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THE FULL TRANSFORMATION SEMIGROUP OF FINITE RANK AND AMALGAMATION BASES FOR FINITE SEMIGROUPS*

KUNITAKA SHOJI (島根大学・総合理工学部 庄司 邦孝)
DEPARTMENT OF MATHEMATICS
SHIMANE UNIVERSITY

In this paper, we prove that the full transformation semigroup of finite rank is an amalgamation bases for finite semigroups.

1 Semigroup amalgamation bases

Definition Let $\mathcal{A}$ be the class of finite semigroups.
An amalgam $[S, T; U]$ of $\mathcal{A}$ is called to be *weakly embedable* in $\mathcal{A}$ if there exist a semigroup K belonging to $\mathcal{A}$ and monomorphisms $\xi_1 : S \to K$, $\xi_1 : T \to K$ such that the restrictions to U of ξ_1 and ξ_2 are equal to each other (that is, $\xi_1(S) \cap \xi_2(T) \supseteq \xi_1(U)$).

An amalgam $[S, T; U]$ of $\mathcal{A}$ is called to be *strongly embeddable* in $\mathcal{A}$ if $\xi_1(S) \cap \xi_2(T) = \xi_1(U)$.

A semigroup U in $\mathcal{A}$ is *amalgamation base* [resp. *weak* amalgamation base] if any amalgam with a core U in $\mathcal{A}$ is strongly embeddable [resp. weakly embeddable] in $\mathcal{A}$.

Result 1 [[3], Theorem 12] *Any finite semigroup U is an amalgamation base for finite semigroups if and only if U is a weak amalgamation base for finite semigroups.*

Result 2[[5], Theorem 1] *If a finite semigroup U is an amalgamation base for finite semigroups, then all $\mathcal{J}$-classes of U form a chain.*

Definition Let U be a semigroup with zero, 0, and $a, b \in S$.
The set $\{s \in U \mid sa = 0\}$ is called the *left annihilator* of a in S and is denoted by $Ann_l(a)$.
In this case, we say that U satisfies the condition Ann_l if $ann_l(a) = ann_l(b)$ implies $aU = bU$.
The *right annihilator* and the condition Ann_r are defined by left-right duality.

Result 3 [[9], Theorem 1.6] *Let U be a finite regular semigroup whose all the $\mathcal{J}$-classes form a chain. Suppose that there is a chain of principal ideals such that U_n is a maximal subgroup and each U_i/U_{i+1} is a completely 0-simple semigroups satisfying the conditions Ann_l and Ann_r $(1 \le i \le n-1)$. Then U is an amalgamation base for finite semigorups.*

*This is an absrtact and the paper will appear elsewhere.

Consider $\mathcal{T}(X)$, where the compostion is from right to left.

The following result is a characterization of semigroups which is amalgamation baseses for finite semigroups.

Result 4. [[6], Lemma 1 and Corollary]*Let U be a finite semigroup. Then the following are equivalent :*

(1) *U is an amalgamation base for finite semigroups ;*

(2) *For any two embeddings ϕ_1, ϕ_2 of U into the full transformation semigroup $\mathcal{T}(X)$, there exist a finite set Y and two embeddings $\delta_1, \delta_2 : \mathcal{T}(X) \to \mathcal{T}(Y)$ such that Y contains X as a subset and $\delta_1\phi_1$ and $\delta_2\phi_2$ coincide on U;*

(3) *For any finite semigroups S, T, any finite faithful left [right] S-set X and any finite faithful left [right] T-set Y, there exist a finite faithful left [right] S-set $X' \supseteq X$ and a finite faithful left [right] T-set $Y' \supseteq Y$ such that the U-sets X', Y' are U-isomorphic to each other.*

2 The main theorem

Consider $\mathcal{T}^{op}(X)$, where the compostion is from left to right.

Let $|X| = n$ and $I_k = \{f \in \mathcal{T}^{op}(X) \mid |(X)f| \leq k\}$. Then $\mathcal{T}^{op}(X) = I_n \supset I_{n-1} \supset \cdots \supset I_2 \supset I_1$ is a chain of ideals of $\mathcal{T}^{op}(X)$ and each factor semigroup I_i/I_{i-1} $(i \geq 2)$is a completely 0-simple semigroup satisfying the conditions Ann_l and Ann_r.

Let R_n denote the set I_1 of contant maps on X and S_n the set of bijective maps on X. Then in $\mathcal{T}^{op}(X). Then S_n \cup R_n$ is a subsemigroup of $\mathcal{T}^{op}(X)$.

Proposition. *The semigroup $S_n \cup R_n$ is an amalgamation bases for finite semigroups.*

By using Proposition and an analogue of Result 3, we obtain

The main theorem. *The full transformation semigroup of finite rank is an amalgamation bases for finite semigroups.*

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