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On two phase problem: compressible-compressible model problem*

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Abstract

We consider the model problem for the two phase problem in cases of compressible - compressible fluid flows without surface tension. In order to prove the local in time existence theorem for our problem, the generation of analytic semigroup for linearized problem and its maximal L_p-L_q regularity are needed in our method. The key step of our method is to prove the existence of $\mathcal{R}$-bounded solution operator to the generalized resolvent problem corresponding to the linearized problem:

1 Introduction

Two phase problem appears in various situations. For example, in order to analyze a motion of raindrops and air bubbles under water, we have to consider the two phase problem. Mathematical analysis for two phase problem has been studied by some mathematicians. We shall introduce the results corresponding to two phase problem.

In two phase problem of compressible and incompressible viscous fluid, Denisova [1] studied a local in time existence theorem for her problem under the technical condition. Recently in Kubo, Shibata and Soga [2], the existence of $\mathcal{R}$-bounded solution operator to generalized resolvent problem corresponding to two phase problem is shown under the natural condition derived from physics. By Weis' operator valued Fourier multiplier theorem with $\mathcal{R}$-boundedness of solution operator, we can show the maximal regularity for the linearized problem for two phase problem. A local in time existence theorem is obtained by applying the maximal regularity to proving the convergence of the successive approximations.

On the other hand, in two phase problem of compressible and compressible viscous fluid, Tani [4],[5] studied a local in time existence theorem under the natural condition in Hölder space framework. In this article, we shall consider the two phase problem of compressible and compressible fluid in L^p-L^q framework and prove the local in time existence theorem of our problem in a similar way as [2].

For this purpose, we shall consider the model problem for the two phase problem in cases of compressible-compressible fluid flows without surface tension. The key step of

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our method is to prove the existence of $\mathcal{R}$-bounded solution operator to the generalized resolvent problem corresponding to the linearized problem:

$$\lambda\rho_\pm + \gamma_1^\pm \mathrm{div}\vec{u}_\pm = f_\pm \quad \text{in } \mathbb{R}^N_\pm, \tag{1.1}$$
$$\lambda\vec{u}_\pm - \mathrm{Div}\, S_\pm(\vec{u}_\pm, \rho_\pm) = \vec{g}_\pm \quad \text{in } \mathbb{R}^N_\pm, \tag{1.2}$$
$$\vec{u}_+|_{x_N=0+} - \vec{u}_-|_{x_N=0-} = \vec{k} \quad \text{on } \mathbb{R}^N_0, \tag{1.3}$$
$$S_+(\vec{u}_+, \rho_+)\vec{n}|_{x_N=0+} - S_-(\vec{u}_-, \rho_-)\vec{n}|_{x_N=0-} = -\vec{h} \quad \text{on } \mathbb{R}^N_0. \tag{1.4}$$

Here, $\rho_\pm, \vec{u}_\pm = (u_{\pm,1}, \ldots, u_{\pm,N})(N \geq 2)$ are unknown mass density and unknown velocity fields. $S_\pm(\vec{u}_\pm, \rho_\pm) = 2\mu_1^\pm D(\vec{u}_\pm) + (\mu_2^\pm \mathrm{div}\vec{u}_\pm - \gamma_2^\pm \rho_\pm)I$ is stress tensor, $D(\vec{u}) = (\nabla\vec{u} +^T \nabla\vec{u})/2$ is $N \times N$ matrix called the Cauchy deformation tensor and I denotes the $N \times N$ identity matrix. Moreover for $N \times N$ matrix function $M = (M_{ij})$, the i th component of $\mathrm{Div}M$ is defined by $\sum_{j=1}^N \partial_j M_{ij}$. $\vec{n} = (0, \ldots, 0, -1)$ is the unit outer normal to $\mathbb{R}^n_-$ and $\mu_i^\pm, \gamma_i^\pm$ $(i = 1, 2)$ are all constants satisfying

$$\mu_1^\pm > 0,\ \mu_1^\pm + \mu_2^\pm > 0,\ \gamma_1^\pm, \gamma_2^\pm \geq 0. \tag{1.5}$$

Here $\mu_1^\pm$ and $\mu_2^\pm$ are 1st and 2nd viscosity constants, respectively, and $\gamma_1^\pm, \gamma_2^\pm$ are constants appearing in the linearization of the original nonlinear problem. The resolvent parameter λ varies in $\Lambda_{\varepsilon,\lambda_0} = \Sigma_{\varepsilon,\lambda_0} \cap K_\varepsilon$,where

$$\begin{aligned} \Sigma_{\varepsilon,\lambda_0} &= \{\lambda \in \mathbb{C} \mid |\arg\lambda| \leq \pi - \varepsilon,\ |\lambda| \geq \lambda_0\}, \\ K_\varepsilon &= \{\lambda \in \mathbb{C} \mid (\mathrm{Re}\lambda + \gamma_m + \varepsilon)^2 + (\mathrm{Im}\lambda)^2 \geq (\gamma_m + \varepsilon)^2\} \end{aligned} \tag{1.6}$$

with $\gamma_m = \max\left(\frac{\gamma_1^+\gamma_2^+}{\mu_1^+ + \mu_2^+}, \frac{\gamma_1^-\gamma_2^-}{\mu_1^- + \mu_2^-}\right)$.

Before stating our main results, we shall introduce several symbols and functional spaces. For the differentations of N-vector $\vec{g} = (g_1, \ldots, g_N)$, we use the following symbols:

$$\nabla\vec{g} = (\partial_i f_j \mid i, j = 1, \ldots, N), \qquad \nabla^2\vec{g} = (\partial_i\partial_j g_k \mid i, j, k = 1, \ldots, N).$$

For any domain Ω, $L_q(\Omega)$ and $W_q^m(\Omega)$ denote the usual Lebesgue space and Sobolev space, while $\|\cdot\|_{L_q(\Omega)}$ and $\|\cdot\|_{W_q^m(\Omega)}$ denote their norms, respectively. For any two Banach spaces X and Y, $\mathcal{L}(X, Y)$ denotes the set of all bounded linear operators from X to Y. $\mathrm{Hol}(U, X)$ denotes the set of all X-valued holomorphic functions defined on U. $\mathbb{N}$ and $\mathbb{C}$ denote the set of all natural and complex numbers, respectively, and we set $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$.

Next we introduce the definition of $\mathcal{R}$-boundedness which is the key word in our method.

Definition 1.1. Let X and Y be Banach spaces. A family of operator $\mathcal{T} \subset \mathcal{L}(X, Y)$ is called $\mathcal{R}$-bounded on $\mathcal{L}(X, Y)$, if there exist constants $C > 0$ and $p \in [1, \infty)$ such that for any $n \in \mathbb{N}$, $\{T_j\}_{j=1}^n \subset \mathcal{T}$, $\{x_j\}_{j=1}^n \subset X$ and sequences $\{r_j(u)\}_{j=1}^n$ of independent, symmetric, $\{-1, 1\}$-valued random variables on $[0, 1]$ there holds the inequality:

$$\left\{\int_0^1 \|\sum_{j=1}^n r_j(u)T_j x_j\|_Y^p du\right\}^{1/p} \leq C\left\{\int_0^1 \|\sum_{j=1}^n r_j(u)x_j\|_X^p du\right\}^{1/p}.$$

The smallest such C is called $\mathcal{R}$-bound of $\mathcal{T}$, which is denoted by $\mathcal{R}_{\mathcal{L}(X,Y)}(\mathcal{T})$.

Then we can obtain the following main result.

Theorem 1.2. *Let $1 < q < \infty$, $0 < \epsilon < \pi/2$ and $\lambda_0 > 0$. Let $\Sigma_{\epsilon,\lambda_0}$ and K_ϵ be the sets defined in* (1.6) *and set $\Lambda_{\epsilon,\lambda_0} = \Sigma_{\epsilon,\lambda_0} \cap K_\epsilon$. Set*

$$
\begin{aligned}
Y_q = \{(f_+, f_-, \vec{g}_+, \vec{g}_-, \vec{h}, \vec{k}) \mid \\
&f_\pm \in W_q^1(\mathbb{R}^N_\pm),\ \vec{g}_\pm \in L_q(\mathbb{R}^N_\pm)^N,\ \vec{h} \in W_q^1(\mathbb{R}^N)^N,\ \vec{k} \in W_q^2(\mathbb{R}^N)^N\},\\
\mathcal{Y}_q = \{(F_{0+}, F_{0-}, F_{1+}, F_{1-}, F_2, F_3, F_4, F_5, F_6) \mid F_{0\pm} \in W_q^1(\mathbb{R}^N_\pm),\\
&F_{1\pm} \in L_q(\mathbb{R}^N_\pm)^N,\ F_2, F_5 \in L_q(\mathbb{R}^N)^{N^2},\ F_3, F_6 \in L_q(\mathbb{R}^N)^N,\ F_4 \in L_q(\mathbb{R}^N)^{N^3}\}.
\end{aligned}
$$

Then, there exist operator families

$$
\mathcal{P}_\pm(\lambda) \in \mathrm{Hol}(\Lambda_{\epsilon,\lambda_0}, \mathcal{L}(\mathcal{Y}_q, W_q^1(\mathbb{R}^N_\pm))),\quad \mathcal{U}_\pm(\lambda) \in \mathrm{Hol}(\Lambda_{\epsilon,\lambda_0}, \mathcal{L}(\mathcal{Y}_q, W_q^2(\mathbb{R}^N_\pm)^N))
$$

such that for any $(f_+, f_-, \vec{g}_+, \vec{g}_-, \vec{h}, \vec{k}) \in Y_q$ and $\lambda \in \Lambda_{\epsilon,\lambda_0}$,

$$
\begin{aligned}
\rho_\pm &= \mathcal{P}_+(\lambda)(f_+, f_-, \vec{g}_+, \vec{g}_-, \nabla\vec{h}, \lambda^{1/2}\vec{h}, \nabla^2\vec{k}, \lambda^{1/2}\nabla\vec{k}, \lambda\vec{k}),\\
\vec{u}_\pm &= \mathcal{U}_\pm(\lambda)(f_+, f_-, \vec{g}_+, \vec{g}_-, \nabla\vec{h}, \lambda^{1/2}\vec{h}, \nabla^2\vec{k}, \lambda^{1/2}\nabla\vec{k}, \lambda\vec{k})
\end{aligned}
$$

solve problem (1.1)-(1.4) *uniquely. Moreover, there exists a constant C depending on ϵ, λ_0, q and N such that*

$$
\begin{aligned}
&\mathcal{R}_{\mathcal{L}(\mathcal{Y}_q, W_q^1(\mathbb{R}^N_\pm)^2)}(\{(\tau\partial_\tau)^\ell\{(\lambda,\gamma)\mathcal{P}_\pm(\lambda)\} \mid \lambda \in \Gamma_{\epsilon,\lambda_0}\}) \le C \quad (\ell = 0, 1),\\
&\mathcal{R}_{\mathcal{L}(\mathcal{Y}_q, L_q(\mathbb{R}^N_\pm)^{N^3+N^2+2N})}(\{(\tau\partial_\tau)^\ell(G_\lambda\mathcal{U}_\pm(\lambda)) \mid \lambda \in \Gamma_{\epsilon,\lambda_0}\}) \le C \quad (\ell = 0, 1),
\end{aligned}
\tag{1.7}
$$

where $G_\lambda u = (\lambda u, \gamma u, \lambda^{1/2}\nabla u, \nabla^2 u)$ and $\lambda = \gamma + i\tau$.

2 Outline of the Proof of Theorem 1.2

In this section, we shall show the outline of the proof of Theorem 1.2. First step of our method is to obtain the solution formula for (1.1)-(1.4) by Fourier transform with respect to $x' = (x_1, \ldots, x_{N-1})$. Second step is to show the $\mathcal{R}$-boundedness for solution operator by using solution formula with technical lemmas.

2.1 Solution formula

In this section, we shall show the solution formula for (1.1)-(1.4). For simplicity, we consider the case where $f_\pm = 0$ and $\vec{g}_\pm = \vec{0}$. Substitute (1.1) into (1.2) and (1.4), we can reduce (1.1)-(1.4) to the following equations:

$$
\lambda v_\pm - \mathrm{Div}\left[2\mu_1^\pm D(v_\pm) + \left(\mu_2^\pm + \frac{\gamma_1^\pm\gamma_2^\pm}{\lambda}\right)(\mathrm{div}\vec{v}_\pm)I\right] = 0 \qquad \text{in } \mathbb{R}^N_\pm \tag{2.1}
$$

$$
v_{+,J} - v_{-,J} = k_J \qquad \text{on } \mathbb{R}^N_0 \tag{2.2}
$$

$$\mu_1^+ \left(D_N v_{+,j} + D_j v_{+,N}\right) - \mu_1^- \left(D_N v_{-,j} + D_j v_{-,N}\right) = -h_j \qquad \text{on } \mathbb{R}_0^N \tag{2.3}$$

$$\begin{aligned} &2\mu_1^+ D_N v_{+,N} + \left(\mu_2^+ + \frac{\gamma_1^+\gamma_2^+}{\lambda}\right) \mathrm{div}\vec{v}_+ \\ &\qquad - \left[2\mu_1^- D_N v_{-,N} + \left(\mu_2^- + \frac{\gamma_1^-\gamma_2^-}{\lambda}\right) \mathrm{div}\vec{v}_-\right] = -h_N \qquad \text{on } \mathbb{R}_0^N \end{aligned} \tag{2.4}$$

Here and hereafter, j and J run from 1 through $N-1$ and N and we set $\delta_\lambda^\pm = \gamma_1^\pm\gamma_2^\pm/\lambda$ for simplicity of notation. In order to obtain the solution formula of (2.1), we prepare the following formula obtained by applying the divergence to (2.1):

$$\left[\lambda - \left(2\mu_1^\pm + \mu_2^\pm + \delta_\lambda^\pm\right) \Delta\right] \mathrm{div}\ v_\pm = 0.$$

By using the formula above and (2.1), we see that

$$\left[\lambda - \left(2\mu_1^\pm + \mu_2^\pm + \delta_\lambda^\pm\right) \Delta\right] (\lambda - \mu_1^\pm\Delta) v_\pm = 0. \tag{2.5}$$

In order to obtain the solution formula of (2.1)-(2.4), we use the partial Fourier transform with respect to $x' = (x_1, \dots, x_{N-1})$ and the partial inverse Fourier transform defined by

$$\mathcal{F}_{x'}[v](\xi', x_N) = \widehat{v} = \int_{\mathbb{R}^{N-1}} e^{-ix'\cdot\xi'} v(x', x_N)dx',$$
$$\mathcal{F}_{x'}^{-1}[w(\xi', x_N)](x') = \left(\frac{1}{2\pi}\right)^{N-1} \int_{\mathbb{R}^{N-1}} e^{ix'\cdot\xi'} w(\xi', x_N)d\xi',$$

respectively. Taking

$$\begin{aligned} 2\mathcal{F}_{x'}[\mathrm{Div}\, D(v_j)](\xi', x_N) &= -|\xi'|^2\widehat{v_j} + D_N^2\widehat{v_j} + i\xi_j(i\xi'\cdot\widehat{v'} + D_N\widehat{v_N}), \\ 2\mathcal{F}_{x'}[\mathrm{Div}\, D(v_N)](\xi', x_N) &= -|\xi'|^2\widehat{v_N} + D_N^2\widehat{v_N} + D_N(i\xi'\cdot\widehat{v'} + D_N\widehat{v_N}) \end{aligned}$$

into account, we obtain the following equations by applying the partial Fourier transform to (2.1)-(2.4) and (2.5):

$$\begin{cases} \lambda\widehat{v_{+,j}} - \mu_1^+ \left[(D_N^2 - |\xi'|^2)\widehat{v_{+,j}} + i\xi_j\widehat{\mathrm{div}\vec{v}_+}\right] - (\mu_2^+ + \delta_\lambda^+)i\xi_j\widehat{\mathrm{div}\vec{v}_+} = 0, \\ \lambda\widehat{v_{+,N}} - \mu_1^+ \left[(D_N^2 - |\xi'|^2)\widehat{v_{+,N}} + D_N\widehat{\mathrm{div}\vec{v}_+}\right] - (\mu_2^+ + \delta_\lambda^+)D_N\widehat{\mathrm{div}\vec{v}_+} = 0, \\ \lambda\widehat{v_{-,j}} - \mu_1^- \left[(D_N^2 - |\xi'|^2)\widehat{v_{-,j}} + i\xi_j\widehat{\mathrm{div}\vec{v}_-}\right] - (\mu_2^- + \delta_\lambda^-)i\xi_j\widehat{\mathrm{div}\vec{v}_-} = 0, \\ \lambda\widehat{v_{-,N}} - \mu_1^- \left[(D_N^2 - |\xi'|^2)\widehat{v_{-,N}} + D_N\widehat{\mathrm{div}\vec{v}_-}\right] - (\mu_2^- + \delta_\lambda^-)D_N\widehat{\mathrm{div}\vec{v}_-} = 0 \end{cases} \tag{2.6}$$

and

$$\left[\lambda + \left(2\mu_1^\pm + \mu_2^\pm + \delta_\lambda^\pm\right)\left(|\xi'|^2 - D_N^2\right)\right]\left[\lambda + \mu_1^\pm(|\xi'|^2 - D_N^2)\right]\widehat{v_{\pm,J}} = 0. \tag{2.7}$$

By (2.7), we see that the characteristic roots of (2.6) are

$$A_\pm = \sqrt{\left(2\mu_1^\pm + \mu_2^\pm + \delta_\lambda^\pm\right)^{-1}\lambda + A^2}, \qquad B_\pm = \sqrt{\left(\mu_1^\pm\right)^{-1}\lambda + A^2}, \qquad A = |\xi'|.$$

By using $B_\pm$ and A, we rewrite (2.6) as follows:

$$\begin{cases} \mu_1^+(B_+^2-D_N^2)\widehat{v_{+,j}} - (\mu_1^+ + \mu_2^+ + \delta_\lambda^+)\, i\xi_j \widehat{\mathrm{div}\vec{v}_+} = 0, \\ \mu_1^+(B_+^2-D_N^2)\widehat{v_{+,N}} - (\mu_1^+ + \mu_2^+ + \delta_\lambda^+)\, D_N \widehat{\mathrm{div}\vec{v}_+} = 0, \\ \mu_1^-(B_-^2-D_N^2)\widehat{v_{-,j}} - (\mu_1^- + \mu_2^- + \delta_\lambda^-)\, i\xi_j \widehat{\mathrm{div}\vec{v}_-} = 0, \\ \mu_1^-(B_-^2-D_N^2)\widehat{v_{-,N}} - (\mu_1^- + \mu_2^- + \delta_\lambda^-)\, D_N \widehat{\mathrm{div}\vec{v}_-} = 0. \end{cases} \tag{2.8}$$

From now, we shall find the solution $\widehat{v_{\pm,J}}$ to (2.6) of the forms:

$$\widehat{v_{+,J}} = \alpha_J^+(e^{-B_+x_N} - e^{-A_+x_N}) + \beta_J^+ e^{-B_+x_N}, \quad \widehat{v_{-,J}} = \alpha_J^-(e^{B_-x_N} - e^{A_-x_N}) + \beta_J^- e^{B_-x_N}. \tag{2.9}$$

We see that $(B_\pm^2 - D_N^2)\widehat{v_{\pm,J}} = (A_\pm^2 - B_\pm^2)\alpha_J^\pm e^{\mp A_\pm x_N}$ and

$$\begin{aligned} \widehat{\mathrm{div}\vec{v}_+} &= (i\xi'\cdot\alpha'_+ + i\xi'\cdot\beta'_+ - B_+(\alpha_N^+ + \beta_N^+))e^{-B_+x_N} + (A_+\alpha_N^+ - i\xi'\cdot\alpha'_+)e^{-A_+x_N}, \\ \widehat{\mathrm{div}\vec{v}_-} &= (i\xi'\cdot\alpha'_- + i\xi'\cdot\beta'_- + B_-(\alpha_N^- + \beta_N^-))e^{B_-x_N} - (A_-\alpha_N^- + i\xi'\cdot\alpha'_-)e^{A_-x_N}, \end{aligned} \tag{2.10}$$

where $\alpha'_\pm = (\alpha_1^\pm, \dots, \alpha_{N-1}^\pm)$ and $\beta'_\pm = (\beta_1^\pm, \dots, \beta_{N-1}^\pm)$.

Substituting (2.10) into (2.8) and equating the coefficients of $e^{\mp B_\pm x_N}, e^{\mp A_\pm x_N}$, we have

$$\begin{cases} i\xi'\cdot\alpha'_+ + i\xi'\cdot\beta'_+ - B_+(\alpha_N^+ + \beta_N^+) = 0, \\ i\xi'\cdot\alpha'_- + i\xi'\cdot\beta'_- + B_-(\alpha_N^- + \beta_N^-) = 0, \\ \mu_1^+(A_+^2 - B_+^2)\alpha_j^+ - (\mu_1^+ + \mu_2^+ + \delta_\lambda^+)\, i\xi_j(A_+\alpha_N^+ - i\xi'\cdot\alpha'_+) = 0, \\ \mu_1^+(A_+^2 - B_+^2)\alpha_N^+ + (\mu_1^+ + \mu_2^+ + \delta_\lambda^+)\, A_+(A_+\alpha_N^+ - i\xi'\cdot\alpha'_+) = 0, \\ \mu_1^-(A_-^2 - B_-^2)\alpha_j^- + (\mu_1^- + \mu_2^- + \delta_\lambda^-)\, i\xi_j(A_-\alpha_N^- + i\xi'\cdot\alpha'_-) = 0, \\ \mu_1^-(A_-^2 - B_-^2)\alpha_N^- + (\mu_1^- + \mu_2^- + \delta_\lambda^-)\, A_-(A_-\alpha_N^+ + i\xi'\cdot\alpha'_-) = 0. \end{cases} \tag{2.11}$$

Since $\mu_1^+(A_+^2 - B_+^2) + (\mu_1^+ + \mu_2^+ + \delta_\lambda^+)A_+^2 = (\mu_1^+ + \mu_2^+ + \delta_\lambda^+)A^2$, the fourth equation in (2.11) implies that $\alpha_N^+ = A^{-2}A_+ i\xi'\cdot\alpha'_+$. By the first equation in (2.11), we have

$$i\xi'\cdot\alpha'_+ = \frac{A^2}{B_+A_+ - A^2}(i\xi'\cdot\beta'_+ - B_+\beta_N^+), \quad \alpha_N^+ = \frac{A_+}{B_+A_+ - A^2}(i\xi'\cdot\beta'_+ - B_+\beta_N^+). \tag{2.12}$$

Similarly, by the sixth equation and the second equation in (2.11), we obtain

$$i\xi'\cdot\alpha'_- = \frac{A^2}{B_-A_- - A^2}(i\xi'\cdot\beta'_- + B_-\beta_N^-), \quad \alpha_N^- = \frac{-A_-}{B_+A_+ - A^2}(i\xi'\cdot\beta'_- + B_-\beta_N^-). \tag{2.13}$$

Next we consider the boundary condition (2.2)-(2.4). By applying the partial Fourier transform to (2.2)-(2.4), we obtain

$$\beta_J^+ - \beta_J^- = \widehat{k_J}, \tag{2.14}$$

$$\mu_1^+\left((A_+ - B_+)\alpha_j^+ - B_+\beta_j^+ + i\xi_j\beta_N^+\right) - \mu_1^-\left((B_- - A_-)\alpha_j^- + B_-\beta_j^- + i\xi_j\beta_N^-\right) = -\widehat{h_j}, \tag{2.15}$$

$$\begin{aligned} &(2\mu_1^+ + \mu_2^+ + \delta_\lambda^\pm)(A_+ - B_+)\alpha_N^+ - 2\mu^+B_+\beta_N^+ + (\mu_2^+ + \delta_\lambda^+)(i\xi'\cdot\beta'_+ - B_+\beta_N^+) \\ &- (2\mu_1^- + \mu_2^- + \delta_\lambda^-)(B_- - A_-)\alpha_N^- - 2\mu_1^-B_-\beta_N^- - (\mu_2^- + \delta_\lambda^-)(i\xi'\cdot\beta'_- + B_-\beta_N^-) = -\widehat{h_N}. \end{aligned} \tag{2.16}$$

Since by (2.12) and (2.13), we have

$$-i\xi'\cdot\widehat{h'} = \mu_1^+\left(\frac{A_+(A^2-B_+^2)}{B_+A_+-A^2}i\xi'\cdot\beta'_+ - \frac{A^2(2A_+B_+-B_+^2-A^2)}{B_+A_+-A^2}\beta_N^+\right)$$
$$-\mu_1^-\left(\frac{A_-(B_-^2-A^2)}{B_-A_--A^2}i\xi'\cdot\beta'_- + \frac{A^2(A^2+B_-^2-2A_-B_-)}{B_-A_--A^2}\beta_N^-\right)$$

and

$$-\widehat{h_N} = \frac{1}{B_+A_+-A^2}\left[2\mu_1^+(A_+^2-A_+B_+)+(\mu_2^++\delta_\lambda^+)(A_+^2-A^2)\right]i\xi'\cdot\beta'_+$$
$$-\left(2\mu_1^++\mu_2^++\delta_\lambda^+\right)\frac{A_+^2-A^2}{B_+A_+-A^2}B_+\beta_N^+ - \left(2\mu_1^-+\mu_2^-+\delta_\lambda^-\right)\frac{A_-^2-A^2}{B_-A_--A^2}B_-\beta_N^-$$
$$-\frac{1}{B_-A_--A^2}\left[2\mu_1^-(A_-^2-A_-B_-)+(\mu_2^-+\delta_\lambda^-)(A_-^2-A^2)\right]i\xi'\cdot\beta'_-.$$

Substituting $\beta_J^+ = \beta_J^- + \widehat{k_J}$ by (2.14) into the formula of $-i\xi'\cdot\widehat{h'}$ and $-\widehat{h_N}$, we obtain

$$-\mu_1^+\frac{A_+(A^2-B_+^2)}{B_+A_+-A^2}i\xi'\cdot\widehat{k'} + \mu_1^-\frac{A^2(2A_+B_+-B_+^2-A^2)}{B_+A_+-A^2}\widehat{k_N} - i\xi'\cdot\widehat{h'}$$
$$= -\left[\mu_1^+\frac{A_+(B_+^2-A^2)}{B_+A_+-A^2} + \mu_1^-\frac{A_-(B_-^2-A^2)}{B_-A_--A^2}\right]i\xi'\cdot\beta'_-$$
$$-A^2\left[\mu_1^+\frac{2A_+B_+-B_+^2-A^2}{B_+A_+-A^2} - \mu_1^-\frac{2A_-B_--A^2-B_-^2}{B_-A_--A^2}\right]\beta_N^-$$

and

$$\frac{-i\xi'\cdot\widehat{k'}}{B_+A_+-A^2}\left[2\mu_1^+(A_+^2-A_+B_+)+(\mu_2^++\delta_\lambda^+)(A_+^2-A^2)\right]$$
$$+\left(2\mu_1^++\mu_2^++\delta_\lambda^+\right)\frac{(A_+^2-A^2)B_+\widehat{k_N}}{B_+A_+-A^2} - \widehat{h_N}$$
$$= \frac{i\xi'\cdot\beta'_-}{B_+A_+-A^2}\left[2\mu_1^+(A_+^2-A_+B_+)+(\mu_2^++\delta_\lambda^+)(A_+^2-A^2)\right]$$
$$-\frac{i\xi'\cdot\beta'_-}{B_-A_--A^2}\left[2\mu_1^-(A_-^2-A_-B_-)+(\mu_2^-+\delta_\lambda^-)(A_-^2-A^2)\right]$$
$$-\left(2\mu_1^++\mu_2^++\delta_\lambda^+\right)\frac{(A_+^2-A^2)B_+\beta_N^-}{B_+A_+-A^2} - \left(2\mu_1^-+\mu_2^-+\delta_\lambda^-\right)\frac{(A_-^2-A^2)B_-\beta_N^-}{B_-A_--A^2}.$$

Here setting

$$L_{11}^+ = -\mu_1^+\frac{A_+(B_+^2-A^2)}{B_+A_+-A^2}, \qquad L_{11}^- = -\mu_1^-\frac{A_-(B_-^2-A^2)}{B_-A_--A^2},$$
$$L_{12}^+ = -\mu_1^+A^2\frac{2A_+B_+-B_+^2-A^2}{B_+A_+-A^2}, \qquad L_{12}^- = \mu_1^-A^2\frac{2A_-B_--A^2-B_-^2}{B_-A_--A^2},$$

$$L_{21}^+ = \frac{1}{B_+A_+ - A^2}\left[2\mu_1^+(A_+^2 - A_+B_+) + (\mu_2^+ + \delta_\lambda^+)(A_+^2 - A^2)\right],$$
$$L_{21}^- = -\frac{1}{B_-A_- - A^2}\left[2\mu_1^-(A_-^2 - A_-B_-) + (\mu_2^- + \delta_\lambda^-)(A_-^2 - A^2)\right],$$
$$L_{22}^+ = -\left(2\mu_1^+ + \mu_2^+ + \delta_\lambda^+\right)\frac{A_+^2 - A^2}{B_+A_+ - A^2}B_+, \qquad L_{22}^- = -\left(2\mu_1^- + \mu_2^- + \delta_\lambda^-\right)\frac{A_-^2 - A^2}{B_-A_- - A^2}B_-$$

and $L_{ij} = L_{ij}^+ + L_{ij}^-$, $L = \begin{pmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{pmatrix}$, we obtain

$$L\begin{pmatrix} i\xi' \cdot \beta'_- \\ \beta_N^- \end{pmatrix} = \begin{pmatrix} -i\xi' \cdot \widehat{h}' - L_{11}^+ i\xi' \cdot \widehat{k}' - L_{12}^+ \widehat{k_N} \\ -\widehat{h_N} - L_{21}^+ i\xi' \cdot \widehat{k}' - L_{22}^+ \widehat{k_N} \end{pmatrix}. \tag{2.17}$$

If $\det L \neq 0$, we have the inverse of L and obtain

$$\begin{pmatrix} i\xi' \cdot \beta'_- \\ \beta_N^- \end{pmatrix} = \frac{1}{\det L}\begin{pmatrix} L_{22} & -L_{12} \\ -L_{21} & L_{11} \end{pmatrix}\begin{pmatrix} -i\xi' \cdot \widehat{h}' - L_{11}^+ i\xi' \cdot \widehat{k}' - L_{12}^+ \widehat{k_N} \\ -\widehat{h_N} - L_{21}^+ i\xi' \cdot \widehat{k}' - L_{22}^+ \widehat{k_N} \end{pmatrix}.$$

Then we get the formula of $i\xi' \cdot \alpha'_\pm, \alpha_N^\pm$ and β_J^+ by (2.12), (2.13) and (2.14). Since we have the formula of $\alpha_j^\mp$ by (2.11), we can obtain the solution formula of (1.1)- (1.4) if $\det L \neq 0$. In next section, we shall consider the Lopantinski determinant $\det L$ when $\lambda \in \Lambda_{\varepsilon,\lambda_0} = \Sigma_{\varepsilon,\lambda_0} \cap K_\varepsilon$.

2.2 Analysis of Lopatinski determinant

In order to analyze Lopatinski determinant, we shall prove the following lemma, which is one of the essential steps in this article.

Lemma 2.1. *Let L be the matrix defined in section 2.1.*

(I) *there exists a positive constant ω depending on $\mu_1^\pm, \mu_2^\pm, \varepsilon, \lambda_0$ and δ_0 such that*

$$|A \det L| \geq \omega(|\lambda|^{1/2} + A)^3 \tag{2.18}$$

for any $(\lambda, \xi') \in \widetilde{\Gamma}_{\varepsilon,\lambda_0}$.

(II) *For any multi-index $\kappa' \in \mathbb{N}_0^{N-1}$ and $(\lambda, \xi') \in \widetilde{\Gamma}_{\varepsilon,\lambda_0}$, the following inequalities hold:*

$$|\partial_{\xi'}^{\kappa'}\{(\tau\partial_\tau)^\ell (A\det L)^{-1}\}| \leq C_{\kappa'}(|\lambda|^{1/2} + A)^{-3}A^{-|\kappa'|}, \quad (\ell = 0, 1) \tag{2.19}$$

Proof. Since we can prove (2.19) by using Leibniz rule and the Bell formula:

$$\partial_{\xi'}^{\kappa'} f(g(\xi')) = \sum_{\ell=1}^{|\kappa'|} f^{(\ell)}(g(\xi')) \sum_{\kappa'_1+\cdots+\kappa'_\ell=\kappa',\ |\kappa'_i|\geq 1} \Gamma_{\kappa'_1,\ldots,\kappa'_\ell}^{\kappa'} (\partial_{\xi'}^{\kappa'_1} g(\xi')) \ldots (\partial_{\xi'}^{\kappa_\ell} g(\xi'))$$

with $f(t)=1/t$ and $g(\xi')=A\det L$ with (2.18), it is sufficient to prove (2.18). In order to prove (2.18), we consider the three cases: (i) $R_1|\lambda|^{1/2}\le A$, (ii) $R_2A\le|\lambda|^{1/2}$, (iii) $R_2^{-1}|\lambda|^{1/2}\le A\le R_1|\lambda|^{1/2}$ for large R_1 and R_2.

First we consider the case: $R_1|\lambda|^{1/2}\le A$ with large $R_1\ge 1$. We see that $|\alpha\lambda+\beta|\ge(\sin\varepsilon/2)(\alpha|\lambda|+\beta)$ for any $\lambda\in\Sigma_\varepsilon$, $\xi\in\mathbb{R}^N$ and $\alpha,\beta>0$ by elemental calculation. By using this inequality, we notice that there exists a very small positive constant δ_3 such that $|(s_1\mu_1^\pm+s_2\mu_2^\pm+\delta_\lambda^\pm)^{-1}\lambda A^{-2}|\le(\sin(\varepsilon/2))^{-1}(s_1\mu_1^\pm+s_2\mu_2^\pm)^{-1}R_1^{-2}\le\delta_3$ for $s_1,s_2\in\mathbb{R}$. Therefore we have $A_\pm=A(1+O(\delta_3)), B_\pm=A(1+O(\delta_3))$ as small δ_3. Therefore we can obtain

$$
\begin{aligned}
L_{11}^\pm&=-\frac{\mu_1^\pm(2\mu_1^\pm+\mu_2^\pm+\delta_\lambda^\pm)}{3\mu_1^\pm+\mu_2^\pm+\delta_\lambda^\pm}A(2+O(\delta_3)), &\quad L_{12}^\pm&=\mp\frac{2(\mu_1^\pm)^2}{3\mu_1^\pm+\mu_2^\pm+\delta_\lambda^\pm}A^2(1+O(\delta_3)),\\
L_{21}^\pm&=\mp\frac{2(\mu_1^\pm)^2}{3\mu_1^\pm+\mu_2^\pm+\delta_\lambda^\pm}(1+O(\delta_3)), &\quad L_{22}^\pm&=-\frac{2(2\mu_1^\pm+\mu_2^\pm+\delta_\lambda^\pm)(\mu_1^\pm)}{3\mu_1^\pm+\mu_2^\pm+\delta_\lambda^\pm}A(1+O(\delta_3)),
\end{aligned}
\tag{2.20}
$$

which imply that

$$
\det L=\left(\mu_1^+ +\frac{\mu_1^-(\mu_1^-+\mu_2^-+\delta_\lambda^-)}{3\mu_1^-+\mu_2^-+\delta_\lambda^-}\right)\left(\frac{\mu_1^+(\mu_1^++\mu_2^++\delta_\lambda^+)}{3\mu_1^++\mu_2^++\delta_\lambda^+}+\mu_1^-\right)A^2(4+O(\delta_3)).
$$

Taking the fact: $\mu_1^\pm>0, \mu_1^\pm+\mu_2^\pm>0$ into account, we see

$$
\left|\mu_1^\pm+\frac{\mu_1^\mp(\mu_1^\mp+\mu_2^\mp+\delta_\lambda^\mp)}{3\mu_1^\mp+\mu_2^\mp+\delta_\lambda^\mp}\right|=\left|\frac{\mu_1^\pm(3\mu_1^\mp+\mu_2^\mp)+\mu_1^\mp(\mu_1^\mp+\mu_2^\mp)+\delta_\lambda^\mp(\mu_1^\pm+\mu_1^\mp)}{3\mu_1^\mp+\mu_2^\mp+\delta_\lambda^\mp}\right|>0.
$$

Summing up, we can show that there exists a positive constant ω such that $|\det L|\ge\omega A^2$. Since the case $R_2A\le|\lambda|^{1/2}$ for large R_2 is shown in a similar way to the case $A\ge R_1|\lambda|^{1/2}$, we omit the case $R_2A\le|\lambda|^{1/2}$.

Finally, we consider the case $R_2^{-1}|\lambda|^{1/2}\le A\le R_1|\lambda|^{1/2}$. Set $\widetilde{\lambda}=\lambda/(|\lambda|^{1/2}+A)^2$ and

$$
\widetilde{A}=\frac{A}{|\lambda|^{1/2}+A},\quad \widetilde{A_\pm}=\sqrt{(2\mu_1^\pm+\mu_2^\pm+\delta_\lambda^\pm)^{-1}\widetilde{\lambda}+\widetilde{A}^2},\quad \widetilde{B_\pm}=\sqrt{(\mu_1^\pm)^{-1}\widetilde{\lambda}+\widetilde{A}^2}
$$

and

$$
\begin{aligned}
&D(R_1,R_2)\\
&=\{(\widetilde{\lambda},\widetilde{A})\mid(1+R_1)^{-2}\le|\widetilde{\lambda}|\le R_2^2(1+R_2)^2,(1+R_2)^{-1}\le\widetilde{A}\le R_1(1+R_1)^{-1}\}.
\end{aligned}
$$

We remark $(\widetilde{\lambda},\widetilde{A})\in D(R_1,R_2)$ if (λ,ξ') satisfies the condition $R_2^{-1}|\lambda|^{1/2}\le A\le R_1|\lambda|^{1/2}$. We also define $\widetilde{L}_{ij}$ by replacing $A_\pm, A$ and $B_\pm$ by $\widetilde{A_\pm},\widetilde{A}$ and $\widetilde{B}$, respectively. And we set $\det\widetilde{L}=\widetilde{L}_{11}\widetilde{L}_{22}-\widetilde{L}_{12}\widetilde{L}_{21}$ and then we have $\det L=(|\lambda|^{1/2}+A)^2\det\widetilde{L}$.

We shall prove that $\det\widetilde{L}\neq 0$ provided that $(\widetilde{\lambda},\widetilde{A})\in D(R_1,R_2)$ and $\widetilde{\lambda}\in\Sigma_\varepsilon$ by contradiction. To this end, we assume that $\det\widetilde{L}=0$, namely $\det L=0$. In this case, in view of (2.17) that $\vec{w}_\pm(x_N)=(w_{\pm,1}(x_N),\dots,w_{\pm,N}(x_N))\neq(0,\dots,0)$ satisfy (2.1)- (2.4) with $\widehat{h}_j(0) = 0, \widehat{h}_N(0) = 0$ and $\widehat{k}_J(0) = 0$, that is, they satisfy the following homogeneous equations:

$$\begin{aligned}
&\lambda w_{\pm,j} - \mu_1^\pm \sum_{\ell=1}^{N-1} i\xi_{\ell'}(i\xi_j w_{\pm,\ell} + i\xi_\ell w_{\pm,j}) \\
&\qquad - \mu_1^\pm D_N(i\xi_j w_{\pm,N} + D_N w_{\pm,j}) - (\mu_2^\pm + \delta_\lambda^\pm) i\xi_j (i\xi' \cdot w'_\pm + D_N w_{\pm,N}) = 0, \qquad (2.21)\\
&\lambda w_{\pm,N} - \mu_1^\pm \sum_{\ell=1}^{N-1} i\xi_\ell (D_N w_{\pm,\ell} + i\xi_\ell w_{\pm,N}) - 2\mu_1^\pm D_N^2 w_{\pm,N} \\
&\qquad\qquad - (\mu_2^\pm + \delta_\lambda^\pm) D_N(i\xi' \cdot w'_\pm + D_N w_{\pm,N}) = 0, \qquad (2.22)\\
&\mu_1^+(D_N w_{+,j} + i\xi_j w_{+,N})|_{x_N=0} - \mu_1^-(D_N w_{-,j} + i\xi_j w_{-,N})|_{x_N=0} = 0, \qquad (2.23)\\
&2\mu_1^- D_N w_{+,N} + (\mu_2^+ + \delta_\lambda^+)(i\xi' \cdot w'_+ + D_N w_{+,N})|_{x_N=0} \\
&\qquad - (2\mu_1^- D_N w_{-,N} + (\mu_2^- + \delta_\lambda^-)(i\xi' \cdot w'_- + D_N w_{-,N})|_{x_N=0} = 0. \qquad (2.24)
\end{aligned}$$

Here we set

$$(a, b)_\pm = \pm \int_0^{\pm\infty} a(x_N)\overline{b(x_N)}dx_N, \qquad \|a\|_\pm = \sqrt{(a, a)_\pm}.$$

Multipling (2.21) by $\overline{w_{\pm,j}}$ and (2.22) by $\overline{w_{\pm,N}}$ and by integration by parts, we obtain

$$\begin{aligned}
&\lambda\|w_{\pm,j}\|_\pm^2 + \mu_1^\pm \sum_{\ell=1}^{N-1} \left((i\xi_\ell, w_{\pm,\ell}, i\xi_j w_{\pm,j})_\pm + \|i\xi_\ell w_{\pm,j}\|_\pm^2\right) + \mu_1^\pm (i\xi_j w_{\pm,N}, D_N w_{\pm,j})_\pm \\
&\qquad + \mu_1^\pm \|D_N w_{\pm,j}\|_\pm^2 + (\mu_2^\pm + \delta_\lambda^\pm)\left((i\xi' \cdot w'_\pm, i\xi_j w_{\pm,j})_\pm + (D_N w_{\pm,N}, i\xi_j w_{\pm,j})_\pm\right) = 0
\end{aligned}$$

and

$$\begin{aligned}
&\lambda\|w_{\pm,N}\|_\pm^2 + \mu_1^\pm \sum_{\ell=1}^{N-1} \left((D_N w_{\ell,\ell}, i\xi_\ell w_{\pm,N})_\pm + \|i\xi_\ell w_{\pm,N}\|_\pm^2\right) + 2\mu_1^\pm \|D_N w_{\pm,N}\|_\pm^2 \\
&\qquad + (\mu_2^\pm + \delta_\lambda^\pm)\left((i\xi' \cdot w'_\pm, D_N w_{\pm,N})_\pm + \|D_N w_{\pm,N}\|_\pm^2\right) = 0.
\end{aligned}$$

Summing up, we see

$$\begin{aligned}
0 =&\lambda \sum_{j=1}^N \|w_{\pm,j}\|_\pm^2 \\
&+ \mu_1^\pm \Bigg(\|i\xi' \cdot w'_\pm\|_\pm^2 + \sum_{\ell,j=1}^{N-1} \|i\xi_\ell w_{\pm,j}\|_\pm^2 + \sum_{j=1}^{N-1} (i\xi_\ell w_{\pm,N}, D_N w_{\pm,j})_\pm \\
&\quad + \sum_{j=1}^{N-1} \|D_N w_{\pm,j}\|_\pm^2 + \sum_{\ell=1}^{N-1} \left((D_N w_{\pm,\ell}, i\xi_\ell w_{\pm,N})_\pm + \|i\xi_\ell w_{\pm,N}\|_\pm^2\right) + 2\|D_N w_{\pm,N}\|_\pm^2 \Bigg) \\
&+ (\mu_2^\pm + \delta_\lambda^\pm)\left(\|i\xi' \cdot w'_\pm\|_\pm^2 + (i\xi' \cdot w'_\pm, D_N w_{\pm,N})_\pm + (D_N w_{\pm,N}, i\xi' \cdot w'_\pm)_\pm + \|D_N w_{\pm,N}\|_\pm^2\right).
\end{aligned} \qquad (2.25)$$

Taking the fact $\delta_\lambda^\pm = \frac{\gamma_1^\pm\gamma_2^\pm}{|\lambda|^2}(\mathrm{Re}\lambda - \mathrm{Im}\lambda)$ and

$$
\begin{aligned}
&\|i\xi_j w_{\pm,j} + D_N w_{\pm,j}\|_\pm^2 \\
&= (i\xi_\ell w_{\pm,N}, D_N w_{\pm,j})_\pm + \|D_N w_{\pm,j}\|_\pm^2 + (D_N w_{\pm,j}, i\xi_j w_{\pm,N})_\pm + \|i\xi_j w_{\pm,N}\|_\pm^2, \\
&\|i\xi'\cdot w'_\pm + D_N w_{\pm,N}\|_\pm^2 \\
&= \|i\xi'\cdot w'_\pm\|_\pm^2 + (i\xi'\cdot w'_\pm, D_N w_{\pm,N})_\pm + (D_N w_{\pm,N}, i\xi'\cdot w'_\pm)_\pm + \|D_N w_{\pm,N}\|_\pm^2,
\end{aligned}
$$

into account and taking the real part and the imaginary part in (2.25), we have

$$
(\mathrm{Im}\lambda)\left(\sum_{j=1}^N \|w_{\pm,j}\|_\pm^2 - \frac{\gamma_1^\pm\gamma_2^\pm}{|\lambda|^2}\|i\xi'\cdot w'_\pm + D_N w_{\pm,N}\|_\pm^2\right) = 0 \tag{2.26}
$$

and

$$
\begin{aligned}
&\mathrm{Re}\lambda \sum_{j=1}^N \|w_{\pm,j}\|_\pm^2 \\
&+ \mu_1^\pm\left(\|i\xi'\cdot w'_\pm\|_\pm^2 + \sum_{\ell,j=1}^{N-1}\|i\xi_\ell w_{\pm,j}\|_\pm^2 + \sum_{j=1}^{N-1}\|i\xi_j w_{\pm,j} + D_N w_{\pm,j}\|_\pm^2 + 2\|D_N w_{\pm,N}\|_\pm^2\right) \\
&+ \left(\mu_2^\pm + \frac{\gamma_1^\pm\gamma_2^\pm}{|\lambda|^2}\mathrm{Re}\lambda\right)\|i\xi'\cdot w'_\pm + D_N w_{\pm,N}\|_\pm^2 = 0.
\end{aligned} \tag{2.27}
$$

When Im $\lambda = 0$ and Re$\lambda \geq 0$, we see $\|w_{\pm,j}\|_\pm = 0$, namely $w_\pm = 0$, which contradict to $w_\pm \neq 0$. When Im $\lambda \neq 0$, by (2.26), (2.27) and

$$
\begin{aligned}
\|i\xi'\cdot w'_\pm\|_\pm^2 + \sum_{\ell,j=1}^{N-1}\|i\xi_\ell w_{\pm,j}\|_\pm^2 + 2\|D_N w_{\pm,N}\|_\pm^2 &\geq 2\left(\|i\xi'\cdot w'_\pm\|_\pm^2 + \|D_N w_{\pm,N}\|_\pm^2\right) \\
&\geq \|i\xi'\cdot w'_\pm + D_N w_{\pm,N}\|_\pm^2,
\end{aligned}
$$

we obtain

$$
\|i\xi'\cdot w'_\pm + D_N w_{\pm,N}\|_\pm^2\left(2\mathrm{Re}\lambda\frac{\gamma_1^\pm\gamma_2^\pm}{|\lambda|^2} + \mu_1^\pm + \mu_2^\pm\right) + \mu_1^\pm\sum_{j=1}^{N-1}\|i\xi_j w_{\pm,j} + D_N w_{\pm,j}\|_\pm^2 \leq 0.
$$

since $\mu_1^\pm > 0$ and

$$
2\mathrm{Re}\lambda\frac{\gamma_1^\pm\gamma_2^\pm}{|\lambda|^2} + \mu_1^\pm + \mu_2^\pm = \frac{\mu_1^\pm + \mu_2^\pm}{|\lambda|^2}\left(\left(\mathrm{Re}\lambda + \frac{\gamma_1^\pm\gamma_2^\pm}{\mu_1^\pm + \mu_2^\pm}\right)^2 + (\mathrm{Im}\lambda)^2 - \left(\frac{\gamma_1^\pm\gamma_2^\pm}{\mu_1^\pm + \mu_2^\pm}\right)\right),
$$

the condition $\lambda \in K_{\varepsilon,\lambda_0}$ implies $\|i\xi'\cdot w'_\pm + D_N w_{\pm,N}\|_\pm = 0$ namely $w_\pm = 0$ by (2.26) which contradict to $w_\pm \neq 0$. Therefore we see that there exists a positive constant c such that $|\det \widetilde{L}| \geq c$. Therefore we obtain $|\det L| \geq c(|\lambda|^{1/2} + A)^2$, which implies Lemma 2.1. □

2.3 Technical Lemma

In this section, we shall introduce one of technical lemmas which we need to prove Theorem 1.2. In order to prove the $\mathcal{R}$-boundedness of solution operator, we use the following lemmas which is proven by Kubo, Shibata and Soga [2].

Lemma 2.2. *Let Λ be a domain in $\mathbb{C}$ and set $\tilde{\Lambda} = \Lambda \times (\mathbb{R}^{N-1} \setminus \{0\})$. Let $n_i(\lambda, \xi')$ $(i = 1, 2)$ be multipliers defined on $\tilde{\Lambda}$ such that*

$$|\partial_{\xi'}^{\kappa'}\{(\tau\partial_\tau)^\ell n_1(\lambda,\xi')\}| \le C_{\kappa'}(|\lambda|^{1/2} + A)^{-2}A^{-|\kappa'|},$$
$$|\partial_{\xi'}^{\kappa'}\{(\tau\partial_\tau)^\ell n_2(\lambda,\xi')\}| \le C_{\kappa'}(|\lambda|^{1/2} + A)^{-1-|\kappa'|}$$

$(\ell = 0, 1)$ for any $\kappa' \in \mathbb{N}_0^{N-1}$ and $(\lambda, \xi') \in \tilde{\Lambda}$. Let $K_i^\pm$ $(i = 1, 2, 3, 4)$ be operators defined by

$$K_1^\pm(\lambda)g = \pm\int_0^{\pm\infty} \mathcal{F}_{\xi'}^{-1}[n_1(\lambda,\xi')AA_\pm M_\pm(x_N + y_N)\hat{g}(\xi', y_N)](x')\,dy_N,$$
$$K_2^\pm(\lambda)g = \pm\int_0^{\pm\infty} \mathcal{F}_{\xi'}^{-1}[n_1(\lambda,\xi')Ae^{\mp B_\pm(x_N+y_N)}\hat{g}(\xi', y_N)](x')\,dy_N,$$
$$K_3^\pm(\lambda)g = \pm\int_0^{\pm\infty} \mathcal{F}_{\xi'}^{-1}[n_2(\lambda,\xi')A_\pm M_\pm(x_N + y_N)\hat{g}(\xi', y_N)](x')\,dy_N,$$
$$K_4^\pm(\lambda)g = \pm\int_0^{\pm\infty} \mathcal{F}_{\xi'}^{-1}[n_2(\lambda,\xi')e^{\mp B_\pm(x_N+y_N)}\hat{g}(\xi', y_N)](x')\,dy_N,$$

where

$$M_\pm(x_N) = \frac{e^{\mp B_\pm x_N} - e^{\mp A_\pm x_N}}{B_\pm - A_\pm}.$$

Then, there exists a constant C such that

$$\mathcal{R}_{\mathcal{L}(L_q(\mathbb{R}_\pm^N), L_q(\mathbb{R}_\pm^N)^{1+N+N^2})}(\{(\tau\partial_\tau)^\ell G_\lambda K_i^\pm(\lambda) \mid \lambda \in \Lambda\}) \le C \quad (\ell = 0, 1, \ i = 1, 2, 3, 4),$$

where G_λ is an operator defined by $G_\lambda u = (\lambda u, \gamma u, \lambda^{1/2}\nabla u, \nabla^2 u)$.

By the Volevich trick;

$$\begin{aligned} a(x_N)b(0) &= -\int_0^\infty \{a'(x_N + y_N)b(y_N) + a(x_N + y_N)b'(y_N)\}dy_N \\ &= \int_{-\infty}^0 \{a'(x_N + y_N)b(y_N) + a(x_N + y_N)b'(y_N)\}dy_N, \end{aligned}$$

we can reduce the solution formula obtained in section 2.1 into the form which we can apply Lemma 2.2. We can check that the multipliers in the solution formula satisfy the condition of Lemma 2.2. Therefore we can prove the $\mathcal{R}$-boundedness of solution operator to problem (1.1)-(1.4), namely we can show the main theorem.

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