{R}^{13}$, where the superscript "T" denotes the transposed. By using these preliminaries, our main result is stated as follows.

Theorem 3.1. *Let $s \geq 3$ and let $\tilde{w}(x)$ be the stationary solution to the system* (1.1), (1.2), (1.3), *which is constructed in Corollary 2.2. Suppose that the initial data satisfy $w_0 - \tilde{w} \in H^s$ and* (3.1). *Then there exists some positive constant ε_0 such that if $\|w_0 - \tilde{w}\|_{H^s} + \|\partial_x \tilde{n}\|_{H^s} \leq \varepsilon_0$, then the initial value problem* (1.1), (1.4) *has a unique global solution $w(t,x)$ satisfying $w - \tilde{w} \in C([0,\infty); H^s) \cap C^1([0,\infty); H^{s-1})$ and*

$$\begin{aligned} &\|(w - \tilde{w})(t)\|_{H^s}^2 + \int_0^t \big(\|(n - \tilde{n}, u, \theta - \theta_\infty)(\tau)\|_{H^s}^2 \\ &\qquad + \|(E - \tilde{E})(\tau)\|_{H^{s-1}}^2 + \|\partial_x B(\tau)\|_{H^{s-2}}^2\big) d\tau \leq C\|w_0 - \tilde{w}\|_{H^s}^2 \end{aligned} \tag{3.2}$$

for $t \geq 0$. Moreover, the solution $w(t,x)$ converges to the stationary solution $\tilde{w}(x)$ uniformly in $x \in \mathbb{R}^3$ as $t \to \infty$. More precisely, we have

$$\begin{aligned} &\|(n - \tilde{n}, u, \theta - \theta_\infty, E - \tilde{E})(t)\|_{W^{s-2,\infty}} \to 0 \qquad \text{as} \qquad t \to \infty, \\ &\|(B - B_\infty)(t)\|_{W^{s-4,\infty}} \to 0 \qquad \text{as} \qquad t \to \infty, \end{aligned} \tag{3.3}$$

where the asymptotic convergence (3.3) *holds true only by assuming the additional regularity $s \geq 4$.*

We note that the uniform energy estimate (3.2) is of the regularity-loss type because we have 1-regularity loss for (E, B) in the dissipation part of (3.2).

In addition to (3.2), we can obtain a similar uniform estimate for time derivative of the solution.

Corollary 3.2. *Let $s \geq 3$ and suppose that the same conditions as in Theorem 3.1 hold true. Let $w(t,x)$ be the solution to (1.1), (1.4) which is constructed in Theorem 3.1. Then the time derivative of the solution $w(t,x)$ satisfies the following uniform estimate:*

$$\|\partial_t w(t)\|_{H^{s-1}}^2 + \int_0^t (\|\partial_t(n,u,\theta)(\tau)\|_{H^{s-1}}^2 + \|\partial_t(E,B)(\tau)\|_{H^{s-2}}^2) d\tau \leq C\|w_0 - \tilde{w}\|_{H^s}^2.$$

The key to the proof of our main Theorem 3.1 is to derive the uniform a priori estimates of the perturbation of solutions to the problem. To state the result on our a priori estimates, we introduce the energy norm $N(t)$ and the corresponding dissipation norm $D(t)$ by

$$N(t) := \sup_{0\leq\tau\leq t} \|(w-\tilde{w})(\tau)\|_{H^s},$$

$$D(t)^2 := \int_0^t (\|(n-\tilde{n}, u, \theta-\theta_\infty)(\tau)\|_{H^s}^2 + \|(E-\tilde{E})(\tau)\|_{H^{s-1}}^2 + \|\partial_x B(\tau)\|_{H^{s-2}}^2) d\tau.$$

We also use the following quantities:

$$M(t) := \sup_{0\leq\tau\leq t} \|(w-\tilde{w})(\tau)\|_{W^{1,\infty}},$$

$$I(t)^2 := \int_0^t (\|(n-\tilde{n}, u)(\tau)\|_{W^{1,\infty}}^2 + \|(\theta-\theta_\infty)(\tau)\|_{L^\infty}^2) d\tau.$$

Proposition 3.3. *Let $s \geq 3$ and suppose that the initial data satisfy $w_0 - \tilde{w} \in H^s$ and (3.1). Let $w(t,x)$ be a solution to (1.1), (1.4) satisfying $w - \tilde{w} \in C([0,T]; H^s) \cap C^1([0,T]; H^{s-1})$ for some $T > 0$. Then there are some positive constants ε_1 and C independent of T such that if $N(T) + \|\partial_x \tilde{n}\|_{H^s} \leq \varepsilon_1$, then the following a priori estimate holds for $t \in [0,T]$:*

$$N(t)^2 + D(t)^2 \leq C\|w_0 - \tilde{w}\|_{H^s}^2.$$

In order to derive the above *a priori* estimate, we obtain the following four energy inequalities.

Lemma 3.4. *Assume the same conditions as in Proposition 3.3. Then we have the following energy estimates for $t \in [0,T]$:*

$$\|(w-\tilde{w})(t)\|_{L^2}^2 + \int_0^t \|(u, \theta-\theta_\infty)(\tau)\|_{L^2}^2 d\tau \leq C\|w_0 - \tilde{w}\|_{L^2}^2 + CM(t)D(t)^2, \tag{3.4}$$

$$\begin{aligned} &\|\partial_x(w-\tilde{w})(t)\|_{H^{s-1}}^2 + \int_0^t \|\partial_x(u, \theta-\theta_\infty)(\tau)\|_{H^{s-1}}^2 d\tau \\ &\qquad \leq C\|\partial_x(w_0-\tilde{w})\|_{H^{s-1}}^2 + C(M(t) + \|\partial_x\tilde{n}\|_{W^{1,\infty}})D(t)^2 \\ &\qquad\qquad + C(N(t) + \|\partial_x\tilde{n}\|_{L^\infty} + \|\partial_x\tilde{n}\|_{H^s})I(t)D(t), \end{aligned} \tag{3.5}$$

$$\begin{aligned} &\int_0^t (\|(n-\tilde{n})(\tau)\|_{H^s}^2 + \|(E-\tilde{E})(\tau)\|_{H^{s-1}}^2) d\tau \\ &\leq \varepsilon \int_0^t \|\partial_x B(\tau)\|_{H^{s-2}}^2 d\tau + C_\varepsilon\big\{\|w_0-\tilde{w}\|_{H^s}^2 + (M(t) + \|\partial_x\tilde{n}\|_{W^{1,\infty}})D(t)^2 \\ &\qquad + (N(t) + \|\partial_x\tilde{n}\|_{L^\infty} + \|\partial_x\tilde{n}\|_{H^s})I(t)D(t)\big\}, \end{aligned} \tag{3.6}$$

$$\int_0^t \|\partial_x B(\tau)\|_{H^{s-2}}^2 d\tau \le C\|w_0 - \tilde{w}\|_{H^{s-1}}^2 + C\int_0^t \|\partial_x (E-\tilde{E})(\tau)\|_{H^{s-2}}^2 d\tau \\ + C(M(t) + \|\partial_x \tilde{n}\|_{W^{1,\infty}})D(t)^2 + C(N(t) + \|\partial_x \tilde{n}\|_{L^\infty} + \|\partial_x \tilde{n}\|_{H^s})I(t)D(t) \tag{3.7}$$

for any $\varepsilon > 0$, *where* C_ε *is a positive constant depending on* ε.

These estimates are derived by the energy method for our problem. Once we get these estimates, we can derive the desired *a priori* estimate as follows.

Proof of Proposition 3.3. First, we substitute (3.7) into (3.6) and take $\varepsilon > 0$ suitably small. This yields

$$\int_0^t \left(\|(n-\tilde{n})(\tau)\|_{H^s}^2 + \|(E-\tilde{E})(\tau)\|_{H^{s-1}}^2\right) d\tau \\ \le C\|w_0 - \tilde{w}\|_{H^s}^2 + C(M(t) + \|\partial_x \tilde{n}\|_{W^{1,\infty}})D(t)^2 \\ + C(N(t) + \|\partial_x \tilde{n}\|_{L^\infty} + \|\partial_x \tilde{n}\|_{H^s})I(t)D(t). \tag{3.8}$$

Next, substituting (3.8) into (3.7), we have

$$\int_0^t \|\partial_x B(\tau)\|_{H^{s-2}}^2 \, d\tau \le C\|w_0 - \tilde{w}\|_{H^s}^2 + C(M(t) + \|\partial_x \tilde{n}\|_{W^{1,\infty}})D(t)^2 \\ + C(N(t) + \|\partial_x \tilde{n}\|_{L^\infty} + \|\partial_x \tilde{n}\|_{H^s})I(t)D(t). \tag{3.9}$$

It then follows from (3.4), (3.5), (3.8) and (3.9) that

$$\|(w-\tilde{w})(t)\|_{H^s}^2 + \int_0^t \left(\|(n-\tilde{n}, u, \theta - \theta_\infty)(\tau)\|_{H^s}^2 + \|(E-\tilde{E})(\tau)\|_{H^{s-1}}^2 + \|\partial_x B(\tau)\|_{H^{s-2}}^2\right) d\tau \\ \le C\|w_0 - \tilde{w}\|_{H^s}^2 + C(M(t) + \|\partial_x \tilde{n}\|_{W^{1,\infty}})D(t)^2 \\ + C(N(t) + \|\partial_x \tilde{n}\|_{L^\infty} + \|\partial_x \tilde{n}\|_{H^s})I(t)D(t).$$

Thus we get the inequality

$$N(t)^2 + D(t)^2 \le C\|w_0 - \tilde{w}\|_{H^s}^2 + C(M(t) + \|\partial_x \tilde{n}\|_{W^{1,\infty}})D(t)^2 \\ + C(N(t) + \|\partial_x \tilde{n}\|_{L^\infty} + \|\partial_x \tilde{n}\|_{H^s})I(t)D(t). \tag{3.10}$$

Here we observe that $M(t) \le CN(t)$ and $I(t) \le CD(t)$ for $s \ge 3$. Therefore (3.10) is reduced to

$$N(t)^2 + D(t)^2 \le C\|w_0 - \tilde{w}\|_{H^s}^2 + C_0(N(t) + \|\partial_x \tilde{n}\|_{H^s})D(t)^2 \tag{3.11}$$

for $s \ge 3$, where C_0 is a positive constant. Now we choose $\varepsilon_1 > 0$ so small that $\varepsilon_1 \le 1/2C_0$, and assume that $N(t) + \|\partial_x \tilde{n}\|_{H^s} \le \varepsilon_1$. Then we get the desired estimate $N(t)^2 + D(t)^2 \le C\|w - \tilde{w}\|_{H^s}^2$ from (3.11). This completes the proof of Proposition 3.3. $\square$

References

[1] G.Q. Chen, J.W. Jerome and D.H. Wang, *Compressible Euler-Maxwell equations*, Transport Theory and Statistical Physics, **29** (2000), 311-331.

[2] S. Kawashima, *Systems of a hyperbolic-parabolic composite type, with applications to the equations of magnetohydrodynamics*, Doctoral Thesis, Kyoto University, 1984. http://repository.kulib.kyoto-u.ac.jp/dspace/handle/2433/97887

[3] Y. Shizuta and S. Kawashima, *Systems of equations of hyperbolic-parabolic type with applications to the discrete Boltzmann equation*, Hokkaido Math. J., **14** (1985), 249–275.

[4] Y. Ueda, R. Duan and S. Kawashima, Decay structure for symmetric hyperbolic systems with non-symmetric relaxation and its application. Arch. Ration. Mech. Anal. **205** (2012), no. 1, 239–266.

[5] Y. Ueda and S. Kawashima, Decay property of regularity-loss type for the Euler-Maxwell system. Methods Appl. Anal. **18** (2011), no. 3, 245–267.

[6] Y. Ueda, S. Wang and S. Kawashima, Dissipative structure of the regularity-loss type and time asymptotic decay of solutions for the Euler-Maxwell system. SIAM J. Math. Anal. **44** (2012), no. 3, 2002–2017

[7] T. Umeda, S. Kawashima and Y. Shizuta, *On the decay of solutions to the linearized equations of electro-magneto-fluid dynamics*, Japan J. Appl. Math., **1** (1984), 435–457.