数理解析研究所講究録
第 1952 巻 2015 年 63-68

Optimal management strategies for rural water resources from a viewpoint of stochastic control

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1. Introduction

Water management in general is to balance the spatio-temporally uneven distribution of water with demands. Water resources for agricultural purposes are mostly found in rural areas, where occurrence of relevant events is dominantly stochastic (Unami et al, 2010). There have been established traditional water management strategies throughout the world which can be understood as stochastic controls. Operation of micro-dams or river currents for irrigation purposes normally abides by "rule curves" prescribing water levels where restriction on intake is imposed (Senga, 1991). While, management of groundwater sometimes involves more complicated issues due to imperfect information to be fed-back (Rahman and Mahbub, 2012). This article discusses how these water management issues are approached in the framework of stochastic control. Then, few examples are presented for demonstration of applicability.

2. Formulation of stochastic control problems

We assume a single water body such as a micro-dam being used for agricultural purposes. The dynamics of its storage volume is represented as the water balance equation

$$\mathrm{d}X_t = Q(t,u)\mathrm{d}t + \sqrt{2D^x}\mathrm{d}B_t^x \tag{1}$$

where X_t is the storage volume at the time t, Q is the deterministic component of water balance, which is controlled by the intake discharge u, D^x is the diffusion coefficient for water balance, and B_t^x is the standard Brownian motion. An admissible set $U=[0,u_{\max}]$ constraining u is considered as well.

Occurrence of events such as drought is the major concern for most water managers. The mean reverting process Y_t, governed by the Langevin equation, is used to represent drought severity as

$$\mathrm{d}Y_t = -b(u)Y_t\mathrm{d}t + \sqrt{2D^y}\mathrm{d}B_t^y \tag{2}$$

where b is the drift coefficient to be manipulated as a result of implementing water intake u, D^y is the diffusion coefficient for drought severity, and the standard Brownian motion B_t^y is assumed to be independent of B_t^x. Occurrence time of an event is modelled as the first exit time from a domain.

Agricultural products are biomass having economic values, whose dynamics is typically considered in terms of the growth model

$$\mathrm{d}Z_t = rZ_t\mathrm{d}t + \sigma Z_t\mathrm{d}B_t^z \tag{3}$$

where Z_t is the value of agricultural products whose growth rate r and volatility coefficient σ may depend on t, u, X_t, Y_t, and Z_t itself. The standard Brownian motion B_t^z is assumed to be independent of B_t^x and B_t^y.

With the system consisting of the stochastic differential equations described above, the performance index $J^u(s,x,y,z)$ to be maximized is prescribed as an expectation

$$J^u(s,x,y,z) = \mathrm{E}\left[\int_s^{\hat{T}} f(s,u,x,y,z)\mathrm{d}t + g(\hat{T}, X_{\hat{T}}, Y_{\hat{T}}, Z_{\hat{T}})\right] \tag{4}$$

where f and g are prescribed benefit functions, and $\hat{T}$ is the first exit time from a predetermined x - y - z -domain Ω of X_t, Y_t, and, Z_t. Then, the maximum $\Phi = \Phi(s,x,y,z)$ of $J^u(s,x,y,z)$, as well as the optimal control u^* attaining it is governed by the Hamilton-Jacobi-Bellman (HJB) equation

$$L^{u^*}\Phi + f(s,u^*,x,y,z) = \sup_{u\in U}\left\{L^u\Phi + f(s,u,x,y,z)\right\} = 0 \tag{5}$$

with terminal and boundary conditions

$$\Phi(s,x,y,z) = g(s,x,y,z) \tag{6}$$

where L^u is the partial differential operator

$$L^u = \frac{\partial}{\partial s} + Q(s,u)\frac{\partial}{\partial x} + D^x\frac{\partial^2}{\partial x^2} - b(u)y\frac{\partial}{\partial y} + D^y\frac{\partial^2}{\partial y^2} + rz\frac{\partial}{\partial z} + \frac{\sigma^2 z^2}{2}\frac{\partial^2}{\partial z^2}. \tag{7}$$

This HJB equation system is analyzed with the help of numerical approaches, though few have been reported in the literature (Kumar and Muthuraman, 2004; Unami et al, 2013).

3. Demonstrative examples

3.1 Optimal operation of rainwater storage tanks

A typical problem is to obtain a rule curve for a rainwater storage tank (RWST) during an irrigation period $t \in [0,T]$, where dry spells with evaporation loss E prevail. Unami et al (2015) discusses the model

$$\mathrm{d}X_t = -(E+u)\mathrm{d}t \tag{8}$$

$$\mathrm{d}Y_t = -Y_t\mathrm{d}t + \sqrt{2}\mathrm{d}B_t^y \tag{9}$$

$$\mathrm{d}Z_t = 0 \tag{10}$$

with $\Omega = (0,V_{\max})\times(-K,K)\times\Box$, where $V_{\max}$ is the maximum storage volume of the RWST and K is a model parameter. The time domain is appropriately scaled. A substantial recharge event of the RWST is assumed to take place if $|Y_t| > K$. A target value for the intake discharge u is set as Q^*, and then $f = -\varepsilon|u - Q^*|$ with an weight ε and $g = 0$ except for $g(s,0,y) = -(T-s)Q^*$ are prescribed, considering water deficit instead of explicitly including the dynamics of the value of agricultural products. When the maximum storage volume $V_{\max}$ of the RWST is large enough, trivial rule curves for the cases of $\varepsilon = 0$ and $\varepsilon = \infty$ are known as $E(T-s)$ and the constant 0, respectively. Rule curves for finite positive values of ε are consistently obtained from numerical solutions to the HJB equation system. The results for model parameters $V_{\max}$ = 4,000 m^3, T = 120 days, E = 9.6 m^3/day, and Q^* = 23.8 m^3/day are shown in Fig. 1. From this figure it is concluded that $V_{\max}$ = 1,000 m^3 is sufficient, and actual rainwater harvesting system including such a RWST with an intake structure has been developed in an arid environment, as shown in Photo 1 (Sharifi et al, 2015). The rule curves will be revised using an improved model, before commencing operation of the RWST in September 2015.

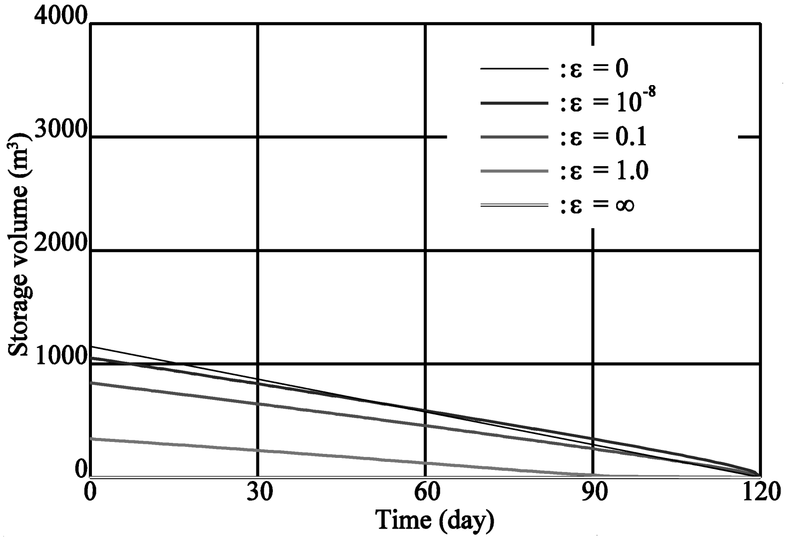

Fig. 1 The rule curves for different ε values (Unami et al. 2015)

Photo 1 Rainwater harvesting system developed in an arid environment

3.2 Groundwater lifting for irrigation

Another example explicitly includes growth of agricultural products but droughts are dealt with as depletion of the water body. Kuwano et al (2014) considers groundwater stored in a Bangladeshi aquifer as the water body, from which water is lifted for dry season irrigation using pumps. The model including parameters σ_x, α, and C is set as

$$\mathrm{d}X_t = -u\mathrm{d}t + \sigma_x\left(X_0 - X_t\right)\exp\left(\sqrt{u}\right)\mathrm{d}B_t^x \tag{11}$$

$$\mathrm{d}Y_t = 0 \tag{12}$$

$$\mathrm{d}Z_t = \alpha u\left(C - Z_t\right)Z_t\mathrm{d}t + \sigma Z_t\mathrm{d}B_t^z \tag{13}$$

with a constant σ. Different cases of $f \mp 0$, $f = -u$, and $f = -u/x$ are tested, while g is set identical to z. Numerical solutions to the HJB equation system in terms of Φ and u^* for the three cases at $t = 0$, 30, 60, and 90 for $T = 100$ are shown in Figs. 2, 3, 4, and 5, respectively. Different restriction of pumping, which varies with time, is well represented.

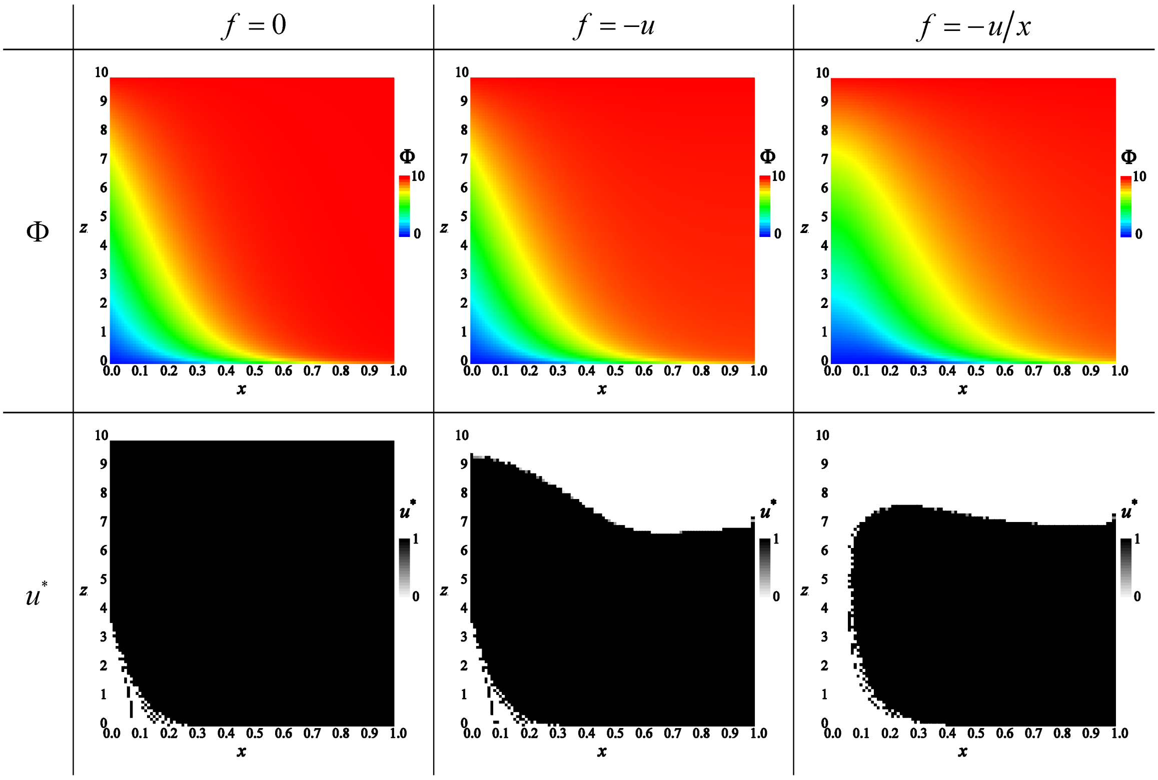

Fig. 2 Computed Φ and u^* for different benefit functions at $t = 0$

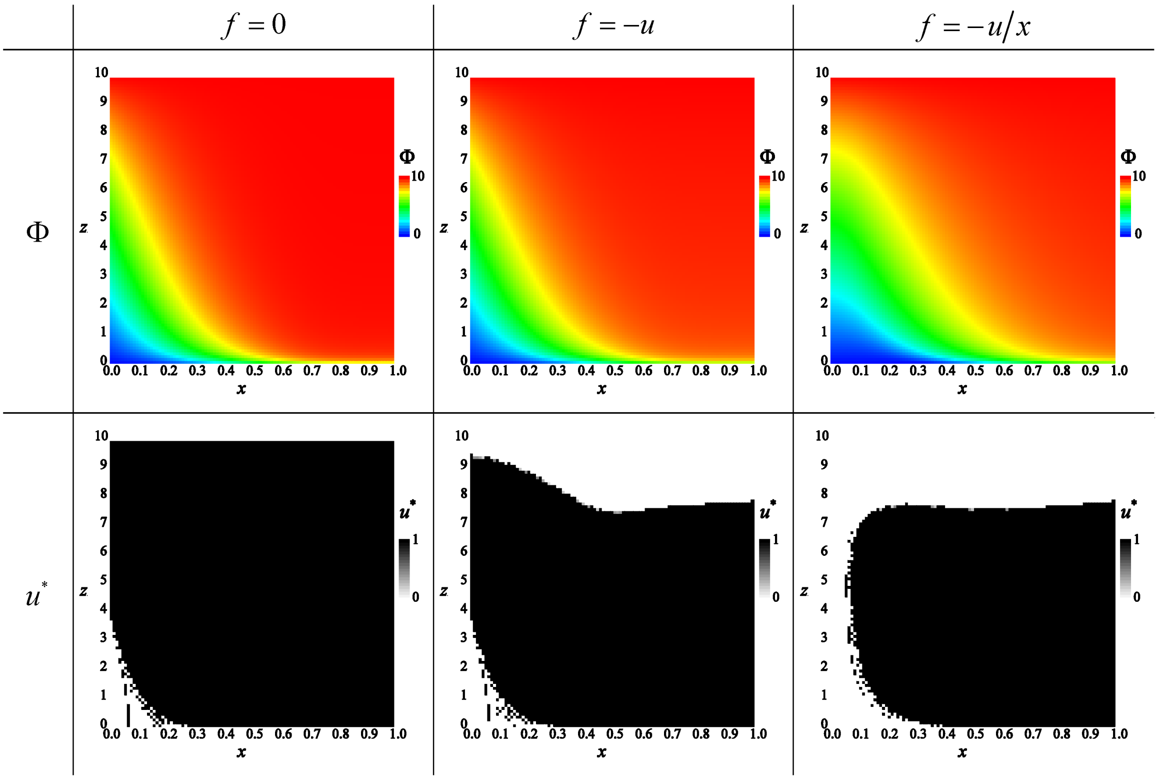

Fig. 3 Computed Φ and u^* for different benefit functions at $t = 30$

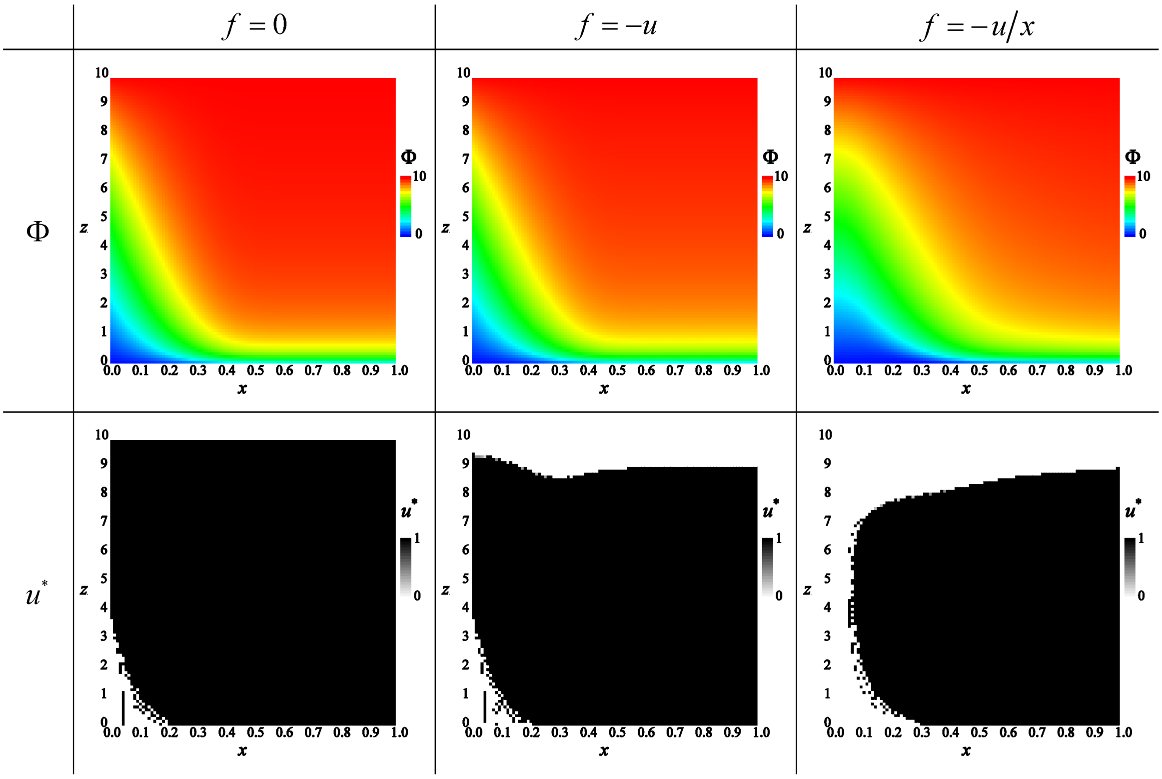

Fig. 4 Computed Φ and u^* for different benefit functions at $t = 60$

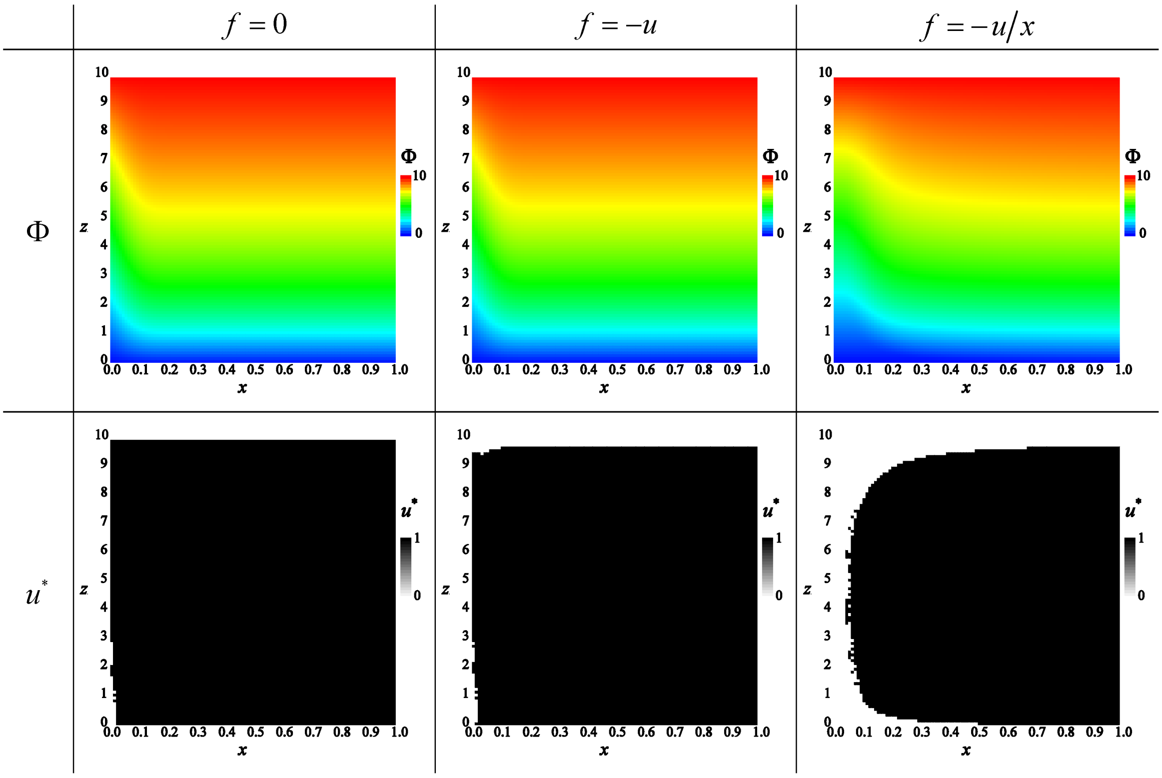

Fig. 5 Computed Φ and u^* for different benefit functions at $t = 90$

4. Conclusions

The basic principles of stochastic control have been applied to practical problems in agricultural water management. The demonstrated optimal management strategies are in good agreement with the authors' field studies. This approach is also being used for developing pilot projects under uncertainty of hydro-environment. Further researches are needed for efficient numerical schemes solving HJB equations of higher dimensions.

Acknowledgments

This research is funded by grant-in-aid for scientific research No. 26257415 from the Japan Society for the Promotion of Science (JSPS). The authors thank Agricultural Research Station at Ghor Mazrah, Mutah University, Jordan, and Godashimla Village, Jamalpur District, Dhaka Division, Bangladesh, for their assistance in field studies.

References

[1] Unami K, Abagale FK, Yangyuoru M, Alam AHMB, Kranjac-Berisavljevic G, 2010. A Stochastic differential equation model for assessing drought and flood risks. *Stochastic Environmental Research and Risk Assessment* 24 (5), 725-733.

[2] Senga Y, 1991. A reservoir operational rule for irrigation in Japan. *Irrigation and Drainage Systems* 5 (2), 129-140.

[3] Rahman M, Mahbub AQM, 2012. Groundwater depletion with expansion of irrigation in Barind Tract: a case study of Tanore Upazila. *Journal of Water Resource and Protection* 4(8), 567-575.

[4] Kumar S, Muthuraman K, 2004. A numerical method for solving singular stochastic control problems. *Operations Research* 52 (4), 563-582.

[5] Unami K, Yangyuoru M, Alam AHMB, Kranjac-Berisavljevic G, 2013. Stochastic control of a micro-dam irrigation scheme for dry season farming. *Stochastic Environmental Research and Risk Assessment* 27 (1), 77-89.

[6] Unami K, Mohawesh O, Sharifi E, Takeuchi J, Fujihara M, 2015. Stochastic modelling and control of rainwater harvesting systems for irrigation during dry spells. *Journal of Cleaner Production* 88, 185-195.

[7] Sharifi E, Unami K, Mohawesh O, Nakamichi T, Kinjo N, Fujihara M, 2015. Design and construction of a hydraulic structure for rainwater harvesting in arid environment, *The 36th IAHR World Congress*, (accepted).

[8] Kuwano R, Unami K, Fujihara M, 2014. Wise-use of shallow aquifer to control the growth of farm products. *Proceedings of Annual Symposium, Applied Hydraulics Division, JSIDRE*, 39-41.