

## Project Financing for Investments in Energy Technologies\*

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### 1 Introduction

Project finance has been widely used for various projects such as natural resource development of oil, natural gas and mineral, and infrastructure construction of pipeline, power plant, and toll road. In particular, project finance is expected to be utilized for the construction project of the infrastructure in developing countries and the resource development project in competition for world's resources.

Although, in the past, project financing has usually related to a greenfield project, which is the project to develop new properties, recently a brownfield project, which is the rehabilitation or the acquisition of an existing property, has increased. This is because that the independent power producers (IPPs) abandon the international projects, and that the infrastructure development firm increases the liquidity of trading assets [4].

For example, in 2004, a joint venture of International Power and Mitsui acquired Edison Mission Energy's international generation assets for US\$2.4 billion. Also, Marubeni and Tokyo Electric Power Company acquired Mirant's independent power generation assets in the Philippines for US\$3.43 billion in 2006. Thus, for investment projects as these projects, evaluations of investment decision and project financing are important. Finnerty [3] evaluates profitability and debt-paying ability of projects using project financing by means of discounted cash flow method. In projects as above examples, however, there exist various uncertainties such as risks of output prices and trading, accidents and disasters in projects, policy changes of countries in which the properties are located, and nonfulfillment of contract. Thus, evaluations under uncertainty are required because there exist many risks in the investment and operation regarding project financing.

Real options theory has recently attracted growing attention for the evaluation method of an investment project under uncertainty [2, 16]. Using this theory, the determination of the optimal investment timing and evaluations of the project value have been studied by many research groups. Brandao and Saraiva [1] develop a real option model allowing the government to analyze the cost and benefit of the project and applied it to BR-163 Toll Road infrastructure linking the Brazilian Midwest to the Amazon River. Ng and Bjornsson [10] also analyze a toll road concession project by means of real options framework. On the other hand, there is a

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growing body of literature analyzing the relation between investment and financing decisions of the firm in real options framework. These include Refs. [8, 9, 7, 13].

In this paper we develop a model to analyze debt guarantee and debt-paying ability in project finance by means of a framework of these real options models. Advantages of project financing are shown by comparing a model in which the investment is financed with equity and debt with a case in which the investment is financed entirely with equity. We also show how catastrophic risks affect the investment timing, the default timing, and debt guarantee. Furthermore, we apply this model to a investment project for electric power plants, and evaluate the acquisition project of Mirant's generation assets in Philippines by the consortium led by Marubeni and Tokyo Electric Power Company. We show the influence of coupon payment and salvage value on the distance to abandon the project.

The remainder of this paper is organized as follows. In Section 2, we describe the model for analyzing the investment decision and project financing. Section 3 derives the solution by numerical calculations and provides some results of numerical analysis. In Section 4, we apply the model developed in Section 2 to a investment project for electric power plants. Finally, Section 5 concludes the paper.

## 2 The model

We show how the project financing affects the investment decisions and the project value. First, we present a benchmark model in which the investment is financed with all equity. Finally, we consider the case in which the investment is financed with equity and debt in order to examine the interactions between investment decision and project financing.

Since there exist various uncertainties in the investment project, the consideration of them is required for the project evaluation. In countries which have a political risk, the catastrophic risks have a large influence on the project such as natural resource development and infrastructure construction. Thus, we assume that the cash flow follows a geometric Brownian motion,

$$dX_t = \mu X_t dt + \sigma X_t dW_t, \quad (2.1)$$

where  $\mu$  is the instantaneous expected growth rate of  $X_t$ ,  $\sigma$  is the associated volatility and  $W_t$  is a standard Brownian motion. The lifetime of the project is a random variable and follows a Poisson process with intensity,  $\lambda$ . Therefore, there exists the probability  $\lambda dt$  that the project will be permanently abandoned during the following short interval of time  $dt$ . The profit flow obtained from the project operating is represented by the following equation,

$$\pi^1(X_t) = (1 - \tau)(X_t - c), \quad (2.2)$$

where  $\tau$  is a constant corporate tax rate and  $c$  is a operating cost of the project.

### 2.1 All-equity financing

In this section, we consider the greenfield project which is financed entirely with equity. Consider a firm that starts operating a plant by incurring investment cost. After the investment decision, the project can generate the profit flow that can be terminated upon abandonment.

### 2.1.1 Operating value after investment

In order to obtain the option value of the investment, we first provide the operating value of the project, that is, the equity value. The firm possessing the greenfield project maximizes the equity value by selecting the abandonment time  $T_d^1$ . Thus, the total value of equity can be represented by the following equation,

$$F(x) = \sup_{T_d^1} \mathbb{E}_t^x \left[ \int_t^{T_d^1} e^{-\rho(s-t)} (1-\tau)(X_s - c) ds \right], \quad (2.3)$$

where  $\rho$  is the discount rate. Given the constant threshold of the abandonment  $x_d^1$ , the optimal abandonment time  $T_d^{1*}$  has the following form,

$$T_d^{1*} = \inf \left\{ T_d^1 > 0 \mid X_{T_d^1} \leq x_d^1 \right\}. \quad (2.4)$$

The following differential equation, which is satisfied by the equity value, is derived from the Bellman equation (See, for example, Dixit and Pindyck (1994)),

$$\frac{1}{2} \sigma^2 x^2 F''(x) + \mu x F'(x) - (\rho + \lambda) F(x) + (1 - \tau)(x - c) = 0. \quad (2.5)$$

The general solutions of this equation are given as follows:

$$F(x) = a_1 x^{\beta_1} + a_2 x^{\beta_2} + \frac{(1-\tau)x}{\rho + \lambda - \mu} - \frac{(1-\tau)c}{\rho + \lambda} \quad (2.6)$$

where  $a_1$  and  $a_2$  is unknown constants, and  $\beta_1$  and  $\beta_2$  are the positive and the negative roots, respectively, of the characteristic equation  $\frac{1}{2} \sigma^2 \beta(\beta - 1) + \mu\beta - (\rho + \lambda) = 0$ . The equity value must satisfy the following boundary conditions,

$$\lim_{x \rightarrow \infty} F(x) = \frac{(1-\tau)x}{\rho + \lambda - \mu} - \frac{(1-\tau)c}{\rho + \lambda} \quad (2.7)$$

$$F(x_d^1) = 0 \quad (2.8)$$

$$F'(x_d^1) = 0 \quad (2.9)$$

Condition (2.7) requires that the option value becomes zero if the cash flow is too large. Therefore, from this condition, we have  $a_1 = 0$ . Conditions (2.8) and (2.9) are the value-matching and smooth-pasting conditions, respectively. From these conditions, we can obtain the threshold value  $x_d^1$  and the unknown constant  $a_2$  as follows,

$$x_d^1 = \frac{\beta_2}{\beta_2 - 1} \frac{\rho + \lambda - \mu}{\rho + \lambda} c \quad (2.10)$$

$$a_2 = \frac{1}{1 - \beta_2} \frac{(1-\tau)c}{\rho + \lambda} \left( \frac{1}{x_d^1} \right)^{\beta_2} \quad (2.11)$$

### 2.1.2 Investment option value

Next, we consider the optimal investment value. By selecting the investment time  $T_p^1$ , the firm maximizes the expected discounted value. Thus, the value of the investment option  $P^1(x)$  is given by

$$P^1(x) = \sup_{T_p^1} \mathbb{E}_0^x [e^{-rT_p^1} \{F(X_{T_p^1}) - (1-\gamma)I\}], \quad (2.12)$$

where  $I$  is the investment cost, and  $\gamma$  is the ratio of the government subsidy. Given the constant threshold of the investment  $x^{1*}$ , the optimal abandonment time  $T_p^{1*}$  has the following form,

$$T_p^{1*} = \inf \left\{ T_p^1 > 0 \mid X_{T_p^1} \leq x^{1*} \right\}. \quad (2.13)$$

Using the standard method as described above, the value of investment option is given by the following equation,

$$P^1(x) = b_1 x^{\beta_1}, \quad (2.14)$$

where  $b_1$  is an unknown constant. For the threshold value of the investment  $x^{1*}$ , the value of investment must satisfy the following value-matching and smooth-pasting conditions,

$$P^1(x^{1*}) = F(x^{1*}) - (1 - \gamma)I, \quad (2.15)$$

$$P^1(x^{1*})' = F(x^{1*})'. \quad (2.16)$$

We can obtain  $x^{1*}$  and  $b_1$  by solving these equations numerically.

## 2.2 Equity and debt financing

### 2.2.1 Setting and assumption

In this section, we consider the following several players: a firm possessing the greenfield project (firm A), a firm possessing the brownfield project (firm B), public and private financial institutions, and the government subsidizing the investment project. The firm A invests in the project by means of project financing and the government subsidy. The public financial institution supports the financing of the private financial institution by means of partial debt guarantee. When the cash flow becomes too small, the firm A abandons the project, and then obtains the salvage value by cashing out assets and rights. The firm B buys out this project, and takes over the right of this project from the firm A. The firm B continues to operate this project until default.

Therefore, from these setting, the profit flow obtained from the project operating can be represented as follows,

$$\pi^2(X_t) = (1 - \tau) \{X_t - (c + k_a + k_b)\}, \quad (2.17)$$

where  $k_a$  and  $k_b$  are the coupon payments to public and private financial institutions, respectively. Suppose that the private financial institution pay a fee to the public financial institution guarantees because of partial debt guarantee at the time of default. Thus, the profit flows of public and private financial institutions is given by the following equations, respectively,

$$\pi_{D_A} = k_a + f, \quad (2.18)$$

$$\pi_{D_B} = k_b - f, \quad (2.19)$$

$$(2.20)$$

where  $f$  is the fee with respect to debt guarantee.

### 2.2.2 Equity and debt values after investment

Likewise the previous section, in order to obtain the option value of the investment, we first provide the equity and the debt values. Let state 1 denote that the firm A possesses assets and rights, and Let state 2 denote that the firm B possesses assets and rights. The firm possessing the greenfield project maximizes the equity value by selecting the abandonment time  $T_a$ . Thus, the total value of equity can be represented by the following equation,

$$E_A^1(x) = \sup_{T_a} \mathbb{E}_t^x \left[ \int_t^{T_a} e^{-\rho(s-t)} (1-\tau) \{X_s - (c + k_a + k_b)\} ds \right]. \quad (2.21)$$

Given the constant threshold of the abandonment  $x_a$ , the optimal abandonment time  $T_a^*$  has the following form,

$$T_a^* = \inf \{T_a > 0 | X_{T_a} \leq x_a\}. \quad (2.22)$$

Using the standard method as described above, the value of equity is given by the following equation,

$$E_A^1(x) = \begin{cases} c_1 x^{\beta_2} + \frac{(1-\tau)x}{\rho + \lambda - \mu} - \frac{(1-\tau)(c + k_a + k_b)}{\rho + \lambda}, & x > x_a \\ A, & x \leq x_a \end{cases} \quad (2.23)$$

where  $c_1$  is a unknown constant, and  $A$  is the salvage value. For  $x_a$  and  $c_1$ , the value of equity must satisfy the following value-matching and smooth-pasting conditions,

$$E_A^1(x_a) = A \quad (2.24)$$

$$E_A^{1'}(x_a) = 0. \quad (2.25)$$

From these conditions, we can obtain the threshold value of the abandonment  $x_a$  and the unknown constant  $c_1$  as follows,

$$x_a = \frac{\beta_2}{\beta_2 - 1} \frac{\rho + \lambda - \mu}{1 - \tau} \left\{ \frac{(1-\tau)(c + k_a + k_b)}{\rho + \lambda} + A \right\}, \quad (2.26)$$

$$c_1 = \frac{1}{1 - \beta_2} \left\{ \frac{(1-\tau)(c + k_a + k_b)}{\rho + \lambda} + A \right\} \left( \frac{1}{x_a} \right)^{\beta_2}. \quad (2.27)$$

Following Leland [5], we provide the debt values. Let  $D_A^I(x)$  and  $D_B^I(x)$  be the debt values which public and private financial institutions hold, respectively. These expected discounted values are given by the following equations,

$$D_A^1(x) = \mathbb{E}_t^x \left[ \int_t^{T_a^*} e^{-\rho(s-t)} (k_a + f) ds \right], \quad (2.28)$$

$$D_B^1(x) = \mathbb{E}_t^x \left[ \int_t^{T_a^*} e^{-\rho(s-t)} (k_b - f) ds \right]. \quad (2.29)$$

Using the standard method, the debt values of public and private financial institutions are given by the following equations,

$$D_A^1(x) = c_2 x^{\beta_2} + \frac{k_a + f}{\rho + \lambda}, \quad x > x_a, \quad (2.30)$$

$$D_B^1(x) = c_3 x^{\beta_2} + \frac{k_b - f}{\rho + \lambda}, \quad x > x_a, \quad (2.31)$$

where  $c_2$  and  $c_3$  are unknown constants.

The expected discounted value of equity for the firm B in state 2 is represented by the following equation,

$$E_B^2(x) = \sup_{T_d^2} \mathbb{E}_t^x \left[ \int_{T_a}^{T_d^2} e^{-\rho(s-T_a)} (1-\tau) \{X_s - (c_b + k_a + k_b)\} ds \right], \quad (2.32)$$

where  $c_b$  is a cost in state 2 which contains the operating cost and the cost to buy the project from firm A. Given the constant threshold of the default  $x_d^2$ , the optimal default time  $T_d^{2*}$  has the following form,

$$T_d^{2*} = \inf \left\{ T_d^2 > 0 \mid X_{T_d^2} \leq x_d^2 \right\}. \quad (2.33)$$

Using the standard method, the equity value of firm B in state 2 is given by the following equation,

$$E_B^2(x) = \begin{cases} d_1 x^{\beta_2} + \frac{(1-\tau)x}{\rho + \lambda - \mu} - \frac{(1-\tau)(c_b + k_a + k_b)}{\rho + \lambda}, & x > x_d^2, \\ 0, & x \leq x_d^2, \end{cases} \quad (2.34)$$

where  $d_1$  is a unknown constant. For  $x_d^2$  and  $d_1$ , the equity value of firm B in state 2 must satisfy the following value-matching and smooth-pasting conditions,

$$E_B^2(x_d^2) = 0, \quad (2.35)$$

$$E_B^{2'}(x_d^2) = 0. \quad (2.36)$$

From these conditions, we can obtain the threshold value of the default  $x_d^2$  and the unknown constant  $d_1$  as follows,

$$x_d^2 = \frac{\beta_2}{\beta_2 - 1} \frac{\rho + \lambda - \mu}{\rho + \lambda} (c + k_a + k_b), \quad (2.37)$$

$$d_1 = \frac{1}{1 - \beta_2} \frac{(1-\tau)(c + k_a + k_b)}{\rho + \lambda} \left( \frac{1}{x_d^2} \right)^{\beta_2}. \quad (2.38)$$

We provide the debt values, which public and private financial institutions hold, in state 2. These expected discounted values are given by the following equations,

$$D_A^2(x) = \mathbb{E}_t^x \left[ \int_{T_a}^{T_d^2} e^{-\rho(s-T_a)} k_a ds + e^{-\rho(T_d^2-T_a)} \frac{(1-\alpha)k_a}{k_a + k_b} (1-\theta)\epsilon(x_d^2) \right], \quad (2.39)$$

$$D_B^2(x) = \mathbb{E}_t^x \left[ \int_{T_a}^{T_d^2} e^{-\rho(s-T_a)} k_b ds + e^{-\rho(T_d^2-T_a)} \frac{k_b + \alpha k_a}{k_a + k_b} (1-\theta)\epsilon(x_d^2) \right], \quad (2.40)$$

where  $\theta$  is the proportional bankruptcy cost,  $0 \leq \theta \leq 1$ ,  $\alpha$  is the ratio of debt guarantee,  $0 \leq \alpha \leq 1$ , and  $\epsilon(x)$  is the operating value of the project without abandonment,

$$\epsilon(x) = \frac{(1-\tau)x}{\rho + \lambda - \mu} - \frac{(1-\tau)c}{\rho + \lambda}. \quad (2.41)$$

Using the standard method, the debt values for public and private financial institutions are given by the following equations,

$$D_A^2(x) = \begin{cases} d_2 x^{\beta_2} + \frac{k_a + f}{\rho + \lambda}, & x > x_d^2, \\ \frac{(1 - \alpha)k_a}{k_a + k_b} (1 - \theta)\epsilon(x_d^2), & x \leq x_d^2, \end{cases} \quad (2.42)$$

$$D_B^2(x) = \begin{cases} d_3 x^{\beta_2} + \frac{k_b - f}{\rho + \lambda}, & x > x_d^2, \\ \frac{k_b + \alpha k_a}{k_a + k_b} (1 - \theta)\epsilon(x_d^2), & x \leq x_d^2, \end{cases} \quad (2.43)$$

where  $d_2$  and  $d_3$  are unknown constants. For  $d_2$  and  $d_3$ , the debt values for public and private financial institutions satisfy the following boundary conditions, From boundary conditions at  $x_d^2$ , we can obtain two unknown constants of  $d_2$  and  $d_3$  as follows,

$$d_2 = \left\{ \frac{(1 - \alpha)k_a}{k_a + k_b} (1 - \theta)\epsilon(x_d^2) - \frac{k_a + f}{\rho + \lambda} \right\} \left( \frac{1}{x_d^2} \right)^{\beta_2}, \quad (2.44)$$

$$d_3 = \left\{ \frac{k_b + \alpha k_a}{k_a + k_b} (1 - \theta)\epsilon(x_d^2) - \frac{k_b - f}{\rho + \lambda} \right\} \left( \frac{1}{x_d^2} \right)^{\beta_2}. \quad (2.45)$$

We determine the equity value and the debt values for for public and private financial institutions in state 1. The boundary conditions at  $x_a$  are represented by the following equations,

$$D_A^1(x_a) = D_A^2(x_a), \quad (2.46)$$

$$D_B^1(x_a) = D_B^2(x_a), \quad (2.47)$$

From these boundary conditions, we can obtain two unknown constants of  $c_2$  and  $c_3$  as follows,

$$c_2 = \left\{ \frac{(1 - \alpha)k_a}{k_a + k_b} (1 - \theta)\epsilon(x_d^2) - \frac{k_a + f}{\rho + \lambda} \right\} \left( \frac{1}{x_d^2} \right)^{\beta_2} = d_2, \quad (2.48)$$

$$c_3 = \left\{ \frac{k_b + \alpha k_a}{k_a + k_b} (1 - \theta)\epsilon(x_d^2) - \frac{k_b - f}{\rho + \lambda} \right\} \left( \frac{1}{x_d^2} \right)^{\beta_2} = d_3, \quad (2.49)$$

### 2.2.3 Investment option value

In this section, we derive the investment option value of the project, which is financed by issuing equity and debt, by means of the values of equity and debt provided in the previous section. From Equations (2.23), (2.30), and (2.31), the project value can be represented by the following equation,

$$\begin{aligned} V(x) &= E_A^1(x) + D_A^1(x) + D_B^1(x) \\ &= (c_1 + c_2 + c_3)x^{\beta_2} + \frac{(1 - \tau)x}{\rho + \lambda - \mu} - \frac{(1 - \tau)c}{\rho + \lambda} + \frac{\tau(k_a + k_b)}{\rho + \lambda}. \end{aligned} \quad (2.50)$$

By selecting  $T^2$  to maximize the project value, the value of the investment option  $P^2(x)$  is given by the following equation,

$$\begin{aligned} P^2(x) &= \sup_{T^2} \mathbb{E}_0^x [e^{-rT^2} [E_A^1(x) - \{(1 - \gamma)I - D_A^1(x) - D_B^1(x)\}]] \\ &= \sup_{T^2} \mathbb{E}_0^x [e^{-rT^2} \{V(x) - (1 - \gamma)I\}]. \end{aligned} \quad (2.51)$$

Given the constant threshold of the investment  $x^{2*}$ , the optimal investment time  $T^{2*}$  has the following form,

$$T^{2*} = \inf \left\{ T^2 > 0 \mid X_{T^2} \geq x^{2*} \right\}. \quad (2.52)$$

Using the standard method, the value of the investment option is given by the following equation,

$$P^2(x) = e_1 x^{\beta_1}, \quad (2.53)$$

where  $e_1$  is a unknown constant. For the threshold value of the investment  $x^{2*}$ , the value of investment must satisfy the following value-matching and smooth-pasting conditions,

$$P^2(x^{2*}) = V(x^{2*}) - (1 - \gamma)I, \quad (2.54)$$

$$P^2(x^{2*})' = V(x^{2*})'. \quad (2.55)$$

We can obtain  $x^{2*}$  and  $e_1$  by solving these equations numerically.

#### 2.2.4 Debt guarantee and debt-paying ability

We consider the decision of financial institutions with respect to financing. The investment project by using project finance can typically be the highly-leveraged project. Therefore, the financial institutions evaluate a safety for the repayment of interest and principal, and then, participate in the project. At the investment time, if the total expected discounted value of the project's cash flows before the repayment of interest and principal are smaller than the expected discounted values of debt which the financial institutions hold, the firm would become overextended with debt. The condition for the debt-paying ability is represented by the following equation,

$$E_A^1(x^{2*}) \geq D_A^1(x^{2*}) + D_B^1(x^{2*}). \quad (2.56)$$

Equation (2.56) is a constraint condition for total coupon payments. Let  $K^*$  be the upper bound of total coupon payments, that is,  $k_a + k_b \leq K^*$ .

Additionally, we consider the relationship between the fee with respect to debt guarantee  $f$  and the ratio of debt guarantee  $\alpha$ . Suppose that the fee with respect to debt guarantee  $f$  is determined for a condition where the ratio of debt values for public and private financial institutions is the same as that of coupon payments. Thus this relationship is represented by the following equation,

$$\frac{D_A(x^{2*})}{D_B(x^{2*})} = \frac{k_a}{k_b} \quad (2.57)$$

By solving Equation (2.57) for the fee with respect to debt guarantee  $f$ , the following equation can be obtained as

$$f = \left[ \frac{(\rho + \lambda) \frac{k_a}{k_a + k_b} (1 - \theta) \epsilon(x_d^2) \left( \frac{x^{2*}}{x_d^2} \right)^{\beta_2}}{1 - \left( \frac{x^{2*}}{x_d^2} \right)^{\beta_2}} \right] \alpha. \quad (2.58)$$

### 3 Numerical analysis

In the previous section we presented a model to analyze the influence of debt guarantee and debt-paying ability on the investment decisions. In this section we provide the numerical analysis with respect to the investment option values and the investment threshold values for both cases of all-equity financing and equity and debt financing. In addition, we show the relationship between the fee with respect to debt guarantee and the ratio of debt guarantee for catastrophic risk and volatility of cash flows. The base case parameters used in this analysis are as follows:  $\mu = 0.03, \sigma = 0.2, \rho = 0.05, \lambda = 0.1, \tau = 0.3, \theta = 0.2, c = 0.5, c_b = 0.5, k_a = 1, k_b = 1, f = 0.1, \alpha = 0.5, \gamma = 0.05, I = 10$ .

Figure 1 shows the option values and threshold values of the investment for both cases of all-equity financing, and equity and debt financing. The dashed-dotted line represents the investment option value of all-equity project, and the dashed line shows the investment option value of the project which is financed with equity and debt. The threshold value of equity and debt financing case turns out to be smaller than that of all-equity. This is because equityholders and debtholders share the risk of project, and projects of equity and debt financing enjoy tax shields likewise corporate financing as in [9, 13].

The threshold values of investment as functions of Poisson intensity for both cases of all-equity financing, and equity and debt financing are shown in Figure 2. The solid line represents the threshold value of all-equity project, and the dashed line shows the threshold value of the project which is financed with equity and debt. As can be seen from Figure 2, the threshold value of all-equity is smaller than that of equity and debt financing when Poisson intensity is small, while the threshold value of equity and debt is smaller than that of all-equity financing when Poisson intensity is large. It seems that for projects in politically-stable countries the firm

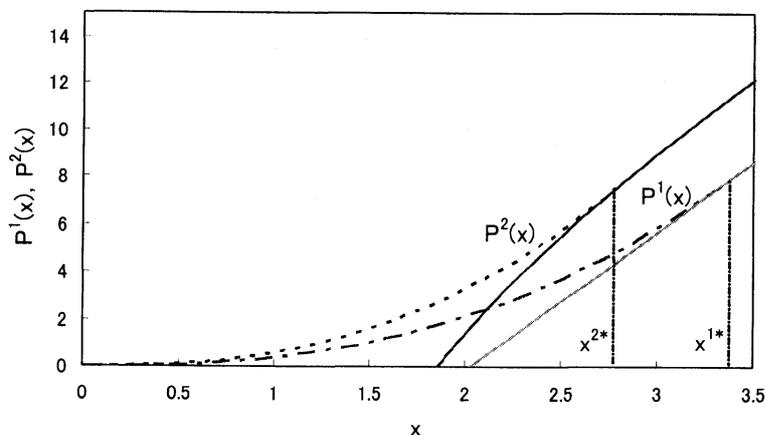


Figure 1: Investment option values and threshold values. The dashed-dotted line represents the investment option value of all-equity project. The dashed line shows the investment option value of the project which is financed with equity and debt.

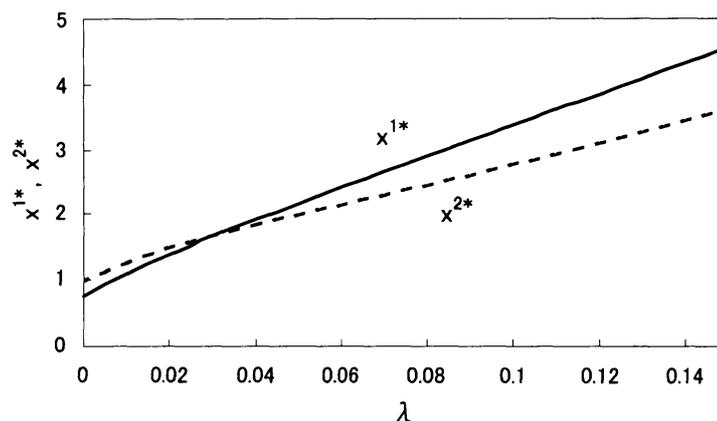


Figure 2: Threshold values of investment as functions of Poisson intensity. The solid line represents the threshold value of all-equity project. The dashed line shows the threshold value of the project which is financed with equity and debt.

is financed with shareholders' equity, whereas for projects with catastrophic risks in politically-unstable countries the firm is financed with equity and debt as project financing. Actually, for many projects of foreign investment with high catastrophic risk, project financing has been utilized [3].

Figure 3 shows the threshold values of investment, abandonment, and default as functions of Poisson intensity. The solid line represents the threshold value of investment, the dashed line shows the threshold value of abandonment, and the dotted line represents the threshold value of default. As Poisson intensity becomes large, threshold values of investment, abandonment, and default increase. Note that the gap between the thresholds of investment, and abandonment, and default grows as Poisson intensity increases. These results show the catastrophic event could occur before the abandonment of firm A and the default of firm B.

Figure 4 shows the relationship between the fee with respect to debt guarantee  $f$  and the ratio of debt guarantee  $\alpha$  for each Poisson intensity of 0.1, 0.15, 0.2. It turns out that the fee with respect to debt guarantee increases with the ratio of debt guarantee, and the slope of  $f$  becomes small as Poisson intensity increases. The project of high catastrophic risk could have larger debt guarantee than that of small catastrophic risk for the same level of the fee.

Figure 5 shows the relationship between the fee with respect to debt guarantee  $f$  and the ratio of debt guarantee  $\alpha$  for each volatility of 0.2, 0.25, 0.3. Unlike the case of catastrophic risk, the slope of  $f$  turns out to become large as volatility increases. Since the opportunity of the default decrease when volatility is large, the increase in debt guarantee is required.

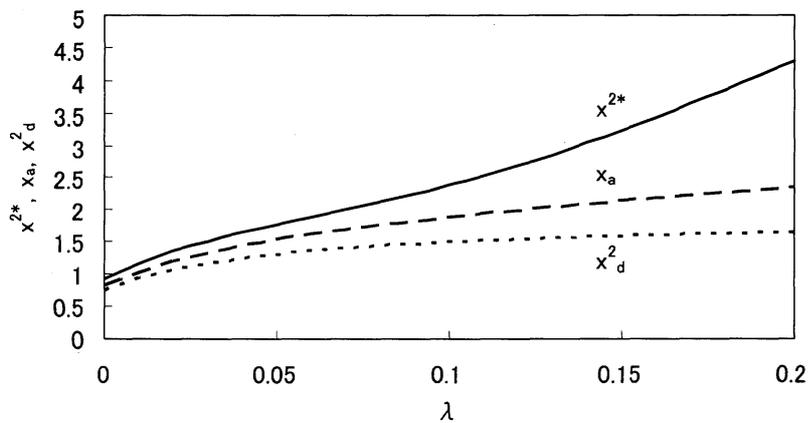


Figure 3: Threshold values of investment, abandonment, and default as functions of Poisson intensity. The solid line represents the threshold value of investment. The dashed line shows the threshold value of abandonment. The dotted line represents the threshold value of default.

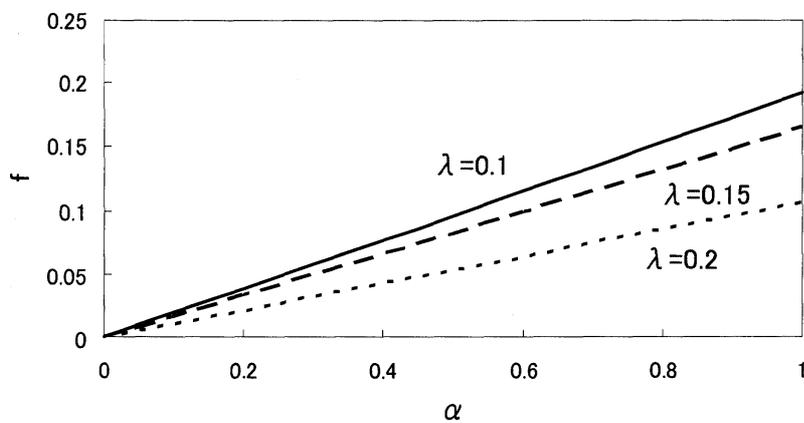


Figure 4: Relationship between the fee with respect to debt guarantee  $f$  and the ratio of debt guarantee  $\alpha$  for each Poisson intensity of 0.1, 0.15, 0.2.

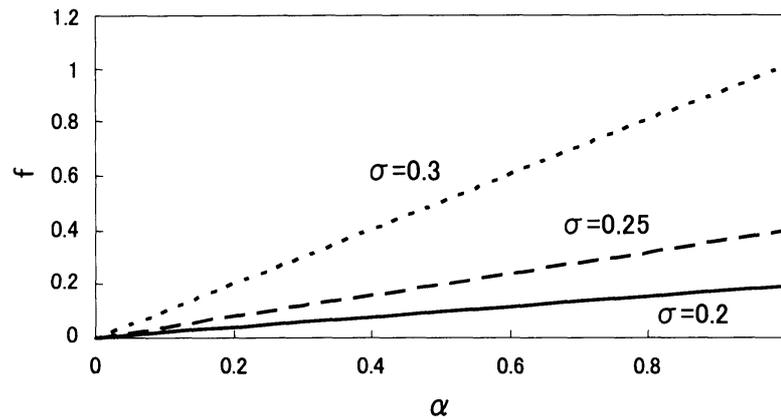


Figure 5: Relationship between the fee with respect to debt guarantee  $f$  and the ratio of debt guarantee  $\alpha$  for each volatility of 0.2, 0.25, 0.3.

#### 4 Application to the investment of electric power plants

This section examines the acquisition project of IPP generation assets by means of the model in this paper. The consortium led by Marubeni and Tokyo Electric Power Company acquired Mirant's independent power generation assets in the Philippines for US\$3.43 billion in 2006. It is likely that the decline in profits by increase in operating cost and high leverage leads to the abandonment of the project. The cause of abandonment also includes increase in political risk in Philippines and significant capital gain. Therefore, we analyze the influence of these causes on the project in this section.

The parameters, which are used in following analyses, are shown in Table 1. The operating and maintenance costs and the construction cost of power plants are set to 20 US\$/MWh and  $8,000 \times 10^6$  US\$, which are based on OECD [11], respectively. Suppose that the expected growth rate and the volatility of the cash flow are 0.03 and 0.1, respectively. Also, we assume that the discounted rate and Poisson intensity are 0.05 and 0.03, respectively. The salvage value of the project, the capacity factor, and the output are set to  $3,400 \times 10^6$  US\$, 90%, and 2,203 MW, by evaluating the value shown in [15], respectively. By using parameters described above, the upper bound of total coupon payments  $K^*$  and the threshold value  $x^{2*}$  become  $1,091 \times 10^6$  US\$, and 64.9 US\$/MWh, respectively. Let  $x_s^{2*}$  be the threshold value for this basic case.

Table 2 shows threshold values of default and abandonment for operating and maintenance costs of 20, 25, and 30. From Table 2, it can be seen that as the operating and maintenance costs increase, the abandonment threshold becomes large, and therefore, the distance from the investment for basic case to the abandonment becomes small. Part of the reason that Mirant abandons the project includes the increases in the operating cost.

The threshold values of default, abandonment and investment for coupon payments of 1,091, 1,150, and 1,200 are shown in Table 3. Although the investment threshold increases as the

Table 1: Parameters

Operating and maintenance costs	$c$	20 (US\$/MWh)
Construction cost	$I$	8,000 ( $10^6$ US\$)
Corporate tax rate	$\tau$	30%
Expected growth rate of price	$\mu$	0.03
Price volatility	$\sigma$	0.1
Discounted rate	$\rho$	0.05
Poisson intensity	$\lambda$	0.03
Salvage value	$A$	3,400 ( $10^6$ US\$)
Capacity factor		90%
Output		2,203 MW

Table 2: Threshold values of default and abandonment for various operating and maintenance costs.

$c$ (US\$/MWh)	$x_d^2$ (US\$/MWh)	$x_a$ (US\$/MWh)	$x_s^{2*} - x_a$
20	45.5	57.7	7.2
25	48.2	60.5	4.4
30	50.9	63.2	1.7

Table 3: Threshold values of default, abandonment and investment for various coupon payments.

$k_a + k_b$ ( $10^6$ US\$)	$x_d^2$ (US\$/MWh)	$x_a$ (US\$/MWh)	$x^{2*}$ (US\$/MWh)	$x^{2*} - x_a$
1,091( $K^*$ )	45.5	57.7	64.9	7.2
1,150	47.3	59.6	65.6	6.0
1,200	48.9	61.2	65.9	4.7

Table 4: Threshold values of default and abandonment for various Poisson intensities.

$\lambda$	$x_d^2(\text{US\$/MWh})$	$x_a(\text{US\$/MWh})$	$x_s^{2*} - x_a$	$x_a - x_d^2$
0.03	45.5	57.7	7.2	12.2
0.035	47.1	60.7	4.2	13.6
0.04	48.7	63.4	1.5	14.7

coupon payment becomes large, the distance from the investment to the abandonment decreases since the abandonment increases. As shown by this result, the other reason that Mirant abandons the project includes large debt financing.

Likewise the cost and the coupon payment, the influence of the catastrophic risk on the project decisions seems to be large. Table 4 shows threshold values of default and abandonment for Poisson of intensities of 0.03, 0.035, and 0.04. As can be seen from Table 4, as Poisson intensity becomes large, the abandonment threshold increases, and therefore, the distance from the investment for basic case to the abandonment becomes small. The other reason that Mirant abandons the project includes the increase in the catastrophic risk. On the other hand, the distance from the abandonment to the default become small as Poisson intensity increases. Therefore, the increase in the catastrophic risk has an impact on long-term holding of the assets for the consortium led by Marubeni and Tokyo Electric Power Company.

The upper bounds of total coupon payments and threshold values of default, abandonment and investment for salvage values of 3,000, 3,400, and 4,000 are shown in Table 5. As the salvage values becomes large, the distance from the investment to the abandonment increases. In the case of large salvage value, the incentive that Mirant possesses that asset becomes large. Likewise, the distance from the abandonment to the default increases with the salvage value. When the salvage value, that is, the amount paid for acquisition is large, the incentive to possess that asset becomes large for the consortium led by Marubeni and Tokyo Electric Power Company.

Table 5: Upper bounds of total coupon payments and threshold values of default, abandonment and investment for various salvage values.

$A(10^6\text{US\$})$	$K^*(10^6\text{US\$})$	$x_d^2(\text{US\$/MWh})$	$x_a(\text{US\$/MWh})$	$x_s^{2*}(\text{US\$/MWh})$	$x_s^{2*} - x_a$	$x_a - x_d^2$
3,000	959	41.3	52.1	58.3	6.2	10.8
3,400	1,091	45.5	57.7	64.9	7.2	12.2
4,000	1,288	51.7	66.1	74.9	8.8	14.4

## 5 Concluding Remarks

In this paper we present a model for analyzing the interaction among investment and project financing decisions of a firm under uncertainty by means of real options framework. Since the catastrophic risks have a large influence on the project in countries which have a political risk, we consider a catastrophic risk in this model, and show the influence of the catastrophic risk on the investment timing, the default timing, and debt guarantee. It turns out that the advantages of project financing become large as the catastrophic risk increases compared to all-equity financing. We also show the relationship between the fee with respect to debt guarantee and the ratio of debt guarantee for catastrophic risk and volatility. We find that the ratio of debt guarantee at same level of the fee becomes high when the Poisson intensity is large, whereas the ratio of debt guarantee at same level of the fee becomes small when the volatility is large. In addition, we apply this model to a investment project for electric power plants, and evaluate the acquisition project of Mirant's generation assets in Philippines by consortium led by Marubeni and Tokyo Electric Power Company. The influence of coupon payment and salvage value on the distance to abandon the project is shown.

Although we study the abandonment decisions for the firm possessing the greenfield project in this paper, the acquisition decisions for the firm possessing the brownfield project are not considered. Thus, in the future, we will extend the model in this paper to introduce the acquisition decisions, and will examine the influence of catastrophic risk on the acquisition. Additionally, we will investigate the case in which the acquisition is financed with equity and debt such as leveraged-buyouts (LBOs).

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