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Report on the chemotaxis-fluid systems

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Abstract. This report deals with the chemotaxis-fluid system (P) with porous-medium diffusion and considers the uniform-in-time L^p-estimate to solutions (n,c,u,p) to (P), where components of the solution denotes the bacteria density, the chemical concentration, the velocity field of the fluid and the pressure in the order of inputting. The analysis of this system is very difficult, however there are few results on the global existence: As to the two dimensional case, the global existence in (P) with the linear diffusion (Δn) on $\mathbb{R}^2$ and with quasilinear degenerate diffusion $(\Delta n^m,\ m>1)$ on bounded convex domains was proved. Moreover in the three dimensional case, there are results on global solvability of (P) with $\Delta n^{\frac{4}{3}}$ on $\mathbb{R}^3$ and with Δn^m $(m>\frac{8}{7})$ on bounded convex domains. As can be seen from these, there are many open interesting problems. This report discusses these problems and gives the observational result.

1. Introduction

The main purpose of our work is to prove the global existence and uniform in time boundedness in the following chemotaxis-fluid system (KSS) on the 3D bounded domain:

$$
\text{(KSS)}\qquad
\begin{cases}
\dfrac{\partial n}{\partial t} = \Delta n^m - \nabla\cdot(n\nabla c) - u\cdot\nabla n, & x\in\Omega,\ t>0,\\[2mm]
\dfrac{\partial c}{\partial t} = \Delta c - nc - u\cdot\nabla c, & x\in\Omega,\ t>0,\\[2mm]
\dfrac{\partial u}{\partial t} = \Delta u - \nabla p - n\nabla\phi, & x\in\Omega,\ t>0,\\[2mm]
\nabla\cdot u = 0, & x\in\Omega,\ t>0,\\[2mm]
\dfrac{\partial n^m}{\partial\nu} = \dfrac{\partial n}{\partial\nu} = 0,\ u=0, & x\in\partial\Omega,\ t>0,\\[2mm]
n(x,0)=n_0(x), c(x,0)=c_0(x), u(x,0)=u_0(x), & x\in\Omega,\ t>0,
\end{cases}
$$

where Ω has a smooth boundary $\partial\Omega$, ν is outward normal derivative on $\partial\Omega$, $\phi\in W^{1,\infty}(\Omega)$, $m>1$ and the initial data are positive functions.

The system (KSS) describes the motion of the swimming bacteria with oxgentaxis which lives in thin fluid. The bacteria moves toward higher concentration of oxygen and oxygen is consumed by bacteria, and moreover, they are transported by the fluid (cf. Tuval, et al. [**9**], Dombrowski, et al. [**1**]). Here, (n,c,u,p) in (KSS) denotes the bacteria density, the chemical concentration, the velocity field of the fluid and the pressure, respectively.

The analysis of this system is difficult because of $u\cdot\nabla c$ in the second equation. When we consider the L^p-estimate of n, getting $\nabla\cdot(n\nabla c)$ in the first equation under control has a key role. However, in order to estimate this term, we run against the term $u \cdot \nabla c$ in the second equation which term is associated directly with the third equation. The second equation looks simple but the estimate of ∇c is hard (at least for the author). In spite of the difficulty such as this, there are few successful results on the global existence. We know the following cases have the global solvability:

- 2D case
  (a) $\Omega = \mathbb{R}^2$, $m = 1$, the small initial data (Duan-Lorz-Markowich [**2**]);
  (b) Ω is the bounded domain, $m \in (\frac{3}{2}, 2]$ (Francesco-Lorz-Markowich [**3**]);
  (c) Ω is the convex bounded domain, $m > 1$ (Tao-Winkler [**7**]);
- 3D case
  (a') $\Omega = \mathbb{R}^3$, $m = 2$ ([**3**]);
  (b') $\Omega = \mathbb{R}^3$, $m = \frac{4}{3}$ (Liu-Lorz [**6**]);
  (c') Ω is the bounded domain, $m \in [\frac{7+\sqrt{217}}{12}, 2]$ ([**3**]);
  (d') Ω is the convex bounded domain, $m > \frac{8}{7}$ (Tao-Winkler [**8**]).

As can be seen from these, there are many open interesting problems.

2. Observation and its proof

This report especially considers functions fulfilling the following Cauchy problem

$$
\text{(P)} \qquad
\begin{cases}
\dfrac{\partial n}{\partial t} = \Delta(n+\varepsilon)^m - \nabla \cdot (n \nabla c) - u \cdot \nabla n, & x \in \mathbb{R}^3,\ t > 0, \\
\dfrac{\partial c}{\partial t} = \Delta c - nc - u \cdot \nabla c, & x \in \mathbb{R}^3,\ t > 0, \\
\dfrac{\partial u}{\partial t} = \Delta u - \nabla p - n \nabla \phi, & x \in \mathbb{R}^3,\ t > 0, \\
\nabla \cdot u = 0, & x \in \mathbb{R}^3,\ t > 0, \\
n(x,0) = n_0(x), c(x,0) = c_0(x), u(x,0) = u_0(x), & x \in \mathbb{R}^3,\ t > 0
\end{cases}
$$

in the classical sense, where $\varepsilon > 0$ and the initial data $n_0, c_0, u_0 \in C_0^\infty(\mathbb{R}^3)$ are positive function.

The definition of the classical solution (n, c, u, p) to (P) is as follows:

Definition 2.1 (classical solutions). Let $T \in (0, \infty)$. A quadruple (n, c, u, p) is called *a classical solution* to (P) on $[0, T)$ if the following conditions are satisfied:

$$n > 0 \quad \text{and} \quad c > 0$$

and

$$
\begin{aligned}
n &\in C^0(\mathbb{R}^3 \times [0, T_{\max})) \cap C^{2,1}(\mathbb{R}^3 \times (0, T_{\max})), \\
c &\in C^0(\mathbb{R}^3 \times [0, T_{\max})) \cap C^{2,1}(\mathbb{R}^3 \times (0, T_{\max})), \\
u &\in C^0(\mathbb{R}^3 \times [0, T_{\max})) \cap C^{2,1}(\mathbb{R}^3 \times (0, T_{\max}))
\end{aligned}
$$

and (n, c, u, p) fulfills the system (P).

Now we present the result.

Proposition 2.1. *Let $T > 0$ and let (n, c, u, p) be a classical solution to* (P) *on* $[0, T)$. *Assume that*

$$m \in \Big[1, \frac{4}{3}\Big).$$

Then there exists a constant $\delta = \delta(m) > 0$ such that if the initial data satisfy the smallness:

$$\|n_0\|_{L^1}, \|n_0\|_{L^3}, \|n_0 \log n_0\|_{L^1}, \|c_0\|_{L^\infty}, \|\nabla c_0\|_{L^2}, \|\Delta c_0\|_{L^4}, \|u_0\|_{D(A_\gamma)} \le \delta$$

where A_γ is the Stokes operator with dense domain $D(A_\gamma)$ and $\gamma := \left(\frac{3-p}{4(p+1)} + \frac{2}{3}\right)^{-1}$ for some $p \in (\frac{3}{2}, 1)$ close to 3 *and moreover if the additional smallness*

(A1) $$\sup_{0<t<T} \Big(- \int_{\mathbb{R}^3} n \log n \, dx \Big) \le \delta,$$

(A2) $$\int_0^T \int_{\mathbb{R}^3} |u|^4 \, dx ds \le \delta,$$

(A3) $$\int_0^T \|u\|_{L^\gamma}^{\frac{4(p+1)}{3-p}} \, ds \le \delta$$

are satisfied, then

$$\sup_{0<t<T} \|n(t)\|_{L^p(\mathbb{R}^3)} \le C \quad (\forall p \in [1, 3))$$

holds, where C does not depend on ε and T.

Remark 2.1. We denote the Stokes operator by $A_p = -P_q \Delta$, where P_q is the continuous projection from $(L^q(\mathbb{R}^3))^3$ onto $L^q_\sigma(\mathbb{R}^3)$, with the domain

$$D(A_q) = \big\{ w \in L^q_\sigma(\mathbb{R}^3);\ \partial_i \partial_j w \in (L^q(\mathbb{R}^3))^3 \ \ (1 \le i, j \le n) \big\}.$$

The space $L^q_\sigma(\mathbb{R}^3)$ is regarded as the closure of $C^\infty_{0,\sigma}(\mathbb{R}^3) := \big\{ w \in (C^\infty_0(\mathbb{R}^3))^3,\ \nabla \cdot w = 0 \big\}$ in $(L^q(\mathbb{R}^3))^3$.

2.1. Preliminaries

The first and second lemmas are the maximal Sobolev regularity for Laplace operator (see e.g., Ladyženskaja-Solonnikov-Ural'ceva [**5**, Chapter IV, Section 3]) and it for Stokes operator (see e.g., Giga-Sohr [**4**]).

Lemma 2.2 (Maximal Sobolev regularity for Laplace operator). *Let $1 < p < \infty$, $N \in \mathbb{N}$ and $T > 0$. Then for every $w \in L^p(0, T; L^p(\mathbb{R}^3))$ and $z_0 \in W^{2,p}(\mathbb{R}^3)$ there exists a solution z to*

$$z_t = \Delta z + w \quad \text{in } \mathbb{R}^3 \times (0, T), \qquad z(x, 0) = z_0(x) \quad \text{in } \mathbb{R}^3$$

which satisfies $z \in L^p(0, T; W^{2,p}(\mathbb{R}^3))$. Moreover, there exists a positive constant $C_{\langle p \rangle} = C_{\langle p \rangle}(p, N)$ such that

$$\|\Delta z\|_{L^p(0,T;L^p(\mathbb{R}^3))} \le \|\Delta z_0\|_{L^p} + C_{\langle p \rangle} \|w\|_{L^p(0,T;L^p(\mathbb{R}^3))}.$$

Lemma 2.3 (Maximal Sobolev regularity for Stokes operator). *Let $1 < p, q < \infty$, $N \in \mathbb{N}$, $T > 0$ and let A_p be a Stokes operator. Then for every $w \in L^q(0,T; L^q_\sigma(\mathbb{R}^3))$ and $z_0 \in D(A_q)$ there exists a solution z to*

$$z_t = -A_p z + w \quad \text{in } \mathbb{R}^3 \times (0,T), \qquad z(x,0) = z_0(x) \quad \text{in } \mathbb{R}^3$$

satisfying $z \in L^q(0,T; D(A_p))$. Moreover z satisfies the following estimate:

$$\int_0^T \|A_p z(t)\|^q_{L^p(\mathbb{R}^3)}\,dt \le \widetilde{C}_{\langle p,q\rangle}\Big(\|z_0\|^q_{D(A_p)} + \int_0^T \|w(t)\|^q_{L^p(\mathbb{R}^3)}\,dt\Big),$$

where $\widetilde{C}_{\langle p,q\rangle} = \widetilde{C}_{\langle p,q\rangle}(p,q,N) > 0$ is a constant.

The next proposition is the mass conservation law for $n(t)$ and the L^∞ bound for $c(t)$. These are proved by integration of the first equation and by the parabolic maximum principle applied to the second equation, respectively.

Proposition 2.4 (L^1-conservation law for n and L^∞-estimate for c). *Let $T > 0$ and let (n, c, u, p) be a classical solution to (P) on $[0, T)$. Then n satisfies*

$$\|n(t)\|_{L^1(\mathbb{R}^3)} = \|n_0\|_{L^1} \quad \text{for all } t \in (0,T)$$

and c satisfies

$$|c| \le \|c_0\|_{L^\infty} \quad \text{for all } (x,t) \in \mathbb{R}^3 \times (0,T).$$

Lemma 2.5 is the energy estimate for (P). The proof is came from [**8**, Lemma 2.3].

Lemma 2.5 (Energy estimate). *Let $T > 0$ and let (n, c, u, p) be a classical solution to (P) on $[0, T)$. Then there exists a positive constant k_1 such that for any $t \in (0, T)$*

$$\frac{d}{dt}\Big\{\int_{\mathbb{R}^3} n\log n + 2\int_{\mathbb{R}^3} |\nabla\sqrt{c}|^2\Big\} + \int_{\mathbb{R}^3} |\nabla n^{\frac{m}{2}}|^2 + \int_{\mathbb{R}^3} c|D^2 \log c|^2 + \frac{1}{2}\int_{\mathbb{R}^3} n\frac{|\nabla c|^2}{c} \le k_1 \int_{\mathbb{R}^3} |u|^4.$$

The following estimate is the estimate to ∇c. The proof is also came from [**8**, Lemmas 2.3, 2.5] and the above energy estimate.

Lemma 2.6 (L^4-estimate to ∇c). *Let $T > 0$ and let (n, c, u, p) be a classical solution to (P) on $[0, T)$. Then there exists a positive constant K_1 such that*

$$\int_0^t \int_{\mathbb{R}^3} |\nabla c|^4\,dxds < K_1, \quad (\forall t \in (0,T))$$

where K_1 depends on the initial data and $\sup\limits_{0<s<t}\big(-\int_{\mathbb{R}^3} n\log n\,dx\big)$, $\int_0^t\int_{\mathbb{R}^3} |u|^4\,dxds$.

Remark 2.2. As to Lemma 2.6, the assumption of smallness means that $\int_0^t\int_{\mathbb{R}^3} |\nabla c|^4$ can be to be small.

2.2. Proof of main proposition

In this subsection, we compute the L^p estimate to $n(t)$ ($\forall p \in [1,3)$). For simplicity, we discuss (P) with $\varepsilon = 0$ throughout the following. Let us start computing.

Let $p > 1$. From the straightforward argument we have

$$\begin{aligned}
\frac{1}{p}\frac{d}{dt}\|n(t)\|_{L^p(\Omega)}^p &= -\int_\Omega \nabla n^m \cdot \nabla n^{p-1} + \int_\Omega n\nabla c \cdot \nabla n^{p-1} + \int_\Omega u\nabla n n^{p-1} \\
&= -\frac{4m(p-1)}{(m+p-1)^2}\int_\Omega |\nabla n^{\frac{m+p-1}{2}}|^2 + (p-1)\int_\Omega \nabla c \cdot n^{p-1}\nabla n \\
&= -\frac{2m(p-1)}{(m+p-1)^2}\int_\Omega |\nabla n^{\frac{m+p-1}{2}}|^2 + \frac{(p-1)}{p}\int_\Omega \nabla c \nabla n^p \\
&\le -\frac{2m(p-1)}{(m+p-1)^2}\|\nabla n^{\frac{m+p-1}{2}}\|_{L^2(\Omega)}^2 + \frac{(p-1)}{p}\int_\Omega |\Delta c| n^p.
\end{aligned}$$

Integrating the above estimate over $(0,t)$, we obtain from the Young inequality with $(\frac{p}{p+1}, \frac{1}{p+1})$ and Lemma 2.2 that

$$\begin{aligned}
&\|n(t)\|_{L^p(\Omega)}^p - \|n_0\|_{L^p(\Omega)}^p \\
&\le -\frac{2mp(p-1)}{(m+p-1)^2}\int_0^t \|\nabla n^{\frac{m+p-1}{2}}\|_{L^2(\Omega)}^2 \\
&\quad + (p-1)\int_0^t\int_\Omega |\Delta c|^{p+1} + (p-1)\int_0^t\int_\Omega |n|^{p+1} \\
&\le -\frac{2mp(p-1)}{(m+p-1)^2}\int_0^t \|\nabla n^{\frac{m+p-1}{2}}\|_{L^2(\Omega)}^2 \\
&\quad + c_1(p-1)\Big\{\|\Delta c_0\|_{L^{p+1}} + \int_0^t\int_\Omega |-nc - u\cdot\nabla c|^{p+1}\Big\} + (p-1)\int_0^t\int_\Omega |n|^{p+1}
\end{aligned}$$

with some positive constant c_1. Hence the estimate $\|c(t)\|_{L^\infty} \le \|c_0\|_{L^\infty}$ ($t > 0$) from Proposition 2.4 ensures that

$$\begin{aligned}
&\|n(t)\|_{L^p(\Omega)}^p - \|n_0\|_{L^p(\Omega)}^p \\
&\le -\frac{2mp(p-1)}{(m+p-1)^2}\int_0^t \|\nabla n^{\frac{m+p-1}{2}}\|_{L^2(\Omega)}^2 + c_1(p-1)\|\Delta c_0\|_{L^{p+1}} \\
&\quad + (p-1)(c_1\|c_0\|_{L^\infty}+1)\int_0^t\int_\Omega |n|^{p+1} + c_1(p-1)\int_0^t\int_\Omega |u\cdot\nabla c|^{p+1}.
\end{aligned} \tag{2.1}$$

Firstly, we consider the third term in the right hand side of (2.1). The Gagliardo-Nirenberg inequality yields for any $p > \frac{m}{2} + 1$

$$\begin{aligned}
\int_0^t\int_\Omega |n|^{p+1} &= \int_0^t \|n^{\frac{p+m-1}{2}}\|_{L^{\frac{2(p+1)}{p+m-1}}}^{\frac{2(p+1)}{p+m-1}} \\
&\le c_2\int_0^t \Big(\|\nabla n^{\frac{p+m-1}{2}}\|_{L^2}^\alpha \|n^{\frac{p+m-1}{2}}\|_{L^{\frac{2}{p+m-1}\cdot\frac{3(p-1)}{2}}}^{1-\alpha}\Big)^{\frac{2(p+1)}{p+m-1}}
\end{aligned}$$

with some constant $c_2 > 0$ and

$$\alpha := \frac{p+m-1}{2}\Big(\frac{2}{3(p-1)} - \frac{1}{p+1}\Big)\Big(\frac{p+m-1}{6(2-m)} - \frac{1}{6}\Big).$$

Note that $\alpha \in (0,1)$ when $m < 2$. Because of $\frac{2\alpha(p+1)}{p+m-1} = 2$, we have

$$\int_0^t \int_\Omega |n|^{p+1}\,dxds \le c_2 \int_0^t \|\nabla n^{\frac{p+m-1}{2}}\|_{L^2}^2 \|n\|_{L^{\frac{3(p-1)}{2}}}^{2-m}\,ds. \tag{2.2}$$

Next, we'd like to estimate the last term in the right hand side of (2.1). The Young inequality with $(\frac{p+1}{4}, \frac{4-(p+1)}{4})$ gives for any $p < 3$

$$\int_0^t \int_\Omega |u\cdot\nabla c|^{p+1} \le \int_0^t \int_\Omega |\nabla c|^4 + \int_0^t \int_\Omega |u|^{\frac{4(p+1)}{4-(p+1)}}. \tag{2.3}$$

In order to estimate the second term in the right hand side of (2.3), we use the embedding $W^{2,\gamma} \hookrightarrow L^{\frac{4p+4}{-p+3}}$ $(\gamma := (\frac{-p+3}{4p+4} + \frac{2}{3})^{-1})$ and Lemma 2.3. Then we see that

$$\begin{aligned}
\int_0^t \int_\Omega |u|^{\frac{4p+4}{-p+3}} &\le c_3 \int_0^t \|u\|_{W^{2,\gamma}}^{\frac{4p+4}{-p+3}} \\
&\le c_3 \int_0^t \|u\|_{L^\gamma}^{\frac{4p+4}{-p+3}} + c_3 \int_0^t \|\Delta u\|_{L^\gamma}^{\frac{4p+4}{-p+3}} \\
&\le c_3 \int_0^t \|u\|_{L^\gamma}^{\frac{4p+4}{-p+3}} + c_4\Big\{\|u_0\|_{D(A_\gamma)} + \int_0^t \|n\nabla\phi\|_{L^\gamma}^{\frac{4p+4}{-p+3}}\Big\} \\
&\le c_3 \int_0^t \|u\|_{L^\gamma}^{\frac{4p+4}{-p+3}} + c_4\|u_0\|_{D(A_\gamma)} + c_4\|\nabla\phi\|_{L^\infty}\int_0^t \|n(s)\|_{L^\gamma}^{\frac{4p+4}{-p+3}}
\end{aligned}$$

due to $\phi \in W^{1,\infty}$, where c_3, c_4 are positive constants. In light of the Gagliardo-Nirenberg inequality we can find a positive constant c_5 such that

$$\int_0^t \int_\Omega |u|^{\frac{4p+4}{-p+3}} \le K_2 + c_4\|u_0\|_{D(A_\gamma)} + c_5\|\nabla\phi\|_{L^\infty}\int_0^t \|\nabla n^{\frac{p+m-1}{2}}(s)\|_{L^2}^2 \|n(t)\|_{L^{q_1}}^{\alpha_2}, \tag{2.4}$$

where $K_2 := c_3 \int_0^t \|u\|_{L^\gamma}^{\frac{4p+4}{-p+3}}\,ds$, $q_1 := 3\{\frac{3-p}{4p+4} - (p+m-1)\}\{1 + \frac{3-p}{2(p+1)}\}^{-1}$ and some $\alpha_2 > 0$. Hence connecting (2.4) into (2.3), we see that for any $p \in [\frac{m}{2}+1, 3)$

$$\begin{aligned}
&\int_0^t \int_\Omega |u\cdot\nabla c|^{p+1} \\
&\le K_1 + K_2 + \|u_0\|_{D(A_\gamma)} + \|\nabla\phi\|_{L^\infty}\int_0^t \|\nabla n^{\frac{p+m-1}{2}}(s)\|_{L^2}^2 \|n(t)\|_{L^{q_1}}^{\alpha_2}\,ds,
\end{aligned} \tag{2.5}$$

where $K_1 > 0$ is the same constant as in Lemma 2.6. Then the estimates (2.1), (2.2) and (2.5) provide

$$
\begin{aligned}
&\|n(t)\|^p_{L^p(\Omega)} - \|n_0\|^p_{L^p(\Omega)} \\
&\le -\frac{2mp(p-1)}{(m+p-1)^2} \int_0^t \|\nabla n^{\frac{m+p-1}{2}}\|^2_{L^2} + c_1(p-1)\|\Delta c_0\|_{L^{p+1}} \\
&\quad + (p-1)(c_1\|c_0\|_{L^\infty} + 1) \int_0^t \|\nabla n^{\frac{p+m-1}{2}}\|^2_{L^2} \|n\|^{2-m}_{L^{\frac{3(p-1)}{2}}}\, ds \\
&\quad + (p-1)\Big\{K_1 + K_2 + c_4\|u_0\|_{D(A_\gamma)} + c_5\|\nabla\phi\|_{L^\infty} \int_0^t \|\nabla n^{\frac{p+m-1}{2}}(s)\|^2_{L^2}\|n(t)\|^{\alpha_2}_{L^{q_1}}\, ds\Big\}.
\end{aligned}
$$

Then the smallness of the initial data gives uniform-in-time L^p-estimate on $[0, t_1)$ for small $t_1 > 0$ and for any $p \in [\frac{m}{2}+1, 3)$. The mass conservation law with this boundedness entails uniform-in-time L^p-estimate on $[0, t_1)$ for any $p \in [1, 3)$. Moreover, repeating the same argument for the sake of a priori estimate and the assumption (A1), (A2), (A3) finally ensures L^p-estimate on $[0, T)$, and therefore this completes the proof.

Remark 2.3. Our main purpose of this work is to find the global solvability in (KSS) especially with $m \in [1, \frac{8}{7})$. Since this report could not achieve it the author would like to research it in the future.

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