数理解析研究所講究録
第1917巻 2014年 55-62

Global stability and influence of feedback controls of delayed Lotka-Volterra systems with patch structure

室谷義昭 (早稲田大学・理工学術院)
Yoshiaki Muroya (Department of Mathematics, Waseda University)

1 Introduction

Motivated by our attention to recent works of Chen [1], Li *et al.* [5] and Faria and Muroya [3]) for Lotka-Voltera systems with feedback controls and Takeuchi *et al.* [8] and Faria [2] for Lotka-Voltera systems with patch structure, we investigate the global dynamics for the following n-species Lotka-Volterra system with infinite delays, feedback controls and patch structure.

$$\begin{cases} x_i'(t) = x_i(t)\Big(b_i - \mu_i x_i(t) - \sum_{j=1}^{n} a_{ij}\int_0^{+\infty} K_{ij}(s)x_j(t-s)ds - c_i u_i(t)\Big) \\ \qquad +\sigma_i u_i(t) + \sum_{j=1}^{n}\Big(\alpha_{ij}\int_0^{+\infty} K_{ij}(s)x_j(t-s)ds - \alpha_{ji}x_i(t)\Big), \\ u_i'(t) = -e_i u_i(t) + d_i x_i(t), \quad i=1,2,\ldots,n, \end{cases} \tag{1.1}$$

with initial conditions of system (1.1):

$$\begin{cases} x_i(\theta) = \varphi_i(\theta),\ u_i(\theta) = \psi_i(\theta),\ \theta \in (-\infty, 0], \\ \varphi_i(0) > 0,\ \psi_i(0) > 0,\ i = 1,2,\ldots,n, \end{cases} \tag{1.2}$$

where $\mu_i,\ e_i > 0,\ c_i\ d_i,\ \sigma_i \geq 0,\ \alpha_{ij} \geq 0$ and $b_i,\ a_{ij} \in \mathbf{R}$, and $\varphi_i,\ \psi_i,\ i,j = 1,2,\ldots,n$ are non-negative and bounded continuous functions on $(-\infty, 0]$.

Hear, $x_i(t)$ $(i = 1,2,\ldots,n)$ denotes the number of species x in the patch i, $\gamma_{ij} \geq 0$ denotes the per capita death rate for the species during dispersion from patch j to i, b_i is the intrinsic rate for the species in patch i, μ_i represents the regulation and α_{ij} is the dispersal coefficient of the species from patch j to patch i, $u_i(t)$ denotes the feedback control variable and the kernels $K_{ij} : [0,+\infty) \to [0,+\infty)$ are L^1 functions, normalized so that $\int_0^{+\infty} K_{ij}(s)\,ds = 1$, for $i,j = 1,2,\ldots,n$. For the species to disperse from patch j to i in the model, for simplicity, we neglect the per capita death rate for the species during dispersion from patch j to i (cf. Takeuchi *et al.* [8]).

The unique solution of (1.1) with initial conditions (1.2) is expressed by $(\mathbf{x}(t), \mathbf{u}(t)) = (\mathbf{x}(t;\varphi), \mathbf{u}(t;\psi))$ with $\mathbf{x}(t) = (x_1(t), x_2(t), \ldots, x_n(t))$ and $\mathbf{u}(t) = (u_1(t), u_2(t), \ldots, u_n(t))$.

Moreover, we suppose that for all i, the linear operators defined by $L_{ii}(\varphi_i) = \int_0^{+\infty} K_{ii}(s)\varphi_i(-s)ds$, for $\varphi_i : (-\infty, 0] \to \mathbf{R}$ bounded, are non-atomic at zero, which amounts to have $K_{ii}(0) = K_{ii}(0^+)$, and

$$\text{an } n \times n \text{ matrix } [\alpha_{ij}] \text{ is irreducible.} \tag{1.3}$$

Put

$$\delta_{ij} = \begin{cases} 1, & \text{if } i = j, \\ 0, & \text{if } i \neq j, \end{cases} \qquad \tilde{\alpha}_{ii} = \sum_{j=1}^{n} (1 - \delta_{ji})\alpha_{ji}, \ i = 1, 2, \ldots, n, \tag{1.4}$$

and

$$M(0) = \begin{bmatrix} b_1 + \frac{\sigma_1 d_1}{e_1} - \tilde{\alpha}_{11} & \alpha_{12} & \cdots & \alpha_{1n} \\ \alpha_{21} & b_2 + \frac{\sigma_2 d_2}{e_2} - \tilde{\alpha}_{22} & \cdots & \alpha_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{n1} & \alpha_{n2} & \cdots & b_n + \frac{\sigma_n d_n}{e_n} - \tilde{\alpha}_{nn} \end{bmatrix}. \tag{1.5}$$

Let the stability modulus of an n $\times$ n matrix M, denoted by $s(M)$, be defined by $s(M) := \max\{Re\lambda : \lambda \ is\ an\ eigenvalue\ of\ M\}$. If M has nonnegative off-diagonal elements and is irreducible, then $s(M)$ is a simple eigenvalue of M with a (component-wise) positive eigenvector. A positive equilibrium $E^* = (\mathbf{x}^*, \mathbf{u}^*)$ of (1.1) with $\mathbf{x}^* = (x_1^*, x_2^*, \ldots, x_n^*)$ and $\mathbf{u}^* = (u_1^*, u_2^*, \ldots, u_n^*)$, satisfies the following equations:

$$\begin{cases} x_i^*\left((b_i - \tilde{\alpha}_{ii}) - \mu_i x_i^* - \sum_{j=1}^{n} a_{ij}x_j^* - c_i u_i^*\right) + \sigma_i u_i^* + \sum_{j=1}^{n}(1 - \delta_{ij})\alpha_{ij}x_j^* = 0, \\ -e_i u_i^* + d_i x_i^* = 0, \quad i = 1, 2, \ldots, n. \end{cases} \tag{1.6}$$

Since $u_i^* = \dfrac{d_i}{e_i}x_i^*$, $i = 1, 2, \ldots, n$, the positive equilibrium of (1.1) is the solution $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ of the system $\mathbf{F}(\mathbf{x}) = \mathbf{0}$ in $\mathbf{R}^n$, where

$$\begin{cases} \mathbf{F}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_n(\mathbf{x}))^T, \quad \mathbf{x} = (x_1, x_2, \ldots, x_n)^T, \\ f_i(x_1, x_2, \ldots, x_n) \equiv -\left[x_i\left\{\left(b_i + \dfrac{\sigma_i d_i}{e_i} - \tilde{\alpha}_{ii}\right) - \left(\mu_i + a_{ii} + \dfrac{c_i d_i}{e_i}\right)x_i - \sum_{j=1}^{n}(1 - \delta_{ij})a_{ij}x_j\right\}\right. \\ \left. + \sum_{j=1}^{n}(1 - \delta_{ij})\alpha_{ij}x_j\right] = 0, \ i = 1, 2, \ldots, n, \\ \dfrac{\partial f_i(x_1, x_2, \ldots, x_n)}{\partial x_j} \\ = \begin{cases} -\left(b_i + \dfrac{\sigma_i d_i}{e_i} - \tilde{\alpha}_{ii}\right) + 2\left(\mu_i + a_{ii} + \dfrac{c_i d_i}{e_i}\right)x_i, \ \text{for } j = i, \ i = 1, 2, \ldots, n, \\ -(\alpha_{ij} - x_i a_{ij}), \ \text{for } j \neq i, \ i = 1, 2, \ldots, n, \end{cases} \end{cases} \tag{1.7}$$

where the Fréchet derivative of $\mathbf{F}(\mathbf{x})$ is $\mathbf{F}'(\mathbf{x}) = \left[\dfrac{\partial f_i(x_1, x_2, \ldots, x_n)}{\partial x_j}\right]$. Hereafter, we use the ordering of vectors and matrices in $\mathbf{R}^n$ as the usual component-wise one in $\mathbf{R}^n$.

Consider a solution $(\bar{\mathbf{x}}(t), \bar{\mathbf{u}}(t)) = (\bar{x}(t;\varphi), \bar{u}(t;\psi))$ of the auxiliary cooperative system with $\bar{\mathbf{x}}(t) = (\bar{x}_1(t), \bar{x}_2(t), \ldots, \bar{x}_n(t))$ and $\bar{\mathbf{u}}(t) = (\bar{u}_1(t), \bar{u}_2(t), \ldots, \bar{u}_n(t))$, given by

$$\begin{cases} \bar{x}_i'(t) = \bar{x}_i(t)\left((b_i - \tilde{\alpha}_{ii} - \alpha_{ii}) - \mu_i \bar{x}_i(t) + \sum_{j=1}^{n} |a_{ij}^-| \int_0^{+\infty} K_{ij}(s)\bar{x}_j(t-s)ds\right) \\ \qquad + \sigma_i \bar{u}_i(t) + \sum_{j=1}^{n} \alpha_{ij} \int_0^{+\infty} K_{ij}(s)\bar{x}_j(t-s)ds, \\ \bar{u}_i'(t) = -e_i \bar{u}_i(t) + d_i \bar{x}_i(t), \quad i = 1, 2, \ldots, n. \end{cases} \tag{1.8}$$

with the same initial conditions

$$\begin{cases} \bar{x}_i(\theta) = \varphi_i(\theta), \ \bar{u}_i(\theta) = \psi_i(\theta), \ \theta \in (-\infty, 0], \\ \varphi_i(0) > 0, \ \psi_i(0) > 0, \ i = 1, 2, \ldots, n, \end{cases} \tag{1.9}$$

where we use the notations $a_{ij}^+ \equiv \frac{|a_{ij}|+a_{ij}}{2} \geq 0$ and $|a_{ij}^-| = \frac{|a_{ij}|-a_{ij}}{2} \geq 0$, $i,j = 1, 2, \ldots, n$.

For $n \times n$ matrices $\tilde{A}^0 = [\tilde{a}_{ij}^0]$, $A^0 = [a_{ij}^0]$ and $A = [a_{ij}]$ and a positive vector $\mathbf{x} = (x_1, x_2, \ldots, x_n)$, put $n \times n$ matrices $\hat{\tilde{M}}^0(\mathbf{x}) = [\delta_{ij} x_i \mu_i - (x_i |\tilde{a}_{ij}^0| + x_j |\tilde{a}_{ji}^0|)/2]$, $\hat{M}^0(\mathbf{x}) = [\delta_{ij} x_i \mu_i - (x_i |a_{ij}^0| + x_j |a_{ji}^0|)/2]$ and $\hat{M}(\mathbf{x}) = [\delta_{ij} x_i \mu_i - (x_i |a_{ij}| + x_j |a_{ji})/2]$, respectively.

In this paper, we obtain the following result.

Theorem 1.1 *Assume that $s(M(0)) \leq 0$. If there exists an $n \times n$ matrix $\tilde{A}^0 = [\tilde{a}_{ij}^0]$ such that*

$$\tilde{a}_{ij}^0 \leq a_{ij}, \ i,j = 1, 2, \ldots, n, \tag{1.10}$$

and for the positive left eigenvector $\tilde{\omega} = (\tilde{\omega}_1, \tilde{\omega}_2, \ldots, \tilde{\omega}_n)$ of $M(0)$, there exist positive constants $(\tilde{\theta}_{i1}, \tilde{\theta}_{i2}, \ldots, \tilde{\theta}_{in})$ with $\tilde{\theta}_{ii} = 1$, $i = 1, 2, \ldots, n$ such that

$$\tilde{\omega}_i(\mu_i - |\tilde{a}_{ii}^0|) \geq \sum_{j=1}^{n} (1 - \delta_{ij}) \frac{1}{2}\left(\tilde{\theta}_{ij} \tilde{\omega}_i |\tilde{a}_{ij}^0| + \frac{1}{\tilde{\theta}_{ji}} \tilde{\omega}_j |\tilde{a}_{ji}^0|\right), \ i = 1, 2, \ldots, n, \tag{1.11}$$

then the trivial solution $E^0 = (\mathbf{0}, \mathbf{0})$ is globally asymptotically stable.
In particular, if $a_{ij} \geq 0$, $i,j = 1, 2, \ldots, n$, then for $s(M(0)) \leq 0$, the trivial solution $E^0 = (\mathbf{0}, \mathbf{0})$ is globally asymptotically stable.

Note that if $a_{ij} \geq 0$, $i,j = 1, 2, \ldots, n$, then for $s(M(0)) \leq 0$, the trivial solution $E^0 = (\mathbf{0}, \mathbf{0})$ is globally asymptotically stable (see Lemma 2). If an $n \times n$ matrix $\hat{\tilde{M}}^0(\tilde{\omega}) = [\delta_{ij} \tilde{\omega}_i \mu_i - (\tilde{\omega}_i |\tilde{a}_{ij}^0| + \tilde{\omega}_j |\tilde{a}_{ij}^0|)/2]$ is diagonally dominant, then for $(\tilde{\theta}_{i1}, \tilde{\theta}_{i2}, \ldots, \tilde{\theta}_{in}) = (1, 1, \ldots, 1)$,

$i = 1, 2, \ldots, n$, (1.11) holds.

Theorem 1.2 *Assume that $s(M(0)) > 0$ and suppose that*

$$\omega_i \left(\mu_i \omega_i - \sum_{j=1}^{n} |a_{ij}^-| \omega_j \right) > 0, \ i = 1, 2, \ldots, n. \tag{1.12}$$

Then, there exists a positive equilibrium $\bar{E}^* = (\bar{\mathbf{x}}^*, \bar{\mathbf{u}}^*)$ of the auxiliary cooperative system (1.8) with $\bar{\mathbf{x}}^* = (\bar{x}_1^*, \bar{x}_2^*, \ldots, \bar{x}_n^*)$ and $\bar{\mathbf{u}}^* = (\bar{u}_1^*, \bar{u}_2^*, \ldots, \bar{u}_n^*)$ which is globally asymptotically stable and satisfy

$$\limsup_{t \to +\infty} x_i(t) \le \bar{x}_i^*, \ \textit{and} \ \limsup_{t \to +\infty} u_i(t) \le \bar{u}_i^*, \ i = 1, 2, \ldots, n, \tag{1.13}$$

and

$$\mathbf{F}(\bar{\mathbf{x}}^*) = \left[\bar{x}_i^* \left\{ \left(a_{ii}^+ + \frac{c_i d_i}{e_i} \right) \bar{x}_i^* + \sum_{j=1}^{n} (1 - \delta_{ij}) a_{ij}^+ \bar{x}_j^* \right\} \right] \ge \mathbf{0}. \tag{1.14}$$

(i) If

$$\begin{cases} \alpha_{ij} > 0, \ \textit{for any } i, j = 1, 2, \ldots, n \ \textit{such that } a_{ij}^+ > 0, \\ \sigma_i > 0, \ \textit{for any } i = 1, 2, \ldots, n \ \textit{such that } c_i > 0, \ \textit{and} \\ (\mu_i + \dfrac{c_i d_i}{e_i}) \omega_i + \displaystyle\sum_{j=1}^{n} a_{ij} \omega_j > 0, \ \textit{for any } i = 1, 2, \ldots, n, \end{cases} \tag{1.15}$$

then the system (1.1) *is permanent and*

$$\begin{aligned} &\min_{1 \le i \le n} \liminf_{t \to +\infty} (x_i(t)/\omega_i) \\ &\ge \underline{\hat{x}} \equiv \min \Bigg\{ \left(\min_{a_{ij}^+ > 0, \ i,j \in \{1,2,\ldots,n\}} \frac{\alpha_{ij}}{\omega_i a_{ij}^+} \right), \left(\min_{c_i > 0, \ i \in \{1,2,\ldots,n\}} \frac{\sigma_i}{\omega_i c_i} \right), \\ &\qquad \left(\min_{1 \le i \le n} \frac{(b_i + \frac{\sigma_i d_i}{e_i} - \tilde{\alpha}_{ii}) \omega_i + \sum_{j=1}^{n} (1 - \delta_{ij}) \alpha_{ij} \omega_j}{\omega_i \left((\mu_i + \frac{c_i d_i}{e_i}) \omega_i + \sum_{j=1}^{n} a_{ij} \omega_j \right)} \right) \Bigg\}, \end{aligned} \tag{1.16}$$

where $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$ is a positive eigenvector corresponding to the spectral radius $\rho(M(0)) = s(M(0)) > 0$ which satisfies

$$\left(b_i + \frac{\sigma_i d_i}{e_i} - \tilde{\alpha}_{ii} \right) \omega_i + \sum_{j=1}^{n} (1 - \delta_{ij}) \alpha_{ij} \omega_j > 0, \ i = 1, 2, \ldots, n. \tag{1.17}$$

(ii) In addition to (i), if

$$\alpha_{ij} - \bar{x}_i^* a_{ij}^+ \ge 0, \ i, j = 1, 2, \ldots, n, \tag{1.18}$$

then there exists a positive equilibrium $E^* = (\mathbf{x}^*, \mathbf{u}^*)$ *of* (1.1) *such that* (1.6) *holds.*
(iii) Moreover, if there exists an $n \times n$ *matrix* $A^0 = [a_{ij}^0]$ *such that*

$$a_{ij}^0 \le a_{ij},\ \alpha_{ij} - x_i^*(a_{ij} - a_{ij}^0) \ge 0,\ i,j = 1,2,\dots,n,\ \text{and } [\alpha_{ij} - x_i^*(a_{ij} - a_{ij}^0)] \text{ is irreducible,} \tag{1.19}$$

and for the positive vector $\mathbf{v} = (v_1, v_2, \dots, v_n)$ *defined by*

$$\sum_{j=1}^{n} v_j(1-\delta_{ji})\{\alpha_{ji} - x_j^*(a_{ji} - a_{ji}^0)\}x_i^* = v_i \sum_{j=1}^{n}(1-\delta_{ij})\{\alpha_{ij} - x_i^*(a_{ij} - a_{ij}^0)\}x_j^*,\ i = 1,2,\dots,n, \tag{1.20}$$

there exist positive constants $(\theta_{i1}, \theta_{i2}, \dots, \theta_{in})$ *with* $\theta_{ii} = 1,\ i = 1,2,\dots,n$ *such that*

$$v_i(\mu_i - |a_{ii}^0|) \ge \sum_{j=1}^{n}(1-\delta_{ij})\frac{1}{2}\left(\theta_i v_i |a_{ij}^0| + \frac{1}{\theta_j} v_j |a_{ji}^0|\right),\ i = 1,2,\dots,n, \tag{1.21}$$

then the positive equilibrium E^* *of* (1.1) *is globally asymptotically stable.*

Note that if an $n \times n$ matrix $\hat{M}^0(\mathbf{v}) = [\delta_{ij} v_i \mu_i - (v_i|a_{ij}^0| + v_j|a_{ji}^0|)/2]$ is diagonally dominant, then for $(\theta_{i1}, \theta_{i2}, \dots, \theta_{in}) = (1,1,\dots,1),\ i = 1,2,\dots,n$, (1.21) holds.

Theorem 1 implies that concerning the global stability of the positive equilibrium of (1.1), there is no influence of the feedback controls.

If we choose the $n \times n$ matrix $A^0 = [a_{ij}^0]$ in (iii) of Theorem 1, then we obtain the following corollaries.

(a) First, we choose $a_{ij}^0 = a_{ij}^-,\ i,j = 1,2,\dots,n$.

Corollary 1.1 *Assume that* $s(M(0)) > 0$ *and the conditions of (i)-(ii) of Theorem 1 hold. If an* $n \times n$ *matrix* $[\alpha_{ij} - x_i^* a_{ij}^+]$ *is irreducible and for a positive vector* $\mathbf{v} = (v_1, v_2, \dots, v_n)$ *such that*

$$\sum_{j=1}^{n} v_j(\alpha_{ji} - x_j^* a_{ji}^+) = v_i \sum_{j=1}^{n}(\alpha_{ij} - x_i^* a_{ij}^+), \quad i = 1,2,\dots,n, \tag{1.22}$$

there exist positive constants $(\theta_{i1}, \theta_{i2}, \dots, \theta_{in})$ *with* $\theta_{ii} = 1,\ i = 1,2,\dots,n$ *such that*

$$v_i(\mu_i - |a_{ii}^-|) \ge \sum_{j=1}^{n}(1-\delta_{ij})\frac{1}{2}\left(\theta_{ij} v_i |a_{ij}^-| + \frac{1}{\theta_{ji}} v_j |a_{ji}^-|\right),\ i = 1,2,\dots,n, \tag{1.23}$$

then the positive equilibrium E^* *of* (1.1) *is globally asymptotically stable.*
In particular, if for an $n \times n$ *matrix* $A^- = [a_{ij}^-]$, $\hat{M}^-(\mathbf{v}) = [\delta_{ij} v_i \mu_i - (v_i|a_{ij}^-| + v_j|a_{ji}^-|)/2]$ *is diagonally dominant, then for* $(\theta_{i1}, \theta_{i2}, \dots, \theta_{in}) = (1,1,\dots,1),\ i = 1,2,\dots,n$, (1.23) *holds.*

Corollary 1.2 *If $c_i = \sigma_i = 0,\ \mu_i - |a_{ii}^-| \geq 0,\ a_{ij} = 0,\ j \neq i,\ i = 1, 2, \ldots, n$, then for $s(M(0)) > 0$, there exists a unique positive equilibrium of* (1.1) *which is globally asymptotically stable.*

The models of Takeuchi *et al.* [8, Theorem 2.1] and Faria [2, Theorem 3.5] satisfies this condition.

(b) Second, we choose $a_{ij}^0 = 0,\ i, j = 1, 2, \ldots, n$.

Corollary 1.3 *Assume that $s(M(0)) > 0$ and* (1.15) *hold. If $a_{ij} \geq 0,\ \alpha_{ij} - \bar{x}_i^* a_{ij} \geq 0,\ i, j = 1, 2, \ldots, n$ and an $n \times n$ matrix $[\alpha_{ij} - \bar{x}_i^* a_{ij}]$ is irreducible, then there exists a positive equilibrium $E^* = (\mathbf{x}^*, \mathbf{u}^*)$ of* (1.1) *which is globally asymptotically stable.*

(c) Third, we choose $a_{ij}^0 = a_{ij},\ i, j = 1, 2, \ldots, n$.

Corollary 1.4 *Assume that* (1.12) *and* (1.15) *hold. Then, if $s(M(0)) \leq 0$ and for the positive left eigenvector $\tilde{\omega}$ of $M(0)$, an $n \times n$ matrix $\hat{M}(\tilde{\omega})$ is diagonally dominant, then the trivial solution $E^0 = (\mathbf{0}, \mathbf{0})$ of* (1.1) *is globally asymptotically stable, and if $s(M(0)) > 0$ and $n \times n$ matrices $\hat{M}(\omega)$ and $\hat{M}(\mathbf{v})$ for the positive eigenvector ω of $M(0)$ and the positive vector $\mathbf{v} = (v_1, v_2, \ldots, v_n)$ defined by* (1.20), *are diagonally dominant, then there exists a positive equilibrium $E^* = (\mathbf{x}^*, \mathbf{u}^*)$ of* (1.1) *which is globally asymptotically stable.*

Next, consider the case that $\mu_i = c_i = \sigma_i = 0$ and $a_{ij} \geq 0,\ i, j = 1, 2, \ldots, n$ of (1.1). Then, (1.1) becomes

$$\begin{cases} x_i'(t) = x_i(t)\left(b_i - \sum_{j=1}^n a_{ij} \int_0^{+\infty} K_{ij}(s) x_j(t-s) ds\right) \\ \qquad \sum_{j=1}^n \left(\alpha_{ij} \int_0^{+\infty} K_{ij}(s) x_j(t-s) ds - \alpha_{ji} x_i(t)\right). \end{cases} \tag{1.24}$$

Corollary 1.5 *For* (1.24), *assume that there exists a positive vector $\bar{\mathbf{x}}^0 = (\bar{x}_1^0, \bar{x}_2^0, \ldots, \bar{x}_n^0)$ such that*

$$M(0)(\bar{\mathbf{x}}^0)^T \leq \mathbf{0},\ \alpha_{ij} - \bar{x}_i^0 a_{ij} \geq 0,\ i, j = 1, 2, \ldots, n, \tag{1.25}$$

and

$$-b_i + \sum_{j=1}^n \bar{x}_j^0 a_{ji} > 0,\ i = 1, 2, \ldots, n. \tag{1.26}$$

If $s(M(0)) \leq 0$, then the trivial equilibrium $\tilde{\mathbf{E}}^0 = (0, 0, \ldots, 0)$ of (1.24) *is globally asymptotically stable, and if $s(M(0)) > 0$, then there exists a positive equilibrium $\tilde{\mathbf{E}}^* = \mathbf{x}^* = (x_1^*, x_2^*, \ldots, x_n^*)$ of* (1.24) *which is globally asymptotically stable. Moreover,* (1.24) *is equivalent to a multi-group SI epidemic model (see Kuniya and Muroya [4]).*

Note that $\tilde{R}_0 > 1$ is equivalent to $s(M(0)) > 0$ and $\tilde{R}_0 \leq 1$ is equivalent to $s(M(0)) \leq 0$.

2 Global stability for $s(M(0)) \leq 0$

We first give a basic result on the positiveness and the auxiliary cooperative system (1.8).

Lemma 2.1 *For system* (1.1) *with initial conditions* (1.2), *there exists a unique solution* $(\mathbf{x}(t), \mathbf{u}(t)) = (\mathbf{x}(t;\varphi), \mathbf{u}(t;\psi))$ *with* $\mathbf{x}(t) = (x_1(t), x_2(t), \ldots, x_n(t))$ *and* $\mathbf{u}(t) = (u_1(t), u_2(t), \ldots, u_n(t))$ *which satisfies* $x_i(t) > 0$, *for any* $i = 1, 2 \ldots, n$, *and* $t > 0$. *For the solution* $(\bar{\mathbf{x}}(t), \bar{\mathbf{u}}(t)) = (\bar{x}(t;\varphi), \bar{u}(t;\psi))$ *of the auxiliary cooperative system* (1.8) *with same initial conditions* (1.2), $\bar{\mathbf{x}}(t) = (\bar{x}_1(t), \bar{x}_2(t), \ldots, \bar{x}_n(t))$ *and* $\bar{\mathbf{u}}(t) = (\bar{u}_1(t), \bar{u}_2(t), \ldots, \bar{u}_n(t))$, *it holds* $x_i(t) \leq \bar{x}_i(t)$, $u_i(t) \leq \bar{u}_i(t)$, *for any* $i = 1, 2, \ldots, n$, $t \geq 0$.

Lemma 2.2 *For* $s(M(0)) \leq 0$, *if there exists an* $n \times n$ *matrix* $\tilde{A}^0 = [\tilde{a}^0_{ij}]$ *such that* (1.10) *and* (1.11) *hold, then the trivial solution* $E^0 = (\mathbf{0}, \mathbf{0})$ *is globally asymptotically stable. In particular, if* $a_{ij} \geq 0$, $i, j = 1, 2, \ldots, n$, *then for* $s(M(0)) \leq 0$, *the trivial solution* $E^0 = (\mathbf{0}, \mathbf{0})$ *is globally asymptotically stable.*

Proof of Theorem 1.2 By Lemma 2.2, we obtain Theorem 1.1.

3 Basic results on the global stability for $s(M(0)) > 0$

Lemma 3.1 *If* $s(M(0)) > 0$ *and* (1.12) *holds, then there exists a unique positive equilibrium* $\bar{E}^* = (\bar{\mathbf{x}}^*, \bar{\mathbf{u}}^*)$ *of* (1.8) *with* $\bar{\mathbf{x}}^* = (\bar{x}^*_1, \bar{x}^*_2, \ldots, \bar{x}^*_n)$ *and* $\bar{\mathbf{u}}^* = (\bar{u}^*_1, \bar{u}^*_2, \ldots, \bar{u}^*_n)$ *which is globally asymptotically stable and* (1.13) *and* (1.14) *hold.*

Lemma 3.2 *If* $s(M(0)) > 0$ *and* (1.15) *are satisfied, then the system* (1.1) *is permanent and* (1.16) *holds.*

Lemma 3.3 *Assume that* $s(M(0)) > 0$ *and* (1.12) *hold, then there exists a positive equilibrium* $\bar{E}^* = (\bar{\mathbf{x}}^*, \bar{\mathbf{u}}^*)$ *of* (1.8) *with* $\bar{\mathbf{x}}^* = (\bar{x}^*_1, \bar{x}^*_2, \ldots, \bar{x}^*_n)$ *and* $\bar{\mathbf{u}}^* = (\bar{u}^*_1, \bar{u}^*_2, \ldots, \bar{u}^*_n)$ *which satisfy* (1.13). *Moreover, if* (1.18) *hold. Then, the system* $\mathbf{F}(\mathbf{x}) = \mathbf{0}$ *has a positive solution* $\mathbf{x}^* = (x^*_1, x^*_2, \ldots, x^*_n)$ *in* $0 < x_i \leq \bar{x}^*_i$, $i = 1, 2, \ldots, n$ *which is equivalent to that* (1.1) *has "at least" one positive equilibrium* $E^* = (\mathbf{x}^*, \mathbf{u}^*)$.

Lemma 3.4 *If an* $n \times n$ *matrix* $[\alpha_{ij} - x^*_i(a_{ij} - a^0_{ij})]$ *is irreducible, then the system* (1.20) *has a positive solution* $(v_1, v_2, \ldots, v_n)$ *defined by* $(v_1, v_2, \cdots, v_n) = (C_{11}, C_{22}, \ldots, C_{nn})$, *where* $\tilde{\beta}_{ij} = \{\alpha_{ij} - x^*_i(a_{ij} - a^0_{ij})\}x^*_j$, $1 \leq i, j \leq n$, *and*

$$\tilde{\mathbf{B}} = \begin{bmatrix} \sum_{j\neq 1} \tilde{\beta}_{1j} & -\tilde{\beta}_{21} & \cdots & -\tilde{\beta}_{n1} \\ -\tilde{\beta}_{12} & \sum_{j\neq 2} \tilde{\beta}_{2j} & \cdots & -\tilde{\beta}_{n2} \\ \cdots & & & \\ -\tilde{\beta}_{1n} & -\tilde{\beta}_{2n} & \cdots & \sum_{j\neq n} \tilde{\beta}_{nj} \end{bmatrix},$$

and C_{ii} *denotes the cofactor of the* i*-th diagonal entry of* $\tilde{\mathbf{B}}$, $1 \leq i \leq n$.

4 Global stability of the positive equilibrium for $s(M(0)) > 0$

Proof of Theorem 1.2 Assume that $s(M(0)) > 0$ and suppose that (1.12) holds. Then, by Lemmas 2.1 and 3.1, there exists a positive equilibrium $\bar{E}^* = (\bar{\mathbf{x}}^*, \bar{\mathbf{u}}^*)$ of the auxiliary cooperative system (1.8) with $\bar{\mathbf{x}}^* = (\bar{x}_1^*, \bar{x}_2^*, \ldots, \bar{x}_n^*)$ and $\bar{\mathbf{u}}^* = (\bar{u}_1^*, \bar{u}_2^*, \ldots, \bar{u}_n^*)$ which is globally asymptotically stable and satisfy (1.13) and (1.14).

(i) Suppose that (1.15) holds. Then, by Lemma 3.2, system (1.1) is permanent and (1.16) holds.

(ii) Suppose that in addition to (i) and (ii), (1.18) holds. Then, by Lemma 3.3 there exists a positive equilibrium $E^* = (\mathbf{x}^*, \mathbf{u}^*)$ of (1.1) with $\mathbf{x}^* = (x_1^*, x_2^*, \ldots, x_n^*)$ and $\mathbf{u}^* = (u_1^*, u_2^*, \ldots, u_n^*)$ such that (1.6) holds.

(iii) Moreover assume that there exists an $n \times n$ matrix $A^0 = [a_{ij}^0]$ such that (1.19) holds and for the positive vector $\mathbf{v} = (v_1, v_2, \ldots, v_n)$ defined by (1.20), there exist positive constants $(\theta_{i1}, \theta_{i2}, \ldots, \theta_{in})$ with $\theta_{ii} = 1,\ i = 1, 2, \ldots, n$ such that (1.21) holds. Then, we can prove Theorem 1.2 by applying Lemma 3.4. The detail proof will be shown in the fort coming paper Muroya [6] or Muroya [7].

参考文献

[1] F. Chen, The permanence and global attractivity of Lotka-Volterra competition system with feedback controls. *Nonlinear Anal. RWA* **7** (2006) 133–143.

[2] T. Faria, Asymptotic behabiour for a class of delayed cooperative models with patch structure, *Discrete and cont. Dynam. Sys. Series B* **18** (2013) 1567-1579.

[3] T. Faria and Y. Muroya, Global attractivity and extinction for Lotka-Volterra systems with infinite delay and feedback controls, to appear in *Proceedings of the Royal Society of Edinburgh: Section A.*

[4] T. Kuniya and Y. Muroya, Global stability of a multi-group SIS epidemic model for population migration, to appear in *Discrete Cont. Dynamic. Sys. Series B.*

[5] Z. Li, M. Han and F. Chen, Influence of feedback controls on an autonomous Lotoka-Volterra competitive system with infinite delays, *Nonlinear Analysis* 14 (2013) 402-413.

[6] Y. Muroya, Influence of feedback controls on the global stability of delayed Lotka-Volterra systems with patch structure, submitted in *Discrete Cont. Dynamic. Syst. Series-B.*

[7] Y. Muroya, "Global stability of a delayed nonlinear Lotka-Volterra system with feedback controls and patch structure" submitted in *Appl. Math. Comput.*

[8] Y. Takeuchi, W. Wang and Y.Saito, Global stability of population models with patch structure, *Nonlinear Analysis RWA* **7** (2006) 235-247.