数理解析研究所講究録
第 1959 巻 2015 年 142-152

Pattern dynamics of a reaction-diffusion-advection system with bistable growth *

Tohru Tsujikawa†

Faculty of Engineering, University of Miyazaki
Miyazaki 889-2192

1 Introduction

From a viewpoint of the pattern formation in Biology, reaction diffusion systems are treated. In this paper we consider the following chemotaxis-growth model (e. g. [2], [8], [20]).

$$\left\{\begin{array}{ll} u_t = \mathcal{D}\left\{u_x - \alpha u v_x\right\}_x + f(u), & \text{in } I \times \mathbb{R}_+, \\ v_t = d v_{xx} + u - v, & \text{in } I \times \mathbb{R}_+, \\ u_x(x,t) = 0, \ v_x(x,t) = 0, & \text{on } \partial I \times \mathbb{R}_+, \\ u(\,\cdot\,,0) = u_0 \geq 0, \quad v(\,\cdot\,,0) = v_0 \geq 0, & \text{in } I \end{array}\right. \tag{1}$$

where $\mathcal{D},\ d,\ \alpha$ are positive constants and $I = (0,1)$. In the absence of growth term $f(u)$, this system becomes Keller-Segel type and many people study about steady states, blow up etc.. On the other hand, for (1) it is known several spatio-temporal patterns due to the Turing and Hopf instability induced by the chemotaxis effect (e. g. [7], [12], [3]). In order to understand mechanism of these pattern formations, we show the existence of global solutions and an exponential attractor with finite dimension of (1) (e. g. [1], [9], [11], [17]) . Moreover, the existence and stability of stationary solutions of (1) is discussed for several growth terms [4, 5, 6, 7]. By using Degree theory, bifurcation method and etc., the existence of the stationary solution is locally shown with respect to parameters $\mathcal{D},\ d, \alpha$ (e. g. [17], [19], [5]). But it is difficult to show the global structure of stationary solutions with respect to parameters. In this paper, we treat the bistable growth term $f(u) = u(1-u)(u-a)$ $(0 < a < 1$) and study the global structure of stationary solutions of (1) as large $\mathcal{D}$. To do so, we assume that (u, v) is uniformly bounded as $\mathcal{D}$ tends to ∞. Formally, it holds that $\{u_x - \alpha u v_x\}_x = 0$. It follows from the boundary conditions of

*This is a joint work with Hirofumi Izuhara (University of Miyazaki) and Kousuke Kuto (The University of Electro-Communicatons)

†e-mail address: tujikawa@cc.miyazaki-u.ac.jp

$u,\ v$ that u is represented by $u = Ee^{\alpha v}$ with any positive constant E. By integrating the first equation of (1) on I, (1) deduces to the following system:

$$\begin{cases} \left(\int_0^1 Ee^{\alpha v}\,dx\right)_t = \int_0^1 f(Ee^{\alpha v})\,dx, & t > 0 \\ v_t = dv_{xx} + g(v,\ E), & x \in I,\ t > 0 \\ v_x(0,t) = v_x(1,t) = 0, & t > 0 \\ u \geq 0,\ v \geq 0, & x \in I,\ t > 0, \end{cases} \tag{2}$$

where $g(v, E) = Ee^{\alpha v} - v$. Here we call (2) a shadow system of (1). We remark that a stationary solution of (1) converges to the corresponding solution of (2) in following sense.

Remark 1.1. [18] *Let $\{\mathcal{D}_n\}$ and $(u_n,\ v_n)$ be a positive sequence with $\lim_{n\to\infty} \mathcal{D}_n = \infty$ and a stationary solution of (1) with $\mathcal{D} = \mathcal{D}_n$. Then there exists a subsequence $\{\mathcal{D}_{n'}\}$ of $\{\mathcal{D}_n\}$ and a stationary solution $(u_\infty,\ v_\infty)$ of (2) such that*

$$\lim_{n'\to\infty} (u_{n'},\ v_{n'}) = (u_\infty,\ v_\infty) \quad in\ C^1(\overline{\Omega}) \times C^1(\overline{\Omega}). \tag{3}$$

Therefore, it is important to treat (2) for understanding stationary solutions of (1) for large $\mathcal{D}$. But we have not any similar result with respect to the evolution problem of (1).

In this paper we consider the stationary problem of (2) as follows:

$$\begin{cases} dv_{xx} + g(v,E) = 0, & x \in I, \\ u_x(x) = v_x(x) = 0, & x \in \partial I \\ u \geq 0,\ v \geq 0, & x \in I \end{cases} \tag{4}$$

and

$$\int_\Omega f(Ee^{\alpha v})\,dx = 0. \tag{5}$$

The organization of this paper is as follows: In Section 2, we summarize known results of the existence of solutions of (4) and our main results for the stability of theses solution is given in Section 3. In Section 4, we demonstrate the global structure of stationary solutions and periodic solution of (2). Finally, introducing a new small parameter, we show the onset of the periodic solution is an infinite-dimensional relaxation oscillation which is governed by slow and fast dynamics.

2 Existence of the stationary solution of Shadow System

We summarize known results of the existence of solutions with respect to (4), (5). Hereafter we describe a solution of (4), (5) by $(v(x,\ d,\ E),\ d,\ E)$. All solution is represented by the scaling, repetition and reflection of monotone solutions. Then we only consider monotone increasing solutions.

First, we treat the boundary value problem (4) without the integral constraint (5). By a simple calculation, we have the following Lemmas.

Lemma 1. [6] *There exists a positive constant $\hat{E}$. Then,* (4) *has two positive constant solution $v_*(E)$, $v^*(E)$ for any $0 < E < \hat{E}$ which satisfy $v_*(E) < v^*(E)$, $v_*(\hat{E}) = v^*(\hat{E})$ and are monotone increasing and decreasing function of E, respectively.*

By using Lemma 1 and the bifurcation theory (e. g. [13], [14], [15]), we have

Lemma 2. *For any $0 < E < \hat{E}$, there exists a monotone decreasing function $d^*(E)$ which satisfies $\lim_{E\to 0} d^*(E) = \infty$, $\lim_{E\to\hat{E}} d^*(E) = 0$. Then there is a solution $v(x,\ d,\ E)$ of* (4) *for any $0 < E < \hat{E}$, $0 < d < d^*(E)$ and it holds*

$$\lim_{d\to d^*(E)} v(x,\ d,\ E) = v^*(E), \quad \lim_{d\to 0} v(x,\ d,\ E) = v^B(x,\ E) = \begin{cases} v_*(E) & 0 \le x < 1 \\ \overline{v}(E) & x = 1 \end{cases} \tag{6}$$

where $\overline{v}(E)$ is a constant satisfying $\displaystyle\int_{v_*(E)}^{\overline{v}(E)} g(v,\ E)\ dv = 0.$

Thanks to the result of Schaaf [13], we show

Lemma 3. *Let $\Lambda := \{(d,\ E) \mid 0 < E < \hat{E},\ 0 < d < d^*(E)\}$. There is not any nonconstant solution of* (4) *for $(d,\ E) \in \mathbb{R}_+^2 \setminus \Lambda$.*

Next, we obtain the existence of solutions of (4) which satisfies the integral constraint. Let $h(d,\ E)$ be a function of d and E defined by

$$h(d,\ E) := \int_0^1 f(Ee^{\alpha v(x,\ d,\ E)})\ dx \tag{7}$$

where $v(x,\ d,\ E)$ is a solution given in Lemma 2. Therefore, the integral constraint (5) is represented by

$$h(d, E) = 0. \tag{8}$$

Then it holds that

Lemma 4. *Let $\Lambda^- := \{(d,\ E) |\ 0 \le d \le d^*(E), 0 < E \le \hat{E}\}$ and $\Lambda^0 := \{(d,\ E) |\ 0 < d \le d^*(E), 0 < E \le \hat{E}\}$. Then, $h(d,\ E)$ satisfies $h(d,\ E) \in C(\Lambda^-) \cap C^1(\Lambda^0)$ and*

$$\lim_{d\to 0} h(d,\ E) = f(Ee^{\alpha v_*(E)}). \tag{9}$$

Setting

$$\Gamma := \{(v(x,\ d,\ E),\ d,\ E) \mid v(x,\ d,\ E) \text{ is a solution of (4), (5) for any } (d,\ E) \in \Lambda\}$$

we can show the existence of stationary solutions as follows:

Theorem 5. [6] *There are two positive constants E_a, $E_1 < \hat{E}$ depending on a, 1 and following three statements hold*

(i) *If $1/\alpha < a$, 1, E_a, E_1 satisfy $E_1 < E_a$ and there exists a function $d(E)$ defined on the interval $(E_1,\ E_a)$ and $(v^s(x,\ d(E),\ E),\ d(E),\ E) \in \Gamma$. Moreover,*

$$\lim_{E\to E_1} v^s(x,\ d(E),\ E) = v^*(E_1), \quad \lim_{E\to E_a} v^s(x,\ d(E),\ E) = v^*(E_a). \tag{10}$$

(ii) *If* $a < 1/\alpha < 1$, *there exists a function* $d(E)$ *defined on* $(E_1,\ E_a)$ *or* $(E_a,\ E_1)$ *and* $(v^s(x,\ d(E),\ E),\ d(E),\ E)\ \in \Gamma$, *which satisfies*

$$\lim_{E\to E_1} v^s(x,\ d(E),\ E) = v^*(E_1), \quad \lim_{E\to E_a} v^s(x,\ d(E),\ E) = v^B(x,\ E_a). \tag{11}$$

(iii) *If* $1/\alpha > a,\ 1$, *then* $E_a,\ E_1$ *satisfy* $E_a < E_1$ *and there exists a function* $d(E)$ *defined on* $(E_a,\ E_1)$ *and* $(v^s(x,\ d(E),\ E),\ d(E),\ E)\ \in \Gamma$, *which satisfies*

$$\lim_{E\to E_1} v^s(x,\ d(E),\ E) = v^B(x,\ E_1), \quad \lim_{E\to E_a} v^s(x,\ d(E),\ E) = v^B(x,\ E_a). \tag{12}$$

We call $v^B(x,\ E)$ a singular solution with a boundary layer. In the next section, we discuss the stability of these singular solutions.

3 Stability of stationary solution of Shadow System

In this section we show the stability of stationary solutions of (2), which are given in Theorem 5. First we discuss a stability of the singular stationary solution with a boundary layer obtained in Theorem 5 as d is small. Let $(v^s(x,d,E),d,E)$ be a solution of (4), (5) for $(d,\ E) \in \Lambda$ in the neighborhood of $(0, E_a)$, which satisfies $\lim_{d\downarrow 0} v^s(x,d,E) = v^B(x,E_a)$. In order to study a stability, we consider the following linearized eigenvalue problem around the solution $(v^s(x,d,E),d,E)$ of (2):

$$\begin{cases} \lambda \displaystyle\int_0^1 (\eta + E\alpha w)\, e^{\alpha v^s}\, dx = \int_0^1 f_u(Ee^{\alpha v^s})(\eta + E\alpha w) e^{\alpha v^s} dx \\ \lambda w = dw_{xx} + g_v(v^s,\ E)w + g_E(v^s,\ E)\eta, \qquad x \in (0,\ 1) \\ w_x(0) = w_x(1) = 0, \end{cases} \tag{13}$$

where $g_v(v,\ E) = E\alpha e^{\alpha v} - \gamma$, $g_E(v,\ E) = e^{\alpha v}$. Here, $(\eta,\ w(x))$ means an eigenfunction of the eigenvalue λ and η is a positive constant.

As $d \to 0$, we show the asymptotic behavior of critical eigenvalue which determines the stability by using the SLEP method (see [10]).

Theorem 6. [6] *Set* $1/\alpha > a$. *Let* $(v^s(x,d,E),d,E)$ *be a stationary solution of* (2) *for* $(d,\ E) \in \Lambda$ *in the neighborhood of* $(0, E_a)$, *which satisfies* $\lim_{d\downarrow 0} v^s(x,d,E) = v^B(E_a)$. *Then, there are two eigenvalues* $\lambda_1(d),\ \lambda_2(d)$ *with positive real part such that*

$$\lim_{d\to 0} \lambda_1(d) = f_u(a) > 0,\ \lim_{d\to 0} \lambda_2(d) = \theta^* > 0 \quad (\theta^* \textit{ is given in Lemma 7}).$$

Moreover, other eigenvalues have negative real part and uniformly away from imaginary axis.

We discuss about this result by intuition. If $w(x)$ has not any singularity in the neighborhood of $x = 1$, we have formally $\lim_{d\to 0} \lambda = f_u(a)$ because of

$$\lim_{d\to 0} f_u(Ee^{\alpha v^s}) = f_u(a) \quad \text{compact uniformly in } [0, 1)$$

$$\int_0^1 (\lambda - f_u(Ee^{\alpha v^s}))(\eta + E\alpha w)e^{\alpha v^s} dx = 0.$$

On the other hand, if $w(x)$ has a singularity in the neighborhood of $x = 1$, then $\eta = 0$ becomes an approximate eigenfunction. Therefore, we treat the following linearized eigenvalue problem corresponding to (4), (5) around $v^s(x, d, E)$:

$$\begin{cases} \lambda w = d w_{xx} + g_v(v^s,\ E)w, & x \in (0,\ 1), \\ w_x(0) = w_x(1) = 0. \end{cases} \tag{14}$$

Then, we show the following Lemma.

Lemma 7. *There is only one eigenvalue $\hat{\lambda}(d)$ with positive real part of the linearized eigenvalue problem* (14)*, which satisfies $\lim_{d\to 0} \hat{\lambda}(d) = \theta^* > 0$.*

By using a similar argument, we have

Theorem 8. [6] *Set $1/\alpha > 1$. Let $(v^s(x, d, E), d, E)$ be a stationary solution of* (2) *for $(d,\ E) \in \Lambda$ in the neighborhood of $(0, E_1)$, which satisfies $\lim_{d\downarrow 0} v^s(x, d, E) = v^B(x, E_1)$. Then, there is only one eigenvalue $\lambda^*(d)$ with positive real part such that*

$$\lim_{d\to 0} \lambda^*(d) = \theta^* > 0.$$

Moreover, other eigenvalues have negative real part and uniformly away from imaginary axis. Here θ^* is a constant depending on $E = E_1$ given in Lemma 7.

Next, we show the stability of the nonconstant bifurcating solutions from the constant solution in the neighborhood of the bifurcation point.

Theorem 9. [6] *If $1/\alpha < a, 1$, the following statements hold.*

(1) *Let $(v^s(x, d, E), d, E)$ be a solution of* (2) *for $(d,\ E) \in \Lambda$ in the neighborhood of $(d^*(E_a), E_a)$, which satisfies $\lim_{d\to d^*(E_a)} v^s(x, d, E) = v^*(E_a)$. Then, there exists only one eigenvalue $\lambda^*(d)$ with positive real part such that*

$$\lim_{d\to 0} \lambda^*(d) = f_u(a) > 0.$$

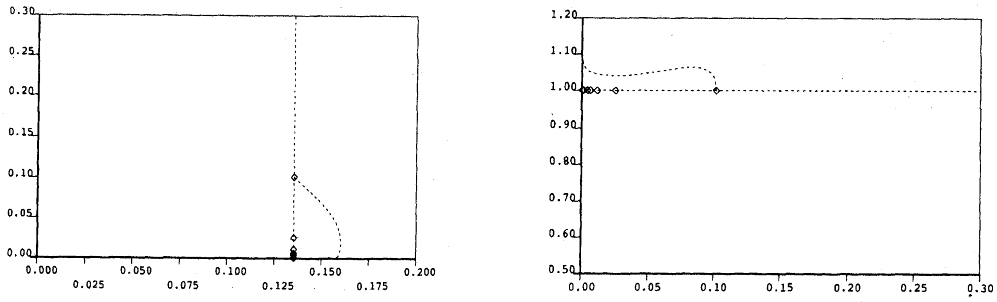
Moreover, other eigenvalues have negative real part and uniformly away from imaginary axis.

(2) *Let $(v^s(x, d, E), d, E)$ be a solution of* (2) *for $(d,\ E) \in \Lambda$ in the neighborhood of $(d^*(E_1), E_1)$, which satisfies $\lim_{d\to d^*(E_1)} v^s(x, d, E) = v^*(E_1)$. Then, all eigenvalues have negative real part and uniformly away from imaginary axis.*

4 Numerical results

In this section, we discuss the existence and stability of stationary solutions of (2) by the numerical approach. When $\alpha = 2.0$ and $a = 0.25$, that is, the case (ii) of Theorem 5, there is a singular solution $v(x, d(E), E)$ with a boundary layer, which satisfies $\lim_{E\to E_a}(v^s(x, d(E), E), d(E)) = (v^B(x, x), 0)$. Moreover, there is a solution $v^s(x, d(E), E)$

which satisfies $\lim_{E\to E_1}(v^s(x,d(E),E),d(E)) = (v^*(E_1),d^*(E_1))$. We show the global bifurcation branch connecting these two solutions $v^B(x,E_a)$ and $v^*(E_1)$ in Figure 1 (a), which is numerically obtained by the AUTO package [1]. Here each point $(d,\ E)$ on the curve of this figure (a) corresponds to a monotone increasing solution of (4), (5). From this result, there is not (static) secondary bifurcation phenomena from nonconstant solution on the bifurcation branch. But, the solution is unstable for small d, that is, there are two eigenvalues with a positive real part from Theorem 9. On the other hand, the solutions for (d,E) in the neighborhood of $(d^*(E_1),E_1)$ are stable from Theorem 9 (2). The number of eigenvalues with respect to stationary solutions on the bifurcation curve changes depending on the parameter d.



(a) (b)

Figure 1: The global solution structure for $\alpha = 2.0$, where the symbol $\diamondsuit$ means a bifurcation point. (a) Vertical and horizontal axis mean parameters d and E, respectively. (b) Vertical and horizontal axis mean the value of v at $x = 1$ and d, respectively.

For the original reaction diffusion system (1), the global bifurcation from the constant solution is obtained by the numerical simulation (see Figure 2). From Figure 2 implies the existence of the Hopf bifurcation from the nonconstant solution in the neighborhood at $d = 0.008$. Moreover, these periodic solutions are stable from Figure 3. Therefore, we suggest that there is a Hopf bifurcation phenomena for the shadow system (2). Therefore, we guess the change of the number of eigenvalues with a positive real part.

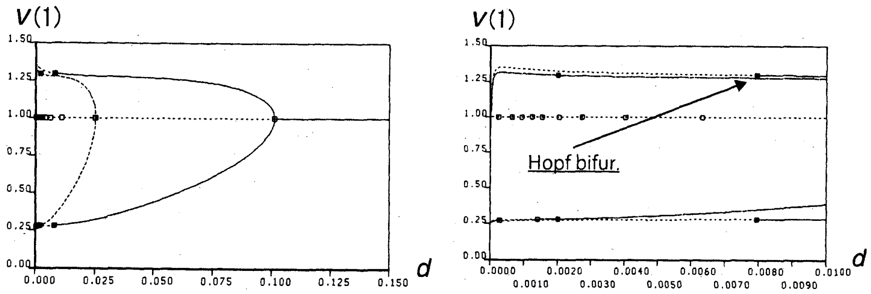


Figure 2: Vertical and horizontal axises mean the maximal value of $v(x)$ $(v(1))$ and d. $\mathcal{D} = 100.$, $\alpha = 2.$, $a = 0.25$.

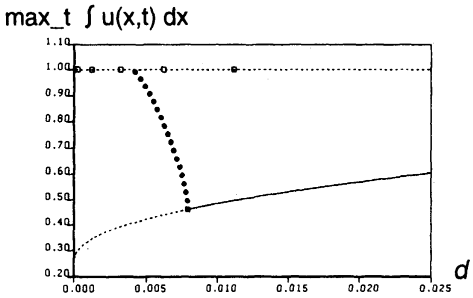


Figure 3: Vertical and horizontal axises mean a maximal value of $\|u(\cdot,t)\|_{L^1}$ and d. $\mathcal{D} = 100.$, $\alpha = 2.0$, $a = 0.25$.

5 Relaxation oscillation

In Section 4, we show that the periodic solution of (1) exists due to the secondary bifurcation phenomena from the monotone solution as large $\mathcal{D}$ and is stable by numerical computations (see Figures 2 - 3). It is difficult to show the Hopf bifurcation from the nonconstant solution for our limiting system (2). But we suggest the appearance of the periodic pattern from the lose of stability in Section 3. Therefore, we discuss about the appearance of this phenomena from the other viewpoint. To do so, we demonstrate the infinite dimensional relaxation oscillation as the onset of these spatio-temporal phenomena (see [3, 4]). In order to explain that, we introduce a new parameter δ which mean the rate of the growth. Then, we have the following system with respect to (2):

$$\begin{cases} \left(\int_0^1 Ee^{\alpha v}\,dx\right)_t = \delta \int_0^1 f(Ee^{\alpha v})\,dx, & t > 0 \\ v_t = dv_{xx} + g(v,\ E), & x \in (0,\ 1),\ t > 0 \\ v_x(0,t) = v_x(1,t) = 0, & t > 0 \end{cases} \tag{15}$$

where $E(t)$, $v(x,t)$ are unknown functions.

Figure 4 implies the existence of periodic solution of (15) for small δ by a numerical simulation.

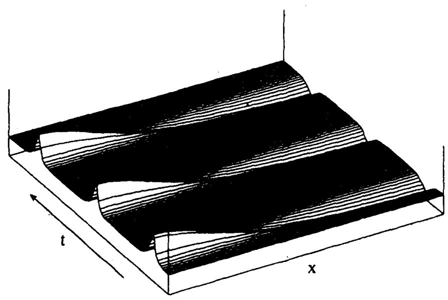


Figure 4: Periodic pattern with respect to $v(x,t)$ of (15) for $d = 0.04$, $\alpha = 2.0$, $\delta = 0.0005$, $a = 0.25$.

According to the argument in [3, 4], we show that the behavior of the solution is described by the fast and slow dynamics derived from (15) as δ is sufficiently small.

Fast dynamics

First, we consider the case of $\delta = 0$ as follows:

$$\begin{cases} \left(\int_0^1 Ee^{\alpha v}\, dx\right)_t = 0, & t > 0 \\ v_t = dv_{xx} + g(v,\ E), & x \in (0,\ 1),\ t > 0 \\ v_x(0,t) = v_x(1,t) = 0, & t > 0. \end{cases} \tag{16}$$

It follows from the first equation of (16) that

$$\int_0^1 Ee^{\alpha v}\, dx = \mu \tag{17}$$

for any positive constant μ.

We already discuss the constant solution of (16) in Lemma 1. On the other hand, the stationary solution $v(x)$ satisfies

$$\int_0^1 Ee^{\alpha v}\, dx = \int_0^1 v\, dx \quad (= \mu\ ) \tag{18}$$

by integrating the first equation of (4) and using the boundary condition of v. Therefore, a constant solution (E_c, v_c) is represented by $E_c = \mu e^{-\alpha\mu}$, $v_c = \mu$ for any $\mu > 0$.

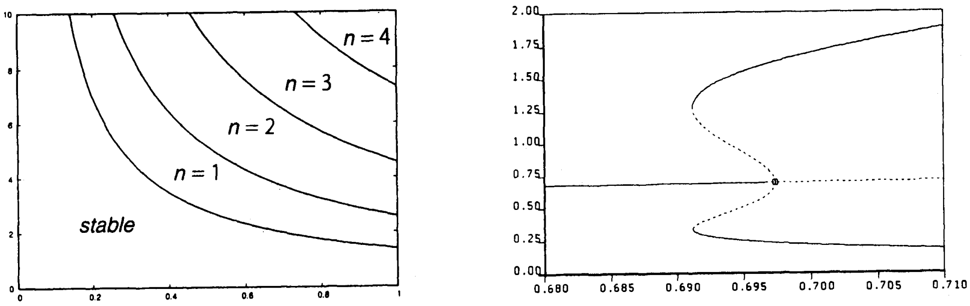
In order to have nonconstant solutions by using the bifurcation method, we consider the linearized eigenvalue problem around (E_c, v_c). By setting $E = E_c + M$ $v = \mu + V$ and substituting this into (16), the linearized eigenvalue problem is written by

$$\begin{cases} M_t = -E_c\alpha\left\{Me^{\alpha\mu} + (\alpha\mu - 1)\int_0^1 V\, dx\right\}, & t > 0 \\ V_t = dV_{xx} + Me^{\alpha V} + (\alpha\mu - 1)V, & x \in (0,\ 1),\ t > 0 \\ V_x(0,t) = V_x(1,t) = 0, & t > 0. \end{cases} \tag{19}$$

Substituting $M = \phi e^{\lambda t}$, $V = \psi e^{\lambda t} \cos n\pi x$ into (19), we have $\phi(\lambda + \alpha\mu) = 0$, $\psi(\lambda + d(n\pi)^2 - \alpha\mu - 1) = 0$. When $\lambda = 0$, it holds that $\phi = 0$, $\alpha\mu = d(n\pi)^2 - 1$. Because of $d > 0$, we get the condition $\alpha\mu - 1 > 0$. There are two constant solutions of (4) in Lemma 1. Then, $v^*(E)$ satisfies this condition. The curve in Figure 5 (a) corresponds to $\lambda = 0$ in $\mu - \alpha$ plane, which is the mode n perturbations. On the other hand , we show the nonconstant solution subcritically bifurcating from the constant solution $v^*(E)$ and subsequently turn the direction due to saddle-node bifurcation in Figure 5 (b). Therefore, there appears a hysteresis phenomena. That is, the bifurcating solution from the constant solution becomes unstable and recovers the stability after the saddle-node bifurcation. We remark that the direction of bifurcating branch is different depending on α.

Therefore, an approximation $(E_c(\mu), v_c(x, \mu, d))$ of stationary solutions of (15) satisfies

$$\begin{cases} 0 = dv_{xx} + g(v,\ E_c(\mu)), & x \in (0,\ 1),\ t > 0 \\ v_x(0,t) = v_x(1,t) = 0, & t > 0 \\ E_c(\mu)\int_0^1 e^{\alpha v}\, dx = \mu, & t > 0. \end{cases} \tag{20}$$



(a) (b)

Figure 5: $f(u) = u(1-u)(u-0.25)$, $d = 0.04$, (a) Vertical and horizontal axises mean α and μ, (b) Vertical and horizontal axises mean $v(0)$ and μ, $\alpha = 2.0$. There is a first bifurcation at $\mu = 0.697$ and saddle-node at $\mu = 0.697$.

Slow dynamics

Next, we assume that the solution of (15) tends to some monotone stationary solution on the upper branch in Figure 5 (b). Then, the solution is governed by the dynamics with the slow time scale $T = \delta t$. Therefore, by using the new time variable T and setting $\delta = 0$, (15) is rewritten as

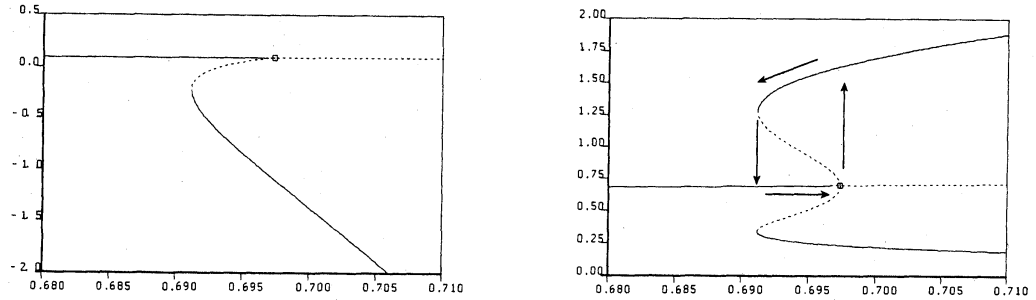
$$\begin{cases} \left(\int_0^1 E e^{\alpha v}\, dx \right)_T = \int_0^1 f(E e^{\alpha v})\, dx, & T > 0 \\ 0 = d v_{xx} + g(v,\ E), & x \in (0,\ 1),\ T > 0 \\ v_x(0, T) = v_x(1, T) = 0, & T > 0. \end{cases} \tag{21}$$

Since the relation (17) from (16) is shown, $\mu(t)$ satisfies

$$\frac{d\mu}{dT} = \int_0^1 f(E e^{\alpha v})\, dx \tag{22}$$

for a solution $(E_s(\mu),\ v^s(x, \mu, d))$ of (21). Therefore, the behavior of the solution is expressed by (22).

Figure 6 (a) shows that $\int_0^1 f(E(\mu) e^{\alpha v_c})\, dx = f(E(\mu) e^{\alpha v_c})$ is positive on the constant solution brunch corresponding to the center line in Figure 6 (b). Therefor, v_c is increasing with respect to T. But the solution is destabilized as it passes the bifurcation point and eventually tends to the stable nonconstant solution. On the other hand, the value of integration $\int_0^1 f(E(\mu) e^{\alpha v_s^+})\, dx$ is negative on the upper branch in Figure 6 (b). Therefore, the solution $\mu(t)$ is decreasing with respect to T and tends to the constant solution through the saddle-node bifurcation point. Thereafter these process is periodically repeated. Therefore the appearance of the relaxation oscillation phenomena is heuristically explained for small δ.



(a) (b)

Figure 6: $f(u) = u(1-u)(u-0.25)$, $d = 0.04, \alpha = 2.0$ (a) Vertical and horizontal axises mean $v(0)$ and μ, (b) Vertical and horizontal axises mean $\int_0^1 f(Ee^{\alpha v})dx$ and μ.

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