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静電場中での液体ジェットの安定性と崩壊 (III)

Instabilities and breakup of a liquid jet in a static electric field (III)

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Abstract

Instabilities and breakup process of a two-dimensional viscous planar jet are analytically investigated when the jet flows between two parallel less conductive walls on which an external electric field is imposed. According to the leaky dielectric model for surface charges and a membrane approximation for long wave deformations, a set of simplified nonlinear evolution equations is derived and solved numerically under the initial-boundary condition that the jet is emanating from a slit nozzle exit. It is found that there exist two types of typical breakup modes: the surface tension and electric force dominant modes and the existing regions of these two modes are determined in terms of two parameters. It is also found that fluctuations on the jet are amplified in the downstream as long as the jet is thick when they are applied to the thickness direction at the nozzle exit, while they are suppressed as the jet becomes extremely thin.

1 Introduction

Behavior of a liquid jet in a static electric field is determined depending on combined effects of electric and fluid forces. In particular, instabilities and breakup have been theoretically and experimentally investigated mainly in a liquid column jet for practical use [1, 2]. However, they have not been sufficiently investigated in a liquid planar jet except for instabilities of periodic disturbances in a uniform sheet when the liquid conductivity is vanishingly small or infinitely large [3], in spite of importance in applications to producing thin liquid sheets.

Therefore, in this paper we analytically deals with unsteady behavior of a two-dimensional planar jet emanating from a slit nozzle. In particular, we consider when the jet flows between less conductive sheath walls on which an external electric field is imposed. In the analysis, we pay attention to competing effects of the surface tension for stabilization and the electric force for destabilization on the emanating jet and reveals effective parameters on the jet behavior.

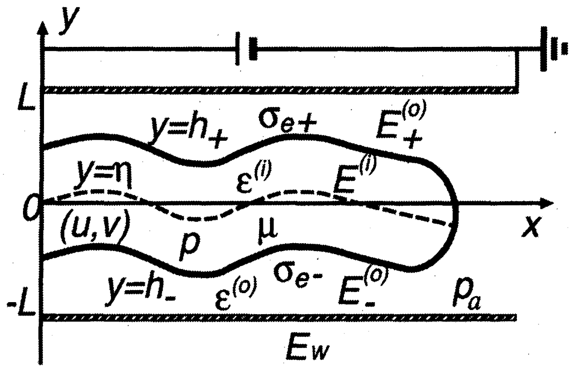


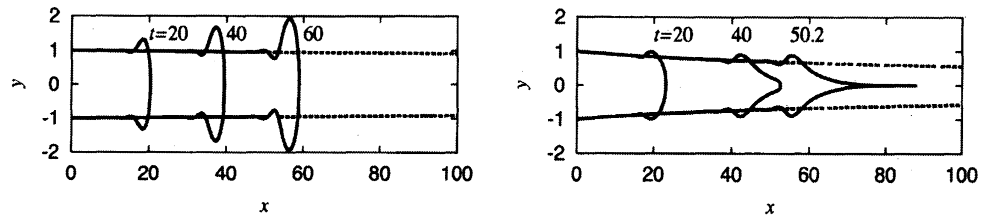
Figure 1: A two-dimensional planar liquid jet between two parallel sheaths on which an external electric field is imposed.

2 Formulation

We consider the two-dimensional planar liquid jet as in Fig.1, where the liquid is viscous and slightly conductive and the jet flows between two parallel sheath walls on which an external electric field E_w (constant) is imposed. In the $x - y$ coordinate system, the sheaths at $y = \pm L$ (constant) are close to the jet surfaces prescribed by $y = h_\pm(x,t)$. The liquid velocity and pressure are denoted by $\boldsymbol{u} = (u(z,r,t), v(z,r,t))$ and $p(x,y,t)$, while the electric fields outside and inside the jet are denoted by $\boldsymbol{E}_\pm^{(o)}(x,y,t)$ and $\boldsymbol{E}^{(i)}(x,y,t)$ and the permittivities are by $\epsilon^{(o)}$ and $\epsilon^{(i)}$ ($\epsilon^{(o)} < \epsilon^{(i)}$). For simplicity, we neglect any motion of surrounding gas and the gravitational force and, for convenience of the analysis, we introduce the mid-plane $y = \eta(x,t)$ ($\equiv (h_+ + h_-)/2$) and thickness $b(x,t)$ ($\equiv h_+ - h_-$) of the jet. According to the leaky dielectric model, we can assume that the surface charge densities $\sigma_{e\pm}$ exist only on the surfaces.

Assuming the jet thickness and gap length between jet and walls are to be thin, we apply a membrane (slender jet) approximation to the basic equations (continuity and momentum equations of the jet and conservation equations of surface charge densities) and the boundary conditions (kinematical and dynamical conditions and continuity of the electric field at the jet surfaces and the imposed electric field on the walls). As a result of this, we obtain rather simplified nonlinear equations of u, v, b, η, $\sigma_{e\pm}$ in the lowest order approximation. We note that these equations include the following non-dimensional parameters: the electric Peclet number Pe (convective current/conductive current) and the electric Euler number Λ (electric pressure /fluid inertial force) together with the Weber number Wb(fluid inertial force / surface tension) and the Reynolds number Re (fluid inertial force / viscous force). We note the Taylor number Ta $\equiv \Lambda$ Wb (electric pressure / surface tension).

3 Numerical Results and Conclude Remarks



(a) (b)

Figure 2: Evolutions of the profiles $h_\pm$ for the symmetric mode, where Pe = 100 and $\Lambda = 0.01$ for the surface tension dominant mode in (a), while Pe = 100 and $\Lambda = 0.1$ for the electric force dominant mode in (b). On the other hand, the steady solutions are shown in the broken lines.

In order to examine the behavior of the jet emanating from a slit nozzle, we numerically solve the equations for the following initial-boundary condition: $b/2 = \sqrt{1-x^2}$, $u = 1$, $v = 0$, $\sigma_{e\pm} = \sigma_{e0}$ for $0 \le x \le 1$ at $t = 0$, while $b/2 = u = 1$, $\sigma_{e\pm}(0,t) = \sigma_{e0}$, $\eta = A\sin\omega t$, $v = A\omega\cos\omega t$ at $x = 0$ for $t > 0$. The numerical calculations are carried out by using the combined methods of the CIP for the advection phase with the time splitting for the non-advection phase. In the present analysis, the parameters are fixed to be Wb = 100, Re = 100, $E_w = 1$, $\sigma_{e0} = 0.1$, $\beta(\equiv \epsilon^{(i)}/\epsilon^{(o)} - 1) = 2$ and $L = 2$ when $A = 0$ or 0.05 and $\omega = \pi/5$, while Pe and Λ are chosen as the control parameters.

We first consider symmetric deformations ($h_+ = -h_-$) of the jet with respect to the mid-plane when $A = 0$ and $\sigma_{e+} = \sigma_{e-}$. Figure 2 shows the typical time evolutions (solid lines) of the profiles $h_\pm$. In this Figure, the steady state evolutions (broken lines) are additionally shown for comparison. In Fig.2(a) for Pe = 100 and $\Lambda = 0.01$, we can see that the tip of the jet grows larger and a blob is produced due to the surface tension as the time increases, which is referred to as the surface tension dominant mode. On the other hand, in Fig. 2(b) for Pe = 100 and $\Lambda = 0.1$, we can see that the thickness rapidly decreases in the downward because the jet is strongly extracted by the imposed electric field, which is referred to as the electric force dominant mode.

Figure 3 shows that these modes are transferred in the Λ – Pe parameter space when $L = 2$ (solid line) and 4 (broken line). In this Figure, the electric force dominant mode (b) prevails in the region above these lines for each values of L, while the surface tension dominant mode (a) appears below the lines. This means that the surface tension mode (a) is more dominant in less conductive liquids and for smaller imposed electric field intensity (large Pe and small Λ or Ta), while the electric force mode (b) is more dominant for more conductive liquids and larger imposed electric field intensity (small Pe and large Λ or Ta). It is also found that the effect of the imposed electric field decreases and, so that, the parameter region of (b) diminishes as L increases.

Finally, we consider more general case of deformations of the jet when sinuous fluctuations are applied to the mid-plane at $x = 0$. Typical evolutions of the jet for such fluctuations in the electric force dominant mode are shown in Fig. 4 for $A = 0.05$ and $\omega = \pi/5$, where Pe = 100 and $\Lambda = 0.1$. We can see from this Figure that the sinuous

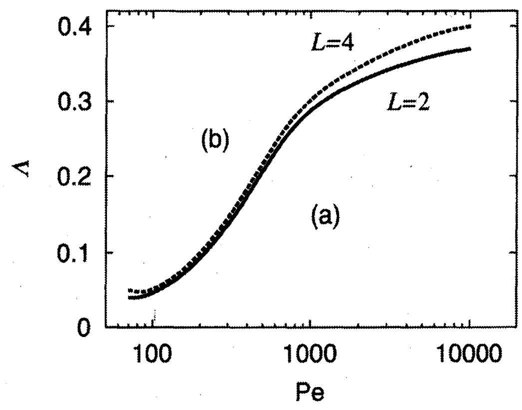


Figure 3: Critical curves discriminating the surface tension dominant mode referred to as (a) and the electric force dominant mode as (b) in (Λ, Pe) space when $L = 2$ (solid line) and 4 (broken line).

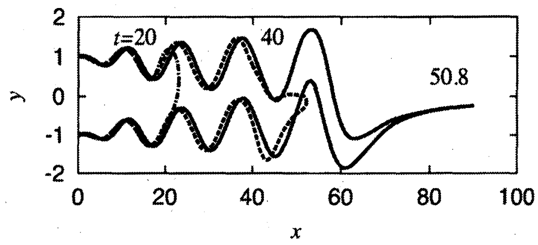


Figure 4: Evolutions of the jet surfaces for the fluctuation $\eta = A \sin \omega t$ and $v = A\omega \cos \omega t$ with $A = 0.05$ and $\omega = \pi/5$ when $\mathrm{Pe} = 100$, $\Lambda = 0.1$.

deformations due to the fluctuations propagate growing to the downward as the time increases. However, we can see that the deformations are suppressed in the region near the top as the thickness becomes thin.

Although the well-known aerodynamic instability due to the surrounding gas is not considered in the present analysis, we note that the electric force becomes dominant and then the sinuous deformations less appear when the sheet thickness becomes thin. Consequently, we expect that sufficiently thin sheets can be stably produced as long as Λ is not so small.

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