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Uniform Semi-Unification *

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Abstract

The notion of uniform semi-unification is extended by unification. We revisited symbolic semi-unification whose solvability coincides with that of uniform semi-unification (Aoto & Iwami, 2013). In this paper, we give the some proofs omitted in our previous work [1] due to the space limitation.

1 Introduction

The notion of semi-unification is extended by unification. If a semi-unifier exists, there exists a most general semi-unifier [3, 5, 9]. However, semi-unification is undecidable in general [5]. Hence, many decidable classes of semi-unification have been studied. For example, uniform semi-unification is decidable [2, 4, 8, 9, 10, 11]. We revisited symbolic semi-unification whose solvability coincides with that of uniform semi-unification [1].

In this paper, we give the some proofs omitted in [1] due to the space limitation. First, we consider symbolic semi-unification in section 2. In section 3, we introduce a rule-based symbolic semi-unification and show its partial correctness. In section 4, we discuss termination of symbolic semi-unification procedure on some derivation strategy. We refer to [1] omitted definitions in this paper.

2 Symbolic Semi-Unification

In this section, we consider a notion of symbolic semi-unification. We defined ∇-term, ∇-equation and ∇-substitution in [1]. We refer to [1] omitted definitions.

Definition 2.1 ([1]) *For a set E of ∇-equations, a* semi-unifier *of E is a ∇-substitution σ such that $s\sigma^* = t\sigma^*$ for all $s \approx t \in E$; if E has a semi-unifier, E is said to be* semi-unifiable. *A* symbolic semi-unification problem *asks whether there exists a semi-unifier for a given set of ∇-equations.*

Lemma 2.2 ([1]) *Let σ be a ∇-substitution and s, t be ∇-terms. If $s\sigma^* = t\sigma^*$ then $\nabla(s)\sigma^* = \nabla(t)\sigma^*$.*

Definition 2.3 ([1]) *For a set E of ∇-equations, the* ∇-equality generated by E, *denoted by $\approx_E$, is the smallest equivalence relation such that (i) $s \approx_E t$ for any $s \approx t \in E$, (ii) $s \approx_E t$ implies $\nabla(s) \approx_E \nabla(t)$, and (iii) for any $f \in \mathcal{F}$, $f(s_1, \ldots, s_n) \approx_E f(t_1, \ldots, t_n)$ iff, for any $i = 1, \ldots, n$, $s_i \approx_E t_i$ holds.*

*This paper is revised version of [1].

Definition 2.4 ([1]) *A set E of ∇-equations is* inconsistent *if either (i) $x^i \approx_E s$ with $x^i \trianglelefteq s \notin \mathcal{V}^*$, or (ii) $f(s_1, \ldots, s_m) \approx_E g(t_1, \ldots, t_n)$ with $f \neq g$ for some $f, g \in \mathcal{F}$. Furthermore, E is* consistent *if it is not inconsistent.*

Since we gave only the proof sketch of the next lemma [1], we give the proof of it in detail.

Lemma 2.5 ([1]) *Let E be a set of ∇-equations. Suppose E is semi-unifiable and let σ be a semi-unifier of E. Then for any ∇-terms u, v, $u \approx_E v$ implies $u\sigma^* = v\sigma^*$.*

Proof. The proof proceeds by induction on the derivation of $u \approx_E v$. If $u \approx v \in E$ then the claim follows by assumption. If $u \approx_E v$ follows from $u' \approx_E v'$ where $u = \nabla(u')$ and $v = \nabla(v')$, then by induction hypothesis $u'\sigma^* = v'\sigma^*$, and hence by Lemma 2.2, $\nabla(u')\sigma^* = \nabla(v')\sigma^*$. Other cases follow easily. □

Theorem 2.6 ([1]) *For any terms $s, t \in \mathrm{T}(\mathcal{F}, \mathcal{V})$, the following are equivalent: (i) $\{\nabla(s) \approx t\}$ is semi-unifiable, (ii) $\{s \leqslant t\}$ is semi-unifiable, and (iii) $\{\nabla(s) \approx t\}$ is consistent.*

3 Partial Correctness of Symbolic Semi-Unification

In this section, we discuss a rule-based symbolic semi-unification procedure and prove its partial correctness. We refer to [1] omitted definitions.

Decompose $\dfrac{\{f(s_1, \ldots, s_n) \approx f(t_1, \ldots, t_n)\} \uplus E}{\{s_1 \approx t_1, \ldots, s_n \approx t_n\} \cup E} \; f \in \mathcal{F}$

Reduce $\dfrac{\{x^i \approx t, C[x^i] \approx u\} \uplus E}{\{x^i \approx t, C[t] \approx u\} \cup E} \; x^i > t$

Delete $\dfrac{\{x^i \approx x^i\} \uplus E}{E}$

Clash $\dfrac{\{f(s_1, \ldots, s_m) \approx g(t_1, \ldots, t_n)\} \uplus E}{\bot} \; f \neq g,\ f, g \in \mathcal{F}$

Check $\dfrac{\{x^i \approx t\} \uplus E}{\bot} \; t \notin \mathcal{V}^*,\ x^i \trianglelefteq t$

Figure 1: Inference rules for symbolic semi-unification ([1])

Definition 3.1 ([1]) *One step derivation using any of inference rules listed in Figure 1 is denoted by $\rightsquigarrow$. Here, the inference rules act on a finite set of ∇-equations and $\uplus$ denotes the disjoint union. For an input of a finite set E_0 of ∇-equations and the relation $>$, a symbolic semi-unification procedure non-deterministically constructs a derivation $E_0 \rightsquigarrow E_1 \rightsquigarrow \cdots$ (possibly following some fixed derivation strategy). The derivation may be finite or infinite, and it is* maximal *if it does not end with E_k for which a further application of an inference rule is possible. A symbolic semi-unification procedure (following a fixed derivation strategy)* terminates *if any derivation (following that derivation strategy) is finite.*

Remark 3.2 ([1]) *We adopt a variant of Reduce using substitution (instead of the replacement):*

$$\text{Reduce}'' \quad \frac{\{x^i \approx t\} \uplus E}{\{x^i \approx t\} \cup \{x^i := t\}(E)} \; x^i > t$$

Rule-based semi-unification calculi in [4, 8] use the replacement, and those in [6, 7, 11] use the substitution. We note that any substitution can be simulated by repeated applications of replacement.

Since we omitted the proof of the next lemma [1], we give the proof of it here.

Lemma 3.3 ([1]) *Suppose $E \overset{*}{\rightsquigarrow} E'$ with $E' \neq \bot$. Then $\approx_E = \approx_{E'}$.*

Proof. We show that $E \rightsquigarrow E'$ with $E' \neq \bot$ implies $\approx_E = \approx_{E'}$. Then the claim follows by induction on the length of $E \overset{*}{\rightsquigarrow} E'$. We distinguish the cases by the inference rule applied to $E \rightsquigarrow E'$. By our assumption that $E' \neq \bot$, inference rules Clash and Check are not used. Suppose that Delete is used. Let $E' \uplus \{x^i \approx x^i\} = E$. Then, since $s \approx_{E'} s$ for any s, the claim follows from the assumption immediately. Suppose Decompose is used. Let $E = F \uplus \{f(s_1, \ldots, s_n) \approx f(t_1, \ldots, t_n)\}$ and $E' = F \cup \{s_1 \approx t_1, \ldots, s_n \approx t_n\}$. ($\approx_E \supseteq \approx_{E'}$) Since $f(s_1, \ldots, s_n) \approx f(t_1, \ldots, t_n) \in E$, $f(s_1, \ldots, s_n) \approx_E f(t_1, \ldots, t_n)$. Hence by the definition of $\approx_E$ ((iii) of Definition 2.3), $s_i \approx_E t_i$ for all $i = 1, \ldots, n$. ($\approx_E \subseteq \approx_{E'}$) Since $s_i \approx t_i \in E'$, $s_i \approx_{E'} t_i$ for all $i = 1, \ldots, n$. Hence by the definition of $\approx_{E'}$ ((iii) of Definition 2.3), $f(s_1, \ldots, s_n) \approx_{E'} f(t_1, \ldots, t_n)$. Suppose Reduce is used. Let $E = F \uplus \{x^i \approx t, C[x^i] \approx u\}$ and $E' = F \cup \{x^i \approx t, C[t] \approx u\}$. ($\approx_E \subseteq \approx_{E'}$) Since $x^i \approx t \in E'$ and $C[t] \approx u \in E'$, $x^i \approx_{E'} t$ and $C[t] \approx_{E'} u$. Then by the definition of $\approx_{E'}$ ((ii) and (iii) of Definition 2.3) $C[x^i] \approx_{E'} C[t]$ and hence by transitivity of $\approx_{E'}$, $C[x^i] \approx_{E'} u$. ($\approx_E \supseteq \approx_{E'}$) Since $x^i \approx t \in E$ and $C[x^i] \approx u \in E$, $x^i \approx_E t$ and $C[x^i] \approx_E u$. Then by the definition of $\approx_E$ ((ii) and (iii) of Definition 2.3) $C[x^i] \approx_E C[t]$ and hence by symmetricity and transitivity of $\approx_E$, $C[t] \approx_E u$. □

Since we omitted the proof of the next corollary [1], we give the proof of it here.

Corollary 3.4 *If $E \overset{*}{\rightsquigarrow} E' \neq \bot$, then E is semi-unifiable iff E' is semi-unifiable.*

Proof. ($\Rightarrow$) Suppose E is semi-unifiable and $E \overset{*}{\rightsquigarrow} E' \neq \bot$. Let σ be a ∇-substitution such that $s\sigma^* = t\sigma^*$ for all $s \approx t \in E$. For any $s \approx t \in E'$, $s \approx_{E'} t$ by the definition of $\approx_{E'}$, and hence by Lemma 3.3, $s \approx_E t$. Hence, by Lemma 2.5, $s\sigma^* = t\sigma^*$ for any $s \approx t \in E'$. Thus E' is semi-unifiable. ($\Leftarrow$) Suppose E' is semi-unifiable and $E \overset{*}{\rightsquigarrow} E' \neq \bot$. Let σ be a ∇-substitution such that $s\sigma^* = t\sigma^*$ for all $s \approx t \in E'$. For any $s \approx t \in E$, $s \approx_E t$ by the definition of $\approx_E$ and hence by Lemma 3.3, $s \approx_{E'} t$. Hence, by Lemma 2.5, $s\sigma^* = t\sigma^*$ for any $s \approx t \in E$. Thus E is semi-unifiable. □

Since we omitted the proof of the next theorem [1], we give the proof of it here.

Theorem 3.5 ([1]) *Let E be a finite set of ∇-equations. (1) If $E \overset{*}{\rightsquigarrow} \bot$ then E is not semi-unifiable. (2) If $E \overset{*}{\rightsquigarrow} E' \neq \bot$ and no inference rules are applicable to E', then E is semi-unifiable.*

Proof. (1) By our assumption, $E \overset{*}{\rightsquigarrow} E' \rightsquigarrow \bot$ for some E'. Then either $f(s_1, \ldots, s_m) \approx g(t_1, \ldots, t_n) \in E'$ with $f \neq g$ or $x^i \approx t \in E'$ with $t \notin \mathcal{V}^*$ and $x^i \trianglelefteq t$. In the former case, $f(s_1, \ldots, s_m) \approx_E g(t_1, \ldots, t_n)$ and in the latter case, $x^i \approx_E f(\ldots, C[x^i], \ldots)$, by Lemma 3.3.

Suppose E is semi-unifiable. Then, by Lemma 2.5, we have $f(s_1,\ldots,s_m)\sigma^* = g(t_1,\ldots,t_n)\sigma^*$ or $x^i\sigma^* = f(\ldots,C[x^i],\ldots)\sigma^*$, for a semi-unifier σ of E. But this is impossible. (2) By our assumption that no inference rules are applicable to E', we have the following observations on E': (a) One side of the equation is of the form x^i. (Otherwise Decompose rule should be applicable.) (b) If $x^i \approx t \in E'$ with $x^i \succ t$ then x^i does not occur in t or in other equations in E'; this is because by (a) and the assumption that Check and Reduce can not be applied. Hence $\sigma = \{s := t \mid s \approx t \in E', s \succ t\}$ is a ∇-substitution, and for any $s \approx t \in E'$, $s\sigma^* = (s\sigma)\sigma^* = t\sigma^*$. Thus E' is semi-unifiable. Hence by Corollary 3.4, E is semi-unifiable. □

4 Termination of Symbolic Semi-Unification Procedure

In this section, we consider termination of symbolic semi-unification procedure on our derivation strategy [1]. We refer to [1] omitted definitions.

Theorem 4.1 **([1])** *Every derivation starting from a consistent finite set of ∇-equations is finite.*

Definition 4.2 ([1]) *A derivation strategy is said to be* refutationally complete *if any maximal derivation starting from an inconsistent set of ∇-equations and following that strategy is finite and ends with $\perp$.*

Lemma 4.3 **([1])** *A derivation strategy subject to using Reduce$''$ in place of Reduce and applying Check whenever possible is refutationally complete.*

Since we omitted the proof of the next theorem [1], we give the proof of it in detail.

Theorem 4.4 **([1])** *The symbolic semi-unification procedure terminates if it follows a refutationally complete derivation strategy; either the input E is semi-unifiable and any maximal derivation ends with a set of ∇-equations or E is not semi-unifiable and any maximal derivation ends with $\perp$.*

Proof. If E is an inconsistent set of ∇-equations, then by the refutational completeness of the derivation strategy then any derivation ends with $\perp$. Otherwise, E is a consistent set of ∇-equations, and hence by Theorem 4.1, it stops. If the derivation ends with $\perp$, by Theorem 3.5, E is not semi-unifiable. Otherwise the derivation ends with a set of ∇-equations and hence by Theorem 3.5, E is semi-unifiable. □

Since we gave only the proof sketch of the next corollary [1], we give the proof of it in detail.

Corollary 4.5 **([1])** *Let E be a finite set of ∇-equations. Then E is consistent iff E is semi-unifiable.*

Proof. ($\Rightarrow$) Suppose E is not semi-unifiable. Take a refutationally complete strategy for the derivation. Then by Theorem 4.4, the derivation ends with $\perp$. Then $E \overset{*}{\leadsto} E' \leadsto \perp$ for some E', and thus either $f(s_1,\ldots,s_m) \approx g(t_1,\ldots,t_n) \in E'$ with $f \neq g$ or $x^i \approx f(\ldots,C[x^i],\ldots) \in E'$. Hence either $f(s_1,\ldots,s_m) \approx_E g(t_1,\ldots,t_n)$ or $x^i \approx_E f(\ldots,C[x^i],\ldots)$ by Lemma 3.3. Thus E is an inconsistent set of ∇-equations. ($\Leftarrow$) Let σ be a semi-unifier of E and suppose E is inconsistent. Then $f(s_1,\ldots,s_m) \approx_E g(t_1,\ldots,t_n)$ with $f \neq g$ or $x^i \approx_E f(\ldots,C[x^i],\ldots)$ by Definition 2.4. Then $f(s_1,\ldots,s_m)\sigma^* = g(t_1,\ldots,t_n)\sigma^*$ or $x^i\sigma^* = f(\ldots,C[x^i],\ldots)\sigma^*$ by Lemma 2.5. But this is a contradiction. □

5 Conclusion

We revisited rule-based calculi for uniform semi-unification [1], on which efficient uniform semi-unification procedures [4, 8] are based. In this paper, we have given the some proofs omitted in [1] due to the space limitation.

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