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Investment Decisions and Debt Priority Structure: Straight Debt and Convertible Debt*

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1 Introduction

Why do firms issue convertible bonds? In corporate finance literature, several reasons for issuing convertible debt has already been confirmed as shown in Eisdorfer [3]. However, little attention has been paid to what underlies the priority status of convertible debt.

Wilson and Fabozzi [11] report that all convertible debt is subordinated to nonconvertible claims. Krishnaswami and Yaman [5] find that 93% of convertible bonds issued between 1983 and 2002 are subordinated. Furthermore, Eisdorfer [3] provides empirical evidence on the priority status of convertible debt, which is based on an investment-related conflict of interests between equity holders and debt holders. He finds that subordinated convertible debt is more likely to be issued by firms with greater potential for investment-related shareholder-bondholder conflicts. Barclay and Smith [1, 2] argue that expected agency costs between shareholders and bondholders affect not only the amount of debt issued but also its priority structure. Using simulation techniques, Siddiqi [9] shows that the agency costs of debt may be reduced or eliminated by convertible debt.

Sundaresan and Wang [10] and Hackbarth and Mauer [4] examine the investment strategies of the firm issuing multiple debt, taking into account the senior-sub structure. Sundaresan and Wang [10] show that the debt seniority structure has significant effects on the firm's default, leverage, and investment decisions, when existing debt is exogenously specified. Hackbarth and Mauer [4] show that priority structure plays an important role in balancing equityholders' over- and underinvestment incentives in the growth option and find that as leverage increases and credit quality deteriorates, firms will allocate priority to future debt issues by choosing a greater proportion of subordinate debt in their current debt structures. We extend the model in Hackbarth and Mauer [4] by incorporating convertible debt financing.

In this paper we examine the investment strategies and the agency cost of debt on the financing decisions and the debt priority. We explore the interaction between financing and investment decisions and analyze the consistency with empirical evidences in Eisdorfer [3].

The remainder of this paper is organized as follows. Sec. 2 describes the model. In Sec. 3, we explore the investment strategies and the agency cost of debt when the investment cost is all-equity financed, straight debt financed and convertible debt financed. Sec. 4 analyzes how the

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debt priority structure among the firm's debt issues affects the investment strategies. Finally, Sec. 5 summarizes this paper.

2 The model

We consider a firm with assets-in-place, which generates earnings of QX_t per unit time, where Q is the quantity produced from the asset-in-place and X_t is the demand shock for its product. We suppose that X_t is given by a geometric Brownian motion

$$dX_t = \mu X_t dt + \sigma X_t dW_t, \quad X_0 = x, \tag{2.1}$$

where μ and σ are the risk-adjusted expected growth rate and the volatility of X_t, respectively, and W_t is a standard Brownian motion defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. We assume that all agents are risk-neutral and discount their future payoffs at a rate $r(> \mu)$.

The firm also has a growth option, which increases earnings from QX_t to αQX_t by paying a fixed investment cost $\xi(\alpha - 1)Q$, where $\alpha > 1$ is the expanding scale of operations and $\xi > 0$ is the investment cost per unit of Q. The optimal investment policy selects the optimal investment time that maximizes the value of equity. Since the firm uses equity and convertible debt financing, equity value maximization may not consist with firm value maximization. Hence, for comparison, we consider the case in which the investment policy maximizes total firm value.

The firm is initially capitalized with equity and straight debt. This initial straight debt has infinite maturity and a coupon payment of s. The firm may issue additional convertible debt to finance the investment cost $\xi(\alpha - 1)Q$. This additional convertible debt also has infinite maturity and a coupon payment of c. These coupon payments are tax-deductible at a constant corporate tax rate τ. Our objective is to study the implication of the convertible debt financing on the timing of investment. By doing this, we assume that the firm's initial leverage level is predetermined. Hence, the coupon rate of the straight debt s is determined endogenously by fixing the initial leverage level. On the other hand, we assume that the firm finances a fraction γ of the investment cost with convertible debt financing. Thus, the coupon payment of the convertible debt c is determined endogenously by equating a part of the investment cost $\gamma\xi(\alpha - 1)Q$ with the convertible debt value at the investment time.

Before the growth option is exercised, since the firm issues the straight debt, the equity holders can receive the instantaneous profit $(1 - \tau)(QX_t - s)$. After exercising the growth option, the equity holders can receive the profit $(1 - \tau)(\alpha QX_t - s - c)$. Once the convertible debt is converted into stock, the firm need not pay coupon payment of convertible debt. Hence, after conversion the equity holders receive the profit $(1 - \tau)(\alpha QX_t - s)$. When the demand shock X_t falls down enough, the firm will default on its debt.

The optimal default policy of the equity holders selects the optimal default time that maximizes the equity's value before and after investment and after conversion. We assume that at default the equity holders receive nothing and the debt holders can receive the liquidation value net of assets in bankruptcy. Before investment, the straight debt holders receive the all net asset value. However, after investment this net asset value is divided between the straight and convertible debt along a debt priority rule. Also, after conversion the straight debt holders can receive the entire net asset value. In this section we examine the case in which the straight and convertible debt have the same priority. In Section 4 we analyze the debt priority structure.

We assume that the holders of convertible debt can convert the debt into a ratio η of the original equity. Our model follows Brennan and Schwartz (1977) and assumes block conversion, that is, all convertible debt holders exercise the conversion option at the same time. The optimal conversion policy of the convertible debt holders selects the optimal conversion time that maximizes the value of the convertible debt.

In what follows, we present security values before and after investment and after conversion.

2.1 Security and firm values after conversion

After the convertible debt is converted into stock, the firm becomes an entity that issues the equity and the straight debt. The equity holders can receive $(1-\tau)(\alpha Q X_t - s)$ until default and nothing at default. Also, the straight debt holders can receive the coupon payment of s until default and are entitled to the liquidation value net of assets after paying the bankruptcy cost.

Denote the values of equity and straight debt after conversion by $E_3(x)$ and $D_{s,3}(x)$. In the region for demand shock where there is no default, the instantaneous change in the equity and the straight debt values satisfy the ordinary differential equations

$$\frac{1}{2}\sigma^2 x^2 \frac{d^2 E_3}{dx^2} + \mu x \frac{dE_3}{dx} - rE_3 + (1-\tau)(\alpha Q x - s) = 0, \tag{2.2}$$

$$\frac{1}{2}\sigma^2 x^2 \frac{d^2 D_{s,3}}{dx^2} + \mu x \frac{dD_{s,3}}{dx} - rD_{s,3} + s = 0, \tag{2.3}$$

respectively. The solutions of (2.2) and (2.3) are given by

$$E_3(x) \quad = \quad a_1 x^{\beta_1} + a_2 x^{\beta_2} + (1-\tau)\left(\frac{\alpha Q x}{r-\mu} - \frac{s}{r}\right) \tag{2.4}$$

$$D_{s,3}(x) \quad = \quad a_3 x^{\beta_1} + a_4 x^{\beta_2} + \frac{s}{r} \tag{2.5}$$

where a_i, $i = 1, \cdots, 4$ are constants to be determined from the boundary conditions and $\beta_1 = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} > 1$ and $\beta_2 = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} < 0$. At the upper boundary, the equity and the straight debt become effectively risk-free, so their values must approach $(1-\tau)(\alpha Q x/(r-\mu) - s/r)$ and s/r, respectively, which imply $a_2 = a_4 = 0$ in Eqs. (2.4) and (2.5). At default, the equity holder can receive nothing. On the other hand, the straight debt holders obtain the all net asset value. These give the boundary conditions

$$E_3(x_d^3) \quad = \quad 0, \tag{2.6}$$

$$D_{s,3}(x_d^3) \quad = \quad (1-\theta)\alpha\epsilon(x_d^3), \tag{2.7}$$

where x_d^3 is the post-conversion default threshold, $\theta \in [0, 1]$ is the proportional bankruptcy cost and $\epsilon(x)$ is the asset value

$$\epsilon(x) = \frac{1-\tau}{r-\mu} Q x. \tag{2.8}$$

The default threshold x_d^3 can be determined by the smooth-pasting condition in which the derivation of $E_3(x)$ with respect to x equals the derivation of the payoff for the equity holders at the default threshold x_d^3. Hence,

$$\frac{dE_3}{dx}(x_d^3) = 0. \tag{2.9}$$

Using the boundary conditions in Eq. (2.6), (2.7) and (2.9), for $x > x_d^3$, the post-conversion values of equity and straight debt, $E_3(x)$ and $D_{s,3}(x)$, are given by

$$E_3(x) = (1-\tau)\left\{\left(\frac{\alpha Qx}{r-\mu} - \frac{s}{r}\right) - \left(\frac{\alpha Qx_d^3}{r-\mu} - \frac{s}{r}\right)\left(\frac{x}{x_d^3}\right)^{\beta_2}\right\}, \tag{2.10}$$

$$D_{s,3}(x) = \frac{s}{r} + \left\{(1-\theta)\alpha\epsilon(x_d^3) - \frac{s}{r}\right\}\left(\frac{x}{x_d^3}\right)^{\beta_2}, \tag{2.11}$$

and the post-conversion default threshold is given by

$$x_d^3 = \frac{\beta_2}{\beta_2 - 1}\frac{r-\mu}{r}\frac{s}{\alpha Q}, \tag{2.12}$$

where $(x/x_d^3)^{\beta_2}$ is the value of a contingent claim that pays \$1 if the demand level hits x_d^3 the first time from above.

Summing equations (2.10) and (2.11), we obtain the post-conversion firm value

$$V_3(x) = \alpha\epsilon(x) + \frac{\tau s}{r}\left\{1 - \left(\frac{x}{x_d^3}\right)^{\beta_2}\right\} - \theta\alpha\epsilon(x_d^3)\left(\frac{x}{x_d^3}\right)^{\beta_2}, \tag{2.13}$$

which is the firm's unlevered value plus the expected present value of the debt tax shields minus the expected present value of the bankruptcy cost.

2.2 Security and firm values after investment

At the investment time, the firm finances the partially investment cost with convertible debt. The convertible debt holders receive the coupon payment of c until default or conversion and can convert the debt into a ratio η of the original equity so as to maximize the convertible debt value. The equity holders receive the value $(1-\tau)(\alpha QX_t - s - c)$ until default or conversion and face the dilution of the equity value at the conversion time. Thus, the equity value becomes diluted to $1/(1+\eta)$ of it at the default time. The straight debt holders receive the coupon payment of s until default.

At the default time, the equity holders receive nothing. In this section since we assume the same debt priority, the straight and convertible debt holders receive the value divided by the size of the coupon payment in default.

Similarly to the post-conversion case , for $x_d^2 < x < x_c$, the post-investment values of equity, the straight and convertible debt, $E_2(x)$, $D_{s,2}(x)$ and $D_c(x)$, are given by

$$E_2(x) = a_5 x^{\beta_1} + a_6 x^{\beta_2} + (1-\tau)\left(\frac{\alpha Qx}{r-\mu} - \frac{s+c}{r}\right), \tag{2.14}$$

$$D_{s,2}(x) = a_7 x^{\beta_1} + a_8 x^{\beta_2} + \frac{s}{r}, \tag{2.15}$$

$$D_c(x) = a_9 x^{\beta_1} + a_{10} x^{\beta_2} + \frac{c}{r}, \tag{2.16}$$

where x_d^2 and x_c are the post-investment default and conversion thresholds, a_i, $i = 5, \cdots, 10$ are constants to be determined from the boundary conditions.

After the investment, there are two boundaries. The upper boundary conditions that come from the conversion policy of the convertible debt holders are given by

$$E_2(x_c) = \frac{1}{1+\eta} E_3(x_c), \tag{2.17}$$

$$D_{s,2}(x_c) = D_{s,3}(x_c), \tag{2.18}$$

$$D_c(x_c) = \frac{\eta}{1+\eta} E_3(x_c). \tag{2.19}$$

Equation (2.17) means that the equity value is diluted to $1/(1+\eta)$ of the post-conversion equity value in conversion. Equation (2.18) implies no wealth transfer from the straight debt holder at the conversion time. Equation (2.19) means that the convertible debt holders receive $\eta/(1+\eta)$ of the post-conversion equity value in conversion.

The lower boundary conditions that relate to the default threshold are given by

$$E_2(x_d^2) = 0, \tag{2.20}$$

$$D_{s,2}(x_d^2) = \frac{s}{s+c}(1-\theta)\alpha\epsilon(x_d^2), \tag{2.21}$$

$$D_c(x_d^2) = \frac{c}{s+c}(1-\theta)\alpha\epsilon(x_d^2). \tag{2.22}$$

Equation (2.20) means that the equity holders receive nothing at default. Equations (2.21) and (2.22) imply that the straight and convertible debt holders receive the net asset value divided by the size of the coupon payment in default.

Substituting Equations (2.14), (2.15) and (2.16) into Equations (2.17)–(2.22), we can determine that

$$\begin{aligned} E_2(x) &= (1-\tau)\left(\frac{\alpha Q x}{r-\mu} - \frac{s+c}{r}\right) \\ &\quad + \left\{\frac{1}{1+\eta}E_3(x_c) - (1-\tau)\left(\frac{\alpha Q x_c}{r-\mu} - \frac{s+c}{r}\right)\right\} p_1(x; x_c, x_d^2) \\ &\quad -(1-\tau)\left(\frac{\alpha Q x_d^2}{r-\mu} - \frac{s+c}{r}\right) p_2(x; x_c, x_d^2), \end{aligned} \tag{2.23}$$

$$\begin{aligned} D_{s,2}(x) &= \frac{s}{r} + \left\{D_{s,3}(x_c) - \frac{s}{r}\right\} p_1(x; x_c, x_d^2) \\ &\quad + \left\{\frac{s}{s+c}(1-\theta)\alpha\epsilon(x_d^2) - \frac{s}{r}\right\} p_2(x; x_c, x_d^2), \end{aligned} \tag{2.24}$$

$$\begin{aligned} D_c(x) &= \frac{c}{r} + \left\{\frac{\eta}{1+\eta}E_3(x_c) - \frac{c}{r}\right\} p_1(x; x_c, x_d^2) \\ &\quad + \left\{\frac{c}{s+c}(1-\theta)\alpha\epsilon(x_d^2) - \frac{c}{r}\right\} p_2(x; x_c, x_d^2), \end{aligned} \tag{2.25}$$

where $p_1(x; x_a, x_b)$ for $x_a > x_b$ is the expected present value of \$1 contingent on X_t first reaching the threshold x_b from above before reaching x_a. The $p_2(x; x_a, x_b)$ is that of \$1 contingent on X_t

first reaching the threshold x_a from bellow before reaching the threshold x_b, that is,

$$p_1(x; x_a, x_b) = E[e^{-rT_a}|T_a < T_b] = \frac{x^{\beta_1} x_b^{\beta_2} - x^{\beta_2} x_b^{\beta_1}}{x_a^{\beta_1} x_b^{\beta_2} - x_a^{\beta_2} x_b^{\beta_1}}, \tag{2.26}$$

$$p_2(x; x_a, x_b) = E[e^{-rT_b}|T_b < T_a] = \frac{x_a^{\beta_1} x^{\beta_2} - x_a^{\beta_2} x^{\beta_1}}{x_a^{\beta_1} x_b^{\beta_2} - x_a^{\beta_2} x_b^{\beta_1}}, \tag{2.27}$$

where $E[\cdot|\cdot]$ denotes conditional expectation, and T_a and T_b are the times when X_t reaches the thresholds x_a and x_b, respectively.

Summing equations (2.23), (2.24) and (2.25), the post-investment firm value $V_2(x)$ is represented by

$$\begin{aligned} V_2(x) &= \alpha\epsilon(x) + \frac{\tau s}{r}\left\{1 - \left(\frac{x_c}{x_d^3}\right)^{\beta_2} p_1(x; x_c, x_d^2) - p_2(x; x_c, x_d^2)\right\} \\ &+ \frac{\tau c}{r}\left\{1 - p_1(x; x_c, x_d^2) - p_2(x; x_c, x_d^2)\right\} \\ &- \theta\left\{\epsilon(x_d^3)\left(\frac{x_c}{x_d^3}\right)^{\beta_2} p_1(x; x_c, x_d^2) + \epsilon(x_d^2) p_2(x; x_c, x_d^2)\right\}, \end{aligned} \tag{2.28}$$

which implies that the first term is the firms's unlevered value, the second and third terms are the expected present value of the straight and convertible debt tax shields and the last term is the expected present value of the bankruptcy cost.

Next, we determine the default and conversion thresholds. The optimal default threshold can be determined by the smooth-pasting condition in which the derivation of $E_2(x)$ with respect to x equals the deviation of the payoff for the equity holders at the default threshold x_d^2. On the other hand, the optimal conversion threshold is derived from the smooth-pasting condition in which the derivation of $D_c(x)$ with respect to x equals the deviation of the payoff for the holders of the convertible debt at the conversion threshold x_c. Hence,

$$\frac{dE_2}{dx}(x_d^2) = 0, \tag{2.29}$$

$$\frac{dD_c}{dx}(x_c) = \frac{\eta}{1+\eta}\frac{dE_3}{dx}(x_c). \tag{2.30}$$

Substituting Equations (2.10), (2.23) and (2.25) into Equations (2.29) and (2.30), we find that

$$\begin{aligned} x_d^2 &= \frac{r-\mu}{\alpha(1-\tau)Q}\left[(1-\tau)\left(\frac{\alpha Q x_d^2}{r-\mu} - \frac{s+c}{r}\right) q_2(x_d^2; x_c, x_d^2)\right. \\ &\left. - \left\{\frac{1}{1+\eta}E_3(x_c) - (1-\tau)\left(\frac{\alpha Q x_c}{r-\mu} - \frac{s+c}{r}\right)\right\} q_1(x_d^2; x_c, x_d^2)\right], \end{aligned} \tag{2.31}$$

$$\begin{aligned} x_c &= \frac{r-\mu}{\alpha(1-\tau)Q}\left[\frac{1+\eta}{\eta}\left\{\left(\frac{\eta}{1+\eta}E_3(x_c) - \frac{c}{r}\right) q_1(x_c; x_c, x_d^2)\right.\right. \\ &\left.\left. + \left(\frac{c}{s+c}(1-\theta)\alpha\epsilon(x_d^2) - \frac{s}{r}\right) q_2(x_c; x_c, x_d^2)\right\} + \beta_2(1-\tau)\left(\frac{\alpha Q x_d^3}{r-\mu} - \frac{s}{r}\right)\left(\frac{x^*}{x_d^3}\right)^{\beta_2}\right] \end{aligned} \tag{2.32}$$

where $q_1(x; x_a, x_b)$ and $q_2(x; x_a, x_b)$ represent

$$q_1(x; x_a, x_b) = x\frac{\partial p_1}{\partial x}(x; x_a, x_b) = \frac{\beta_1 x^{\beta_1} x_b^{\beta_2} - \beta_2 x^{\beta_2} x_b^{\beta_1}}{x_a^{\beta_1} x_b^{\beta_2} - x_a^{\beta_2} x_b^{\beta_1}}, \tag{2.33}$$

$$q_2(x; x_a, x_b) = x\frac{\partial p_2}{\partial x}(x; x_a, x_b) = \frac{\beta_2 x_a^{\beta_1} x^{\beta_2} - \beta_1 x_a^{\beta_2} x^{\beta_1}}{x_a^{\beta_1} x_b^{\beta_2} - x_a^{\beta_2} x_b^{\beta_1}}. \tag{2.34}$$

2.3 Security and firm values before investment

Before the investment, the firm has the straight debt issue and the equity holders receive the value $(1-\tau)(QX_t - s)$ until investment or default.

As shown in Appendix B, for $x_d^1 < x < x^*$, the pre-investment values of equity and the straight debt, $E_1(x)$ and $D_{s,1}(x)$ are given by

$$E_1(x) = a_{11}x^{\beta_1} + a_{12}x^{\beta_2} + (1-\tau)\left(\frac{Qx}{r-\mu} - \frac{s}{r}\right) \tag{2.35}$$

$$D_{s,1}(x) = a_{13}x^{\beta_1} + a_{14}x^{\beta_2} + \frac{s}{r}, \tag{2.36}$$

where x_d^1 and x^* are the pre-investment default and investment thresholds, a_i, $i = 11, \cdots, 14$ are constants to be determined from the boundary conditions.

As in the case of the post-investment, there are two boundaries before the investment. The upper boundary conditions that come from the investment policy are given by

$$E_1(x^*) = E_2(x^*) + D_c(x^*) - \xi(\alpha-1)Q, \tag{2.37}$$

$$D_{s,1}(x^*) = D_{s,2}(x^*) \tag{2.38}$$

Equation (2.37) implies that the investment option is exercised by financing with convertible debt and then paying the investment cost $\xi(\alpha-1)Q$. Equation (2.38) implies no wealth transfer from the straight debt holder at the investment time.

The lower boundary conditions that relate to the default threshold are given by

$$E_1(x_d^1) = 0, \tag{2.39}$$

$$D_{s,1}(x_d^1) = (1-\theta)\epsilon(x_d^1). \tag{2.40}$$

Equation (2.39) means that the equity holders receive nothing at default. Equation (2.40) implies that the straight debt holders receive the all net asset value in default.

Substituting Equations (2.35) and (2.36) into Equations (2.37)–(2.40), we determine that

$$\begin{aligned} E_1(x) &= (1-\tau)\left(\frac{Qx}{r-\mu} - \frac{s}{r}\right) \\ &+\left\{E_2(x^*) + D_c(x^*) - \xi(\alpha-1)Q - (1-\tau)\left(\frac{Qx^*}{r-\mu} - \frac{s}{r}\right)\right\}p_1(x; x^*, x_d^1) \\ &-(1-\tau)\left(\frac{Qx_d^1}{r-\mu} - \frac{s}{r}\right)p_2(x; x^*, x_d^1), \end{aligned} \tag{2.41}$$

$$D_{s,1}(x) = \frac{s}{r} + \left(D_{s,2}(x^*) - \frac{s}{r}\right)p_1(x; x^*, x_d^1) + \left((1-\theta)\epsilon(x_d^1) - \frac{s}{r}\right)p_2(x; x^*, x_d^1). \tag{2.42}$$

Summing equations (2.41) and (2.42), the pre-investment firm value $V_1(x)$ is represented by

$$\begin{aligned} V_1(x) &= \epsilon(x) + (\alpha-1)(\epsilon(x^*) - \xi Q)p_1(x;x^*,x_d^1) \\ &+ \frac{\tau s}{r}\left[1 - \left\{\left(\frac{x_c}{x_d^3}\right)^{\beta_2} p_1(x^*;x_c,x_d^2) + p_2(x^*;x_c,x_d^2)\right\} p_1(x;x^*,x_d^1) - p_2(x;x^*,x_d^1)\right] \\ &+ \frac{\tau c}{r}\left\{1 - p_1(x^*;x_c,x_d^2) - p_2(x^*;x_c,x_d^2)\right\} p_1(x;x^*,x_d^1) \\ &- \theta\left[\left\{\epsilon(x_d^3)\left(\frac{x_c}{x_d^3}\right)^{\beta_2} p_1(x^*;x_c,x_d^2) + \epsilon(x_d^2)p_2(x^*;x_c,x_d^2)\right\} p_1(x;x^*,x_d^1) + \epsilon(x_d^1)p_2(x;x^*,x_d^1)\right] \end{aligned} \tag{2.43}$$

which implies that the first term is the firms's unlevered value, the second term is the investment option value, third and fourth terms are the expected present value of the straight and convertible debt tax shields and the last term is the expected present value of the bankruptcy cost.

2.3.1 Second-best investment policy

We determine the pre-investment default and investment thresholds. As noted above, the investment policy maximizes the equity value. Since this policy may not maximize total firm value, we refer to it as the second-best investment threshold. The optimal default threshold can be determined by the smooth-pasting condition in which the derivation of $E_1(x)$ with respect to x equals the deviation of the payoff for the equity holders at the default threshold x_d^1. On the other hand, the optimal second-best investment threshold is derived from the smooth-pasting condition in which the derivation of $E_1(x)$ with respect to x equals the deviation of the payoff for the equity holders at the second-best investment threshold x_{SB}^*. Therefore,

$$\frac{dE_1}{dx}(x_d^1) = 0, \tag{2.44}$$

$$\frac{dE_1}{dx}(x_{SB}^*) = \frac{dE_2}{dx}(x_{SB}^*) + \frac{dD_c}{dx}(x_{SB}^*). \tag{2.45}$$

Substituting Equations (2.23), (2.25) and (2.41) into Equations (2.44) and (2.45), we find that

$$\begin{aligned} x_d^1 = \frac{r-\mu}{(1-\tau)Q}\Bigg[&(1-\tau)\left(\frac{Qx_d^1}{r-\mu} - \frac{s}{r}\right) q_2(x_d^1;x_{SB}^*,x_d^1) \\ &- \left\{E_2(x_{SB}^*) - \xi(\alpha-1)Q - (1-\tau)\left(\frac{Qx_{SB}^*}{r-\mu} - \frac{s}{r}\right)\right\} q_1(x_d^1;x_{SB}^*,x_d^1)\Bigg], \end{aligned} \tag{2.46}$$

$$\begin{aligned} x_{SB}^* = \frac{r-\mu}{(\alpha-1)(1-\tau)Q}\Bigg[&\left\{E_2(x_{SB}^*) - \xi(\alpha-1)Q - (1-\tau)\left(\frac{Qx_{SB}^*}{r-\mu} - \frac{s}{r}\right)\right\} q_1(x_{SB}^*;x_{SB}^*,x_d^1) \\ &- \left\{\frac{1}{1+\eta}E_3(x_c) - (1-\tau)\left(\frac{\alpha Q x_c}{r-\mu} - \frac{s+c}{r}\right)\right\} p_1(x_{SB}^*;x_c,x_d^2) \\ &- (1-\tau)\left(\frac{Qx_d^1}{r-\mu} - \frac{s}{r}\right) q_2(x_{SB}^*;x_{SB}^*,x_d^1) + (1-\tau)\left(\frac{\alpha Q x_d^2}{r-\mu} - \frac{s+c}{r}\right) p_2(x_{SB}^*;x_c,x_d^2)\Bigg]. \end{aligned} \tag{2.47}$$

2.3.2 First-best investment policy

For comparison, we determine the first-best investment threshold, which maximizes total firm value. Thus, we find the first-best investment threshold x^*_{FB} that satisfies the following smooth-pasting condition

$$\frac{dV_1}{dx}(x^*_{FB}) = \frac{dV_2}{dx}(x^*_{FB}). \tag{2.48}$$

Substituting Equations (2.28) and (2.43) into Equation (2.48), we find that

$$\begin{aligned} x^*_{FB} &= \frac{r-\mu}{(\alpha-1)(1-\tau)Q}\Bigg[\left\{V_2(x^*_{FB}) - \xi(\alpha-1)Q - \epsilon(x^*_{FB}) - \frac{\tau s}{r}\right\} q_1(x^*_{FB}; x^*_{FB}, x^1_d) \\ &\quad - \left\{V_3(x_c) - \alpha\epsilon(x_c) - \frac{\tau(s+c)}{r}\right\} q_1(x^*_{FB}; x_c, x^2_d) \\ &\quad - \left(\theta\epsilon(x^1_d) + \frac{\tau s}{r}\right) q_2(x^*_{FB}; x^*_{FB}, x^1_d) + \left(\theta\alpha\epsilon(x^2_d) + \frac{\tau s}{r}\right) q_2(x^*_{FB}; x_c, x^2_d)\Bigg]. \end{aligned} \tag{2.49}$$

The default threshold for this first-best case is analytically identical to that in Equation (2.46), but because the expressions for the first- and second-best investment thresholds in Equations (2.49) and (2.47) are different, the capital structure, the conversion threshold, and the post-conversion and post-investment default thresholds will be different. Hence, we predict that the pre-investment default thresholds will also be different.

2.4 Agency cost of debt

The investment policy maximizes the equity value. Since this policy may not maximize total firm value, we refer to it as the second-best investment policy. For comparison, we determine the first-best investment policy, which maximizes total firm value. We compute the agency cost of debt as the difference between the current firm values under first-and second-best policies. Hence, the agency cost of debt is given by

$$AC(\%) = \frac{V_1^{FB}(x_0) - V_1^{SB}(x_0)}{V_1^{SB}(x_0)} \times 100, \tag{2.50}$$

where $V_1^{FB}(x_0)$ and $V_1^{SB}(x_0)$ are the firm values for first- and second-best investment policies.

2.5 Capital structure

The leverage ratios for the initial, post-investment and post-conversion are given by

$$l_1 = \frac{D_{s,1}(x_0)}{V_1(x_0)}, \tag{2.51}$$

$$l_2 = \frac{D_{s,2}(x^*) + D_c(x^*)}{V_2(x^*)}, \tag{2.52}$$

$$l_3 = \frac{D_{s,3}(x_c)}{V_3(x_c)}. \tag{2.53}$$

We suppose that the coupon payment for the straight debt s is determined endogenously by fixing initial leverage ratio l_1. On the other hand, since the firm finances a fraction γ of the investment cost with convertible debt financing, the coupon payment of the convertible debt c is determined endogenously by satisfying

$$D_c(x^*) = \gamma\xi(\alpha - 1)Q. \tag{2.54}$$

3 Financing Decisions

We explore the first- and second-best investment strategies and the agency cost of debt when the investment cost is all-equity financed, straight debt financed, and convertible debt financed on the same debt priority. We use the following base case parameters: The initial demand level, x_0, is one, the quantity produced from the asset-in-place, Q, is 1.0, the expanding scale of operations in investment, α, is 2.0 and the investment cost per unit of Q, ξ, is 25.0. The volatility of X_t, σ, is 20% per year, the drift rate of X_t, μ, is 0% per year, the risk-free rate, r , is 5% per year, the corporate tax rate, τ, is 30%, the proportional bankruptcy costs, θ, are 30% of the value of assets-in-place at the time of bankruptcy, the conversion ratio, η, is 30% of the original equity, the ratio of the investment cost ratio with convertible debt financing, γ, is 1.0, and the initial leverage ratio, l_1 is 50% of total initial firm value.

Table 1 reports the calculation results of the first- and second-best investment thresholds, x^*_{FB} and x^*_{SB} and the agency cost of debt, $AC(\%)$ for the first- and second-best investment strategies when the investment cost is all-equity financed, straight debt financed such in Lyandres and Zhdanov [6] and Hackbarth and Mauer [4], and convertible debt financed.

Table 1: First- and second-best investment and financing decisions

	First-best x^*_{FB}	Second-best x^*_{SB}	Agency cost AC
All-equity financing	3.2997	3.3489	0.002
Straight debt financing	2.6457	2.2687	0.361
Convertible debt financing	2.6822	2.1452	0.659

The investment thresholds in all-equity financing are larger than those in debt financing. The all-equity financing leads to underinvestment relative to the debt financing as shown in Myers [8]. Also, in the debt financing the first-best investment threshold is larger than the second-best investment threshold. As shown in Mauer and Sarkar [7], since the second-best investment strategy is the equity-value maximization, the equity holders are not affected by the benefit of the debt holders at investment, and are hence indifferent to increased risk of default resulted from the earlier investment. Therefore, the first-best investment strategy leads to underinvestment compared with the second-best investment strategy.

For the second-best investment strategy, the investment threshold in the convertible debt financing is smaller than that in the straight debt financing. In order to avoid to a wealth transfer from the equity holder to convertible debt holders in conversion, the equity holders invest at lower investment threshold relative to that in the straight debt financing. Therefore, the convertible debt financing leads to overinvestment compared to the straight debt financing.

On the other hand, for the first-best investment strategy, the investment threshold in the convertible debt financing is larger than that in the straight debt financing. Once the investment is financed with debt, the firm can enjoy the interest tax shields, and so can have a tax incentive to accelerate the investment. Since convertible debt includes the option to convert the debt into stocks, the presence of its option reduces the magnitude of the tax shield effect. The firm with the convertible debt financing invests at the higher investment threshold compared with that in the straight debt financing and maximizes the firm value. Hence, the convertible debt financing leads to underinvestment relative to the straight debt financing.

The agency cost of debt in the convertible debt financing is the highest relative to those in the all-equity and debt financing. Hence, the firms financed with the convertible debt have greater potential on the investment-related agency conflicts for the same debt priority.

4 Debt priority structure

In this section, we relax the assumption of the same debt priority in bankruptcy and analyze how the priority among the firm's debt issues influences the investment strategy, the capital structure, the firm value, and the agency cost of debt in the convertible debt financing. We compare the results for three types of the debt priority structure; the same debt priority, the outstanding straight senior debt and the convertible subordinated debt financing, and the outstanding straight subordinated debt and the convertible senior debt financing.

In the case of the same debt priority, the straight and convertible debt holders receive the liquidation value net of assets in bankruptcy divided by the size of the coupon payment in default after the issue of the convertible debt. That is, when the demand level is x_d^2, the straight and convertible debt holders receive $\frac{s}{s+c}(1-\theta)\epsilon(x_d^2)$ and $\frac{c}{s+c}(1-\theta)\epsilon(x_d^2)$, respectively. In the case of the outstanding straight senior debt and the convertible subordinated debt financing, the straight debt holders either are guaranteed the par price or receive the total liquidation value net of assets in bankruptcy. That is, when the demand level is x_d^2, the straight debt holders receive

$$D_s(x_d^2) = \min\left(F_s, (1-\theta)\epsilon(x_d^2)\right), \tag{4.1}$$

where F_s is the par value of the straight debt and is equal to $F_s = D_s(x_0)$. Convertible debt holders are guaranteed payments at default. They receive either the par price or the remaining value calculated by subtracting the par price of straight debt from the liquidation value net of assets in bankruptcy. If the remaining value is negative, convertible debt holders cannot receive nothing at default. That is, the convertible debt holders receive

$$D_c(x_d^2) = \min(F_c, \max\left((1-\theta)\epsilon(x_d^2) - F_s, 0\right)). \tag{4.2}$$

Let F_c denote the par value of convertible debt issued at investment time. The convertible debt is also issued at par, and thus we have $F_c = D_c(x^*)$.

On the other hand, in the case of the outstanding straight subordinated debt and the convertible senior debt financing, the straight debt holders receive

$$D_s(x_d^2) = \min\left(F_s, \max\left((1-\theta)\epsilon(x_d^2) - F_c, 0\right)\right), \tag{4.3}$$

and the convertible debt holders receive

$$D_c(x_d^2) = \min(F_c, (1-\theta)\epsilon(x_d^2)) \tag{4.4}$$

in bankruptcy.

Table 2 reports the calculation results for the first- and second-best investment thresholds, x_{FB}^* and x_{SB}^*, and the agency cost of debt, AC in the case of the same priority, the convertible subordinated debt financing and the convertible senior debt financing.

Table 2: Debt priority structure

	First-best x_{FB}^*	Second-best x_{SB}^*	Agency cost AC
Same priority	2.6822	2.1452	0.659
Convertible subordinated debt financing	2.6649	2.2478	0.303
Convertible senior debt financing	2.6903	2.0932	0.910

4.1 Investment strategies

For the second-best investment strategy, the investment threshold in the convertible senior debt financing is smaller than that in the same priority. As shown in Hackbarth and Mauer (2012), when the outstanding straight debt does not dominate, the equity holders have an incentive to dilute the initial debt holders' claim by accelerating the debt-financed investment. Hence, the investment occurs earlier in the convertible senior debt financing. On the other hand, the second-best investment threshold in the convertible subordinated debt financing is larger than that in the same priority. Protecting the outstanding straight debt from dilution by means of the convertible subordinated debt financing guarantees that the initial debt it will benefit from the investment and hence provides equity holders with an incentive to underinvest.

Also, the first-best investment threshold in the convertible subordinated debt financing is smaller than that in the convertible senior debt financing. Since the additional debt used to finance its investment does not dilute the outstanding debt if the outstanding straight debt dominates, the firm exercises the investment option earlier. Hence, the convertible subordinated debt financing leads to underinvestment relative to the convertible senior debt financing.

The convertible subordinated debt financing leads to the decrease to the agency cost of debt.

4.2 Agency Cost of Debt

In order to explore the effect of the convertible subordinated debt financing on the agency cost of debt, we next focus on the uncertainty and the capital structure.

Figure 1 presents the agency cost of debt with respect to volatility σ in the cases of the all-equity financing, the straight debt financing, the same debt priority of the outstanding straight debt and the financing by the convertible debt, the outstanding straight senior debt and the convertible subordinated debt financing, and the outstanding straight subordinated debt and the convertible senior debt financing. The agency cost of debt in the case of the convertible subordinated debt financing is lower than that in the case of the straight debt financing. This is consistent with empirical evidence in Eisdorfer [3] that subordinated convertible debt is more likely to be issued by firms with greater potential for investment-related agency conflicts.

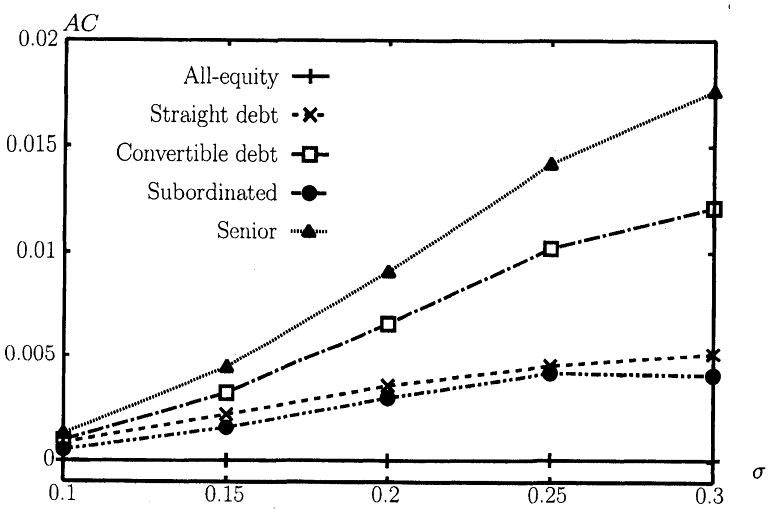

Figure 1: The agency cost of debt for volatility σ

In Fig. 2 the agency cost of debt is drawn as a function of the initial leverage ratio l_1 when the volatility σ is 0.2. When the initial leverage ratio is high, the agency cost of debt in the case of the convertible subordinated debt financing is lower than that in the case of the straight debt financing. This is consistent with empirical evidence in Eisdorfer [3]. On the other hand, when the initial leverage ratio is low, the agency cost of debt in the case of the convertible subordinated debt financing is higher than that in the case of the straight debt financing. This result is our key finding that has not been obtained in empirical evidences.

5 Conclusion

In this paper we explore the investment strategies and the agency cost of debt on the financing decisions and the debt priority and analyze the consistency with empirical evidences.

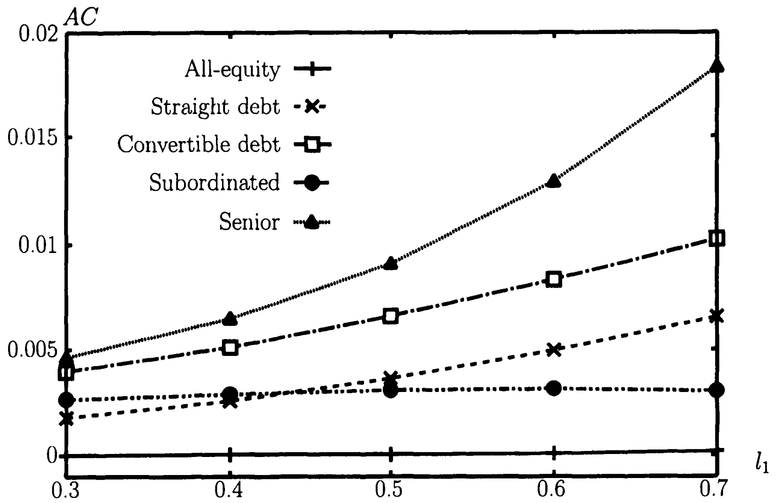

Figure 2: The agency cost of debt for initial leverage ratio l_1

In the convertible debt financing, we show that the investment of the firm with the first-best firm value-maximization policy occurs earlier than with the second-best equity value-maximization policy. Furthermore, the convertible debt financing leads to the higher agency cost of debt compared to the straight debt financing.

However, when the leverage ratio is higher, the agency cost of debt in the case of the convertible subordinated debt financing is lower than that in the case of the straight debt financing. This result is consistent with empirical evidence in Eisdorfer [3] that subordinated convertible debt is more likely to be issued by firms with greater potential for investment-related agency conflicts. On the other hand, when the leverage ratio is lower, the agency cost of debt in the case of the convertible subordinated debt financing is higher than that in the case of the straight debt financing. This result is our key finding that has not been obtained in empirical evidences.

In future work, we would like to address the investment strategies and the agency cost of debt for the firm with outstanding convertible debt.

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