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Strategy for Five in a Row on small boards

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Abstract

Board game Five in a Row, or Gomoku, is an abstract strategy board game played by two players. It is a two-person, zero-sum, and deterministic game with perfect information. The author proposes a technique to analyze strategy board games like Five in a Row by computer experiments. In the technique, a board game is transformed into a game represented with objects in graph theory. The author is intending to solve the problem whether the player having the first move has winning strategy or not in another board game Pentago by applying the technique for Five in a Row to the analysis of Pentago. The author states only a research plan.

KEYWORDS. strategy board games, graph theory, computer experiments, the graph isomorphism problem.

1 Introduction

Board game Five in a Row, or Gomoku, is an abstract strategy board game played by two players. It is a two-person, zero-sum, and deterministic game with perfect information. We will propose a technique to analyze strategy board games like Five in a Row by computer experiments. In the technique, a board game is transformed into a game represented with objects in graph theory. Then, we will apply the technique for Five in a Row to the analysis of board game Pentago. Pentago is a new strategy board game sold by the Swedish company Mindtwister. Our goal is to solve the problem whether the player having the first move has winning strategy or not in Pentago. We will states only a research plan.

First, the rules of Five in a Row are described as follows[1]. Five in a Row is played on the 225 intersections of 15 horizontal and 15 vertical lines. Two players, Black and White, move in turn by placing a stone of their own color on an empty intersection, henceforth called a square. Black starts the game. The player who first makes a line of five consecutive stones of his color (horizontally, vertically or diagonally) wins the game. The stones once placed on the board during the game never move again nor can they be captured. If the board is completely filled, and no one has five-in-a-row, the game is drawn. Notice that if a line of six or more consecutive stones of the same color occurs after a move, then the color's player win in this paper.

It is known that the player having the first move, namely Black, has winning strategy in Five in a Row[1][2]. However, it is convincing that the smaller the board is, the more advantageous White is. We are expecting that Five in a Row games on small boards is drawn even if there is no restrictions on Black's moves. Notice that White cannot have winning strategy in Five in a Row, if there is no restrictions on Black's moves. This is the case for Pentago. If White has winning strategy, then Black also has the following winning strategy, that is, a contradiction occurs. In a game, Black always designates just one black stone placed on the board as the deposit. Black places a black stone at an arbitrary square as the first move, and designates the stone as the deposit. Then, Black plays as if he/she were White according to the winning strategy for White. If a black stone has been already placed at the square at which Black is to place a black stone, then the stone is the deposit. In that case, Black places a black stone at an arbitrary null square and designates the stone that has been just placed as the new deposit. Let P be a position in the game above. We obtain a position in a game according to the winning strategy for White by removing the deposit from P, then applying the following procedure: for each stone on the board in P, if it is a white stone, then replace it with a black stone, otherwise replace it with a white stone. Thus, Black must reach a winning position in the game above.

We transform a position in a board game like Five in a Row into a bipartite graph with some information, and a move into an operation to the graph. In a search of a transformed game, it is desirable that multiple occurrences of equivalent positions are put into a hash table as a unique entry in respect of performance. However, there is no trouble in the case where equivalent positions are put into two or more entries in a hash table except for the performance of the search. We therefore will devise a practical graph isomorphism algorithm and use it in the search for winning strategy in the games.

2 Transformation of games like Five in a Row

In this section, we describe a method of transforming a game like Five in a Row into a game represented with objects in graph theory.

We will define below a strategy game called a vertex removal game. The game are played by two players called Black and White. A position is a bipartite graph $G = (A \cup B, E)$ with edge labels $L : E \to \{0, 1, 2\}$, where all edges connect a vertex in A and one in B, and if the label of edge e is 1 then Black dominates e, else if that is 2 then White dominates e, else e is dominated by neither Black nor White. The label of each edge is 0 in the initial position. Black starts the game, and Black and White move in turn by choosing a vertex v in A and then dominating all the edges incident to v by labeling them. If an edge with label 1 and one with label 2 are incident to a vertex w in B, then w must be removed. The player who first dominates all the edges incident to a vertex in B wins the game.

We can make a vertex removal game equivalent to Five in a Row as follows. Set A consists of all the squares and set B consists of all the lines of five consecutive squares in Five in a Row, For any vertices $v \in A$ and $w \in B$, $vw \in E$ if and only if square v belongs to line of five squares w. Every vertex in B therefore has exactly five neighbors in A. For example, the bipartite graph corresponding to Five in a Row played on a board with 5×5 squares has $37 = 25 + 12 = |A| + |B|$ vertices and $60 = 5|B|$ edges. If a game of Five in a Row is drawn, then the number of all the moves in the game equals to the number of all the squares on the board. However, the number of all the moves until a draw is determined in a drawn vertex removal game may be less than the size of set A. Several vertices $v_1, v_2, \ldots, v_k$ in A may be of degree zero, since all the vertices in B adjacent to a vertex in $\{v_1, v_2, \ldots, v_k\}$ have been removed in the game. No edge labels change by choosing a vertex in A of degree zero as a move in a vertex removal game. Such a move therefore is most disadvantageous to both players. If a position with $A = \varnothing$ occurs, then the game must be drawn even if $B \neq \varnothing$.

3 A variant of a vertex removal game

We will define a variant of a vertex removal game. In the variants, White is more disadvantageous than in original games. Drawing strategy for White in a original game therefore follows from that in the corresponding variant to the original game. Furthermore, the variants are more simple than original games. We are intending to obtain drawing strategy in Pentago by analyzing the corresponding variant.

If the rule that White cannot win, that is, White's winning does not follow from any line of five white stones, is added, then labeling of the edges L is unnecessary in vertex removal games. Black starts the game. In Black's turn, Black chooses a vertex v in A and removes v. If Black has made a isolated vertex in B, then Black wins. In White's turn, White chooses a vertex v in A, removes all the vertices in B that adjacent to v, and removes v. If White has made a null graph, then the game is drawn, else if White has made isolated vertices in A, then those are removed. The variant is clearly corresponds to an original game with the rule that White cannot win.

The rules of Pentago are like Five in a Row. The board of Pentago consists of 6×6 squares, but it is partitioned into four 3×3 small boards. Although White has the first move in Pentago, we follow the same player's colors as in Five in a Row from now on, that is, Black has also the first move in Pentago. Each player must turn a small board 90 degrees clockwise or anti-clockwise after he/she placed his/her stone on the board. A player wins if five stones of the player's color contiguously line up vertically, horizontally, or diagonally, before or after a small board is turned. If the board is completely filled after a small board is turned, and no players won, then the game is drawn.

Pentago cannot correspond to any vertex removal game, since a small board must be turned in each move. However, if White has winning strategy in Pentago that includes the following procedure:

> White chooses the small board that was chosen in the just previous move and turns it so as to cancel just previous Black's turn,

then we may obtain such winning strategy by analyzing the variant of vertex removal game without an edge labeling.

The bipartite graph $G_{\text{Pentago}} = (A \cup B, E)$ in the variant of a vertex removal graph that corresponds to Pentago is very large in comparison with the bipartite graph $G_{\text{Five in a Row}} = (A' \cup B', E')$ in the vertex removal game that corresponds to Five in a Row with a 6×6 squares board. The number of vertices and the number of edges of G_{Pentago} are 192 and 780, respectively, whereas those numbers of $G_{\text{Five in a Row}}$ are 68 and 160, respectively. More precisely, concerning the vertices, $|B| = 156$, $|B'| = 32$, and $|A| = |A'| = 36$ hold. The degree of every vertex in A and A' is shown in Fig. 1.

In our plan to obtain White's drawing strategy, first we are to design a proper evaluation function for Pentago. We are expecting that we can find a simple one with which we

15	22	18	18	22	15
22	24	24	24	24	22
18	24	28	28	24	18
18	24	28	28	24	18
22	24	24	24	24	22
15	22	18	18	22	15

G_{Pentago}

3	4	3	3	4	3
4	6	5	5	6	4
3	5	7	7	5	3
3	5	7	7	5	3
4	6	5	5	6	4
3	4	3	3	4	3

$G_{\text{Five in a Row}}$

Figure 1: The degree of a vertex corresponding to a square on a board of Pentago or Five in a Row

can choose a best move in almost all the turns. Then, we are to develop a practical graph isomorphism algorithm to avoid multiple occurrences of equivalent positions in multiple entries in a hash table. It is not determined whether the graph isomorphism problem has a polynomial-time algorithm or not. However, it is sufficiently possible to develop a practically fast algorithm that solves the graph isomorphism problem with some errors. A proposal of a practically fast graph isomorphism algorithm will be published in IRCET in June 2014.

4 Concluding Remarks

We has described an idea to analyze strategy board games like Five in a Row by computer experiments. In the analysis, the game is transformed into a game called a vertex removal game, which is represented with objects in graph theory. Then, we has proposed a method to obtain a drawing method for the player to put the second move called White in Pentago. In the method, Pentago is transformed into a variant of vertex removal game, in which White is more disadvantageous but graphs representing positions are simpler. We has also mentioned the effectiveness of a practically fast graph isomorphic algorithm in computer experiments to analyze Pentago.

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