Quay crane allocation problem with the internal truck capacity constraint in container terminals

A. Karam*1a, A. B. Eltawil†1b, T. Mizutani2c, K. Nakata2d, N. A. Harraz3f

1Department of Industrial Engineering and Systems Management, Egypt - Japan University of Science and Technology (E-JUST), Alexandria, Egypt
2Department of industrial engineering and management, Tokyo Institute of Technology, Japan
3Production engineering Department, Faculty of Engineering, Alexandria University

aahmed.mostafa@ejust.edu.eg, beltawil@ejust.edu.eg, cmizutani.t.ab@m.titech.ac.jp,
dnakata.k.ac@m.titech.ac.jp, fnharraz@gmail.com

Abstract

In container terminals, discharging and loading of vessels are critical planning decisions which highly depend on the interaction between the quay cranes and internal trucks, they also have a significant impact on the terminal performance and revenue. Decisions on the assignments of quay cranes and internal trucks to the berthed vessels are typically made sequentially. However, the applicability of the handling plan can be improved when these two decisions are made simultaneously. This paper introduces an approach for assigning quay cranes and internal trucks to the berthed vessels simultaneously with consideration of the internal truck limited availability. A two phase solution methodology is proposed. In the first phase, a mixed integer programming model is formulated which provides the number of quay cranes assigned to each vessel, as well as the number of internal trucks assigned to each quay crane at each time period. In the second phase, a heuristic is used to solve the specific quay crane assignment problem. Using a numerical example, the benefits of the proposed approach are demonstrated.

Keywords: Quay crane assignment, internal trucks, container terminal.

1 Introduction

Due to the rapid increase in containerized trade in the last few years, container terminals are becoming an important connection in the global supply chain. Containerized trade grew with an average annual rate of 6.5% from 1996 to 2013 as shown in figure 1 [1]. The growth rate of the containerized trade decreased in 2009 due to the global financial crisis.

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1 Corresponding author and PhD exchange student at department of industrial engineering and management, Tokyo Institute of Technology, Japan (from April, 2014 to December, 2014).

1 On leave from Alexandria University, Egypt.
The competitiveness of a container terminal depends mainly on the length of the service time for vessels as vessel operators want their vessels to be discharged and/or loaded as fast as possible in the terminal. However, fast service operations are constrained by the limited assets of the terminals. Consequently, optimization of handling operations at the container terminal received great attention in both the developed and developing countries [2].

Figure 2 shows the typical layout of the container terminal which consists of three main areas, namely the quay side, yard and gates [3]. Once the vessel is berthed along the quay, the containers of the vessels are loaded and/or discharged by assigned number of quay cranes (QCs). A set of internal trucks (ITs) is used to transport the containers between the quay side and the storage yard. At the yard, the yard cranes (YCs) stack the containers in the storage blocks.

![Figure 1: Global containerized trade between 1996 and 2013[1].](image1)

![Figure 2: Typical layout of the container terminal [3].](image2)

This paper focuses on the quay side operations which include four main problems, namely the Berth Allocation Problem (BAP), the Quay Crane Assignment Problem (QCAP), the Specific Quay Crane Assignment Problem (SQCAP) and the Quay Crane Scheduling Problem (QCSP) [4]. The BAP specifies berthing positions, berthing times and departure times for incoming vessels. The QCAP determines the number of the quay cranes to be assigned to each vessel such that its discharge/loading operations are completed without violating its promised departure time. As the quay cranes are mounted on rails, they can’t pass each other. Therefore, the Specific Quay Crane Assignment Problem (SQCAP) is
considered to determine the specific quay cranes assigned for each vessel respecting the non-crossing constraint of the quay cranes. The Quay Crane Scheduling Problem (QCSP) determines the schedules of the assigned quay cranes by providing the sequence of container unloading and loading operations that a quay crane is supposed to perform on the different hatches of the vessel.

Previous studies paid a lot of attention to expensive assets such as berths and quay cranes. Most studies [5]-[8] implicitly or explicitly assumed that other resources, which work with quay crane such as the internal trucks, are available to operate all quay cranes with a specific productivity. However, this is generally not the case in reality for all container terminals. For example, in some terminals, the excessive utilization of internal trucks may increase their frequency of breakdowns. Therefore, the full availability of internal trucks becomes scarce. The handling rate of a quay crane is highly dependent on how many internal trucks are allocated to it [9]. This in turn implies that the limited number of available internal trucks could be the bottleneck for fast handling operations of the vessel. Hence, in this paper a new approach for the integrated assignment of quay cranes and internal trucks in a container terminal is proposed. The proposed approach comprises two phases. In the first phase, an MIP model is used to determine the quay crane assignments to each vessel and the internal truck assignments to each quay crane simultaneously, taking into consideration the internal truck limited availability, the SQCAP is solved using a special heuristic procedure in the second phase.

The remainder of this paper is organized as follows: Section 2 provides a review of related recent literature. Section 3 describes the proposed modeling approach. A numerical example is shown in section 4. The conclusions and recommendations are in the last section.

2 Review of literature

Recently, many studies have been devoted to scheduling and allocation of resources such as berths and handling equipment in container terminals [3].

In this section, we review only studies related to the Quay Crane Assignment Problem (QCAP) and related Internal Trucks assignment (ITs). The QCAP aims at determining the number of quay cranes that must be assigned for each vessel in order to complete handling of containers on the vessel during its berthing stay. A good solution for the QCAP can have a strong impact on the handling time of vessels. Therefore, the QCAP is usually studied with the Berth Allocation Problem (BAP) by employing different integration approaches [2]. However, a Limited number of researchers [10]-[12] investigated the QCAP as a single problem. Legato et al. [12] proposed a two-phase approach for assignment and deployment of quay cranes. In the first phase, an Integer Programming (IP) model for the QCAP was proposed to provide the number of QCAs assigned for each vessel at each time period. This formulation aims at minimizing the total number of activated quay cranes, the handling time and the change in the number of assigned QCAs during handling operations. In the second phase, they proposed an IP model for deploying the assigned QCAs respecting the non-crossing constraint. They applied their approach to a berth schedule provided by Park et al. [5]. They were able to produce a better solution by activating 7 quay cranes against 9 quay cranes found by Park et al. [5]. Moreover, they improved the overall handling time of vessels as well as the utilization of QCAs compared to the results reported by Park et al. [5]. Daniela et
al. [10] proposed an integrated simulation-optimization frame work for the QCAP and Quay Crane Scheduling Problem (QCSP). They proposed an IP model for the QCAP with objective to minimize the berthing costs and QCs cost. Furthermore, they considered the cost of gang which varies according to the working shift. They defined a gang as the team of human and associated handling equipments that works with QCs. Then, the solution of QCAP is taken as input to the discrete event simulation model of the QCSP. However, this paper lacks numerical analysis to show the value of the findings and significance of the proposed approach.

Internal trucks (ITs) transfer containers between the quay side and the yard side and they are critical resources in handling operations of vessels. In a recent survey of F. Meisel [13], he stated that horizontal transport management in container terminal includes two decisions. The first decision is the allocation of ITs to QCs while the second decision is the scheduling of ITs. Studies related to the second decision can be found in [14]. Fewer researchers studied the allocation of ITs to QC. G. Murty et al. [9] developed a decision support system for the daily operations in container terminal. They used simulation to show how the productivity of a QC depends on its allocated number of ITs. In order to minimize the waiting time of QCs and maximize the utilization of ITs, they allocated from 4 to 5 ITs to each QC. Furthermore, they formulated an IP model that provides IT requirements in each half-hour interval of the day. However, this paper implicitly assumed that ITs are available to provide each assigned QC by 4 or 5 ITs. Moreover, the interrelation between QCs productivity and the available number of ITs is ignored due to solving the QCAP as well as determining IT requirements sequentially.

D. Chang et al. [7] proposed a MIP model for the integrated BAP and QCAP. The berthing position, berthing time and quay crane assignments are determined for each vessel. Moreover, the workload of each quay crane is determined. Practically, the workload of each quay crane is determined based on the number of ITs allocated to each QC. Therefore, they implicitly assumed that ITs are enough to transfer containers.

To sum up, the issue of interrelation between the quay crane assignment and the available number of ITs is not adequately considered in literature. This often leads to handling plans of poor adaptability to the real situation. Furthermore, assigning the same workload for all QCs results in lower utilization of QCs as well as interrupting the smooth handling operations. Finally, the setup time of QCs is not considered when estimating the time of vessel berthing stay.

This paper introduces a two phase approach for the integrated assignment of QCs to vessels and ITs to QCs. The first phase is a proposed mathematical model which optimally produces the assignments of QCs and the IT requirements for each vessel in each time period with the objective of minimizing the average berthing stay per vessel. The solution of the first phase is used as an input to the second phase. In the second phase, a heuristic is proposed to schedule the assignments of quay cranes to vessels as well as allocating internal trucks to each quay crane with objective of minimizing the number of crane shifts.

3 Problem description and modeling approach
3.1 The problem description
The BAP is solved to specify the berthing position, berthing time and promised departure time for each incoming vessel. A penalty cost is often incurred if the departure of a vessel occurs later than its previously committed departure time. As an example, Figure 3 shows a berth schedule of two vessels which can be represented in a space–time-diagram in which the quay is divided into equally sized berthing positions and the planning horizon is divided into periods of 1 hour. The height of each of the rectangles corresponds to the length of a vessel and the width corresponds to the maximum berthing stay. The lower-right vertex of a rectangle gives the vessel’s berthing position and berthing time.

![Figure 3: Berth schedule of the vessels](image)

The handling plan is determined based on the berth schedule by assigning the quay cranes and internal trucks to the incoming vessels such that the discharge/loading operations, for each vessel, are completed without violating its promised departure time. A typical objective of the handling plan is to minimize the time that the vessels stay at the container terminal. The handling rate of the quay crane depends on how many internal trucks are allocated to it because there is often no buffer space below the quay crane [9]. If the internal truck is not available to pick up or deliver the container from or to the quay crane, the operation of the quay crane will be interrupted. Therefore, a minimum number of internal trucks is required to transport containers in order to keep the cranes busy all the time. In practice, the number of internal trucks assigned to each quay crane should be within a specific range which is determined by the terminal planner such that the idle times of the cranes and the internal trucks are practically accepted. At the peak periods, two or more vessels are serviced simultaneously and all the quay cranes and internal trucks available at the container terminal are often activated. Therefore, if the total available number of internal trucks is not enough to provide each assigned quay crane with its minimum truck requirements, hence, the handling rate of some quay cranes, assigned at the peak periods, will decrease and so, additional time periods will be required to complete the discharge/loading operations of the vessels. In practice, the availability of internal trucks is taken into consideration when estimating the vessel handling time to guarantee more robustness to the handling plan[15]. Therefore, an efficient and reliable handling plan can be achieved by assigning the quay cranes and internal trucks simultaneously, taking into consideration the capacities of the quay cranes and internal trucks which are available at the container terminal while traditional methods assume that internal trucks are always available.
3.2 The modeling approach

The proposed approach consists of two phases as shown in figure 4. In the first phase, an MIP model is used to provide the number of QCs assigned to each vessel as well as the IT requirements for the assigned QCs in each period with the objective of minimizing the average berthing stay per vessel. In the second phase, a heuristic is used to solve the SQCAP with the objective of minimizing the total number of crane setups.

![Berth schedule](image)

**Phase 1:** Assignments of QCs to each vessel.
- Assignments of ITs to each QC.

**Phase 2:** Assignments of specific QCs to each vessel.

Figure 4: The proposed two phase solution approach

3.2.1 Phase 1: the proposed MIP model

The model formulation is based on the following assumptions:

1. Each vessel has been previously planned for a berthing time, a promised departure time and a berthing location.
2. All quay cranes are identical.
3. A Quay crane can’t be included in schedule until at least its required minimum number of internal trucks is available.
4. The handling operations of a vessel start only if its minimum number of quay cranes is available.

The following notations are used

Sets:
- $T$: Set of time periods.
- $V$: Set of vessels.
- $K$: Set of quay cranes.

Parameters:
- $n$: Number of vessels arriving within the planning period $T$.
- $S_v$: The berthing time of vessel $v$.
- $C_v$: The promised departing time of vessel $v$.
- $N_v$: The number of the loading and discharging containers of vessel $v$ in (TEU).
- $l_v$: The length of vessel $v$.
- $r_v$: The berthing position of vessel $v$. 
$p$ : Average productivity of each quay crane per one internal truck (TEU/internal truck) per hour.

$q_{\text{max}}^v$ : Maximum number of quay cranes that can be assigned simultaneously to vessel $v$.

$q_{\text{min}}^v$ : Minimum number of quay cranes that can be assigned simultaneously to vessel $v$.

$Q$ : Available total number of quay cranes.

$I$ : Available total number of internal trucks.

$i_{\text{max}}$ : Maximum number of internal trucks that can be allocated to each quay crane.

$i_{\text{min}}$ : Minimum number of internal trucks that can be allocated to each quay crane.

$c$ : Average time consumed in quay crane shift from a vessel to another.

$M$ : A sufficiently large constant.

Decision variables

$q_{vt}^k$ : $I$, if quay crane $k$ is assigned to vessel $v$ in time period $t$, $0$ otherwise.

$i_{vt}^k$ : Integer number to represent the number of internal trucks allocated to quay crane $k$ when it works on vessel $v$ in time period $t$.

$f_{vt}$ : Integer number to represent the difference between the number of quay cranes assigned at time period $(t+1)$ and those assigned at time period $t$.

$x_{vt}$ : $I$, if vessel $v$ is handled at time period $t$, $0$ otherwise.

The model is formulated as follows:

Minimize $\frac{1}{n} \left( \sum_{v \in V} \max_{t = s_v, C_v} (t \cdot x_{vt} - S_v + 1) + c \sum_{v \in V} \sum_{t = S_v}^{C_v-1} |f_{vt}| \right)$

Subject to

$x_{vt} \geq x_{vt+1}$ \hspace{1cm} \forall v \in V, \forall t = S_v, \ldots, C_v \hspace{1cm} (1)$

$\sum_{t = s_v}^{C_v} p \cdot i_{vt}^k \geq N_v$ \hspace{1cm} \forall v \in V \hspace{1cm} (2)$

$\sum_{k \in K} \sum_{i = 1}^{s_v-1} q_{vt}^k = 0$ \hspace{1cm} \forall v \in V \hspace{1cm} (3)$

$\sum_{k \in K} \sum_{t = C_v+1}^{T} q_{vt}^k = 0$ \hspace{1cm} \forall v \in V \hspace{1cm} (4)$

$\sum_{k \in K} \sum_{v \in V} i_{vt}^k \leq I$ \hspace{1cm} \forall t \in T \hspace{1cm} (5)$

$i_{vt}^k \leq i_{\text{max}} \cdot q_{vt}^k$ \hspace{1cm} \forall v \in V, t \in T, k \in K \hspace{1cm} (6)$

$i_{vt}^k \geq i_{\text{min}} \cdot q_{vt}^k$ \hspace{1cm} \forall v \in V, t \in T, k \in K \hspace{1cm} (7)$

$\sum_{v \in V} q_{vt}^k \leq 1$ \hspace{1cm} \forall k \in K, t \in T \hspace{1cm} (8)$

$\sum_{k \in K} q_{vt}^k \leq q_{\text{max}}^v \cdot x_{vt}$ \hspace{1cm} \forall v \in V, \forall t = s_v, \ldots, C_v \hspace{1cm} (9)$

$\sum_{k \in K} q_{vt}^k \geq q_{\text{min}}^v \cdot x_{vt}$ \hspace{1cm} \forall v \in V, \forall t = s_v, \ldots, C_v \hspace{1cm} (10)$

$\sum_{k \in K} \sum_{v \in V} q_{vt}^k \leq Q$ \hspace{1cm} \forall t \in T \hspace{1cm} (11)$
\begin{align*}
\sum_{k \in K} q_{vt}^k - \sum_{k \in K} q_{vt}^k & \leq f_{vt} \quad \forall v \in V, \forall t = S_v, ..., C_v - 1 \\
x_{vt}, q_{vt}^k & \in \{0, 1\} \\
f_{vt}, q_{vt}^k & \geq 0 \text{ and Integer}
\end{align*}

The objective function aims at minimizing the average berthing stay per vessel. The berthing stay of the vessel can be defined as the length of time between the berthing time at which the vessel is berthed and the time at which the discharge/loading operations of the vessel are completed. The average berthing stay per vessel can be defined as the sum of the berthing stays for all the vessels divided by the number of vessels. The berthing stay includes three times which are the waiting time until the handling operation is started, the handling time and the time incurred in setting up the quay cranes. The first term of the objective function represents both handling time and waiting time while the second term represents the setup time incurred when there is change in the number of QCs during handling operation. The proposed model can be represented as MIP model, as shown in the appendix, to be solved using any MIP solver. Constraint (2) guarantees that the vessel operation can’t be interrupted once it has started. Constraint (3) ensures that the containers on each vessel must be completely handled within its berthing time. Constraints (4) and (5) ensure that no internal trucks or quay cranes are used in handling containers on any vessel outside its allocated time window. Constraint (6) ensures that the number of internal trucks used in any time period does not exceed the available number. Constraints (7) and (8) specify that the number of internal trucks assigned to each quay crane is between the minimum and maximum numbers allowed. Constraint (9) guarantees that any quay crane can be assigned to only one vessel at each time period. Constraint (10) and (11) ensure that the number of quay cranes assigned to each vessel is between the maximum and minimum specified limits. Constraint (12) ensures that at each time period, the number of quay cranes assigned to all vessels do not exceed the total available number of quay cranes. Constraint (13) defines the variable \( f_{vt} \). Constraints (14) and (15) are binary and non-negativity constraints respectively.

3.2.2 Phase II: the proposed heuristic for the specific quay crane assignments

The output of the first phase is used as input data to the proposed heuristic which comprises two steps. In step 1, data input and preparation are made, while in step 2, the assignments of specific quay cranes, for each vessel, are developed. This means that the assigned QCs are deployed along the vessel with objective of minimizing the number of crane setups from one vessel to another.
The following mathematical notations will be used in the proposed heuristic.

\[ QC \] : Set of available quay cranes arranged along the quay, \( QC = \{ q_1, q_2, ..., q_k \} \).

\[ B \] : Set of berthing slots, \( B = \{ b_1, b_2, ..., b_n \} \).

\( y_v \) : Starting berth slot of vessel \( v \).

\( M \) : Middle berth slot, \( M = b_n/2 \).

\( QC_{vt} \) : Number of quay cranes assigned to vessel \( v \) in time period \( t \).

\( C_t \) : Number of quay cranes used in time period \( t \), \( C_t \leq k \), where \( k \) is the total number of available quay cranes.

\( Q_vt \) : Sequence of QCs assigned to vessel \( v \) in time period \( t \), e.g. QC4, QC5 and QC6.

\( BV_t \) : Set of berthed vessels at time period \( t \).

\( VC_t \) : An array which contains the set of berthed vessels \( BV_t \) at time period \( t \) in the first column and the corresponding \( y_v \) and \( QC_{vt} \), for each vessel, in the second and third columns respectively, e.g. \( VC_2 = \begin{array}{ccc} A & 12 & 2 \\ B & 2 & 3 \end{array} \). This means that vessel \( A \) is berthed at time period 2 and its corresponding \( y_v \) and \( QC_{vt} \) are 12, 3 respectively.

The detailed flow chart of the proposed heuristic is shown in figure 5.

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**Figure 5: The detailed flow chart of the heuristic procedure**

**4 Numerical example**

In this section, an example is used to show the benefits of the proposed model compared to the traditional method of quay crane assignments. Table 1 shows the data of two incoming vessels.
Table 1: Data of incoming vessels

<table>
<thead>
<tr>
<th>Vessels</th>
<th>Expected Arrival time</th>
<th>Berthing position</th>
<th>Promised departure time</th>
<th>Length</th>
<th>Max. no. of QCs</th>
<th>Min. no. of QCs</th>
<th>Number of containers</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2</td>
<td>6</td>
<td>7</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>330</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>2</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>180</td>
</tr>
</tbody>
</table>

Traditional quay crane assignment methods estimate the handling time of the vessel based on its work load, its assigned number of quay cranes and the handling rate of the quay crane, measured in containers/hour [6]. It is also assumed that a set of internal trucks is always available for each quay crane e.g. 5 [7]. Thus, each quay crane can operate at a constant handling rate e.g. 30 TEU/hour. For instance, the total available numbers of quay cranes and internal trucks are 5 and 25 respectively and the average time consumed in quay crane shift from a vessel to another is set to 15 minutes. Figure 6a shows the assignments of quay cranes and internal trucks to the berth schedule by the traditional quay crane assignment method. The solution of the QC assignments is then used as input to solve the SQCAP which solution is shown in figure 6b.

![Figure 6a](image1)

![Figure 6b](image2)

**Figure 6:** The solution of the example using the traditional quay crane assignments.

It can be noted from figure 6a that if there is any shortage in the truck requirements at time periods 3 and 4 which are called peak periods because the vessels are served simultaneously by activating all the available resources, the handling rate of some quay cranes utilized at these periods may be lower than 30 TEU/hr and so, additional time periods are needed to handle all the containers of the vessel. Therefore, considering the availability of the internal trucks when estimating the handling times of the vessels, would improve the applicability of the handling plan.

To solve this example by the proposed simultaneous method, $i_{\min}$, $i_{\min}$ and $p$ are set to 3, 5 and 6 respectively. Figure 7 shows the solution of the example for the cases when the full capacity of internal trucks is available (a) and with a shortage of 5 internal trucks such that the total available number of internal trucks is 20 (b). The middle part of figure 7 shows...
the QC assignments to each vessel which specifies the length of the handling time for each vessel, the number of quay cranes and the specific quay cranes to serve each vessel at each time period. Moreover, the top part shows the internal truck assignments to each quay crane with consideration of the internal truck capacity constraints. Compared to the traditional quay crane assignment method results in Figure 6, the handling plan can be made with consideration of the shortage in the truck requirements as shown in figure 7b from which we can see that the handling time of vessel A increased by 1 hour compared to the case with 25 trucks in figure 7a.

![Figure 7: Solution of the example with 25 trucks (a) and with 20 trucks (b) respectively by the proposed approach.](image)

This implies that neglecting the internal trucks available capacity may lead to inaccurate plans and shorter than possible handling time. This may lead to penalties for the container terminal due to delays of the vessels departures. The proposed method considering the trucks available capacity will increase the reliability of the handling plan.

5 Conclusions and future research

This paper addressed a new approach for simultaneous quay crane assignment and internal truck assignment to each QC in container terminals, taking into consideration the internal truck capacity constraint. The proposed approach comprises two phases; the first phase is an MIP model for determining the assignments of QCs as well as the IT requirements for each QC in each time period while the second phase is a proposed heuristic for solving the SQCAP.

Using a numerical example, it was demonstrated that the proposed approach is more suitable for real situations compared to the traditional method in which the QCAP and the
internal truck assignments are solved sequentially. Finally, unifying the two phases into a single model may be addressed in future research.

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Appendix

Reduction of the proposed model to MIP model

The following variables are introduced $z_{vt}$ and $d_{vt}$

$$\text{Minimize } \frac{1}{n} \left( \sum_{v \in V} z_{vt} + c \cdot \sum_{v \in V} \sum_{t=S_v}^{C_v-1} d_{vt} \right)$$

Subject to

Constraints (1)-(13) in addition to the following constraints

$$z_{vt} \geq t \cdot x_{vt} - S \quad \forall v \in V, t = S_v, ..., C_v \quad (14)$$

$$-d_{vt} \leq f_{vt} \leq d_{vt} \quad \forall v \in V, t = S_v, ..., C_v \quad (15)$$

$$z_{vt}, d_{vt} \geq 0 \text{ and Integer} \quad (16)$$

References


