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Energy harvesting based on bio-inspired fluid-structure interaction

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Abstract

This work is concerned with flag-like structures in a fluid flow. If a waving (fluttering, i.e. linearly unstable) flag is covered with piezoelectric elements it can be used to harvest energy from the flow. This problem has been considered by many researchers and it is known that the power generation efficiency is not high. The purpose of the present work is to consider a large number of interacting flags and determine the optimum pattern (positions) of these flags under the assumption that they can utilize the vortex wake from the other flags to increase the efficiency of the power generation, just as individual fish in a large school of similar fish can utilize the surrounding wakes with benefit. The present paper first gives a review of the classical, fundamental flapping flag problem. It then goes on to develop a simple model for the main problem, as just described.

1 Introduction

The interest in renewable energy is experiencing a boom in recent years. Many types of renewable energy sources have been considered in the past, such as tides and water waves, and geothermal heat; but at present the most active fields are those concerned with sunlight and wind. As to the latter, large 'farms' of wind turbines have been, or are being, constructed in many parts of the World. In Denmark, for example, 40% of the total electric power was generated by the wind in 2015, and this percentage is on the rise.

At the same time there is a growing interest in harnessing the energy of the wind on a smaller scale. The energy in a fluttering flag, for example, can be harnessed if the flag is accommodated with piezoelectric elements that generate electricity when being bent. This is the subject of the present paper, which reports on an ongoing work.

The problem has, of course, been considered already by several researchers; and it is known that the efficiency of a piezo-element-covered flag is not high. The main purpose of the present work is to consider a large number of interacting flags and determine the optimum pattern (positions) of these flags under the assumption that they can utilize the vortex wake from the other flags to increase the efficiency of the power generation, just as individual fish in a large school of similar fish - or the individual birds in a large formation - can utilize the surrounding wakes with benefit.

The fundamental physical problem - that of the flapping flag - is, however, a most fascinating one, and we will start out by giving a historical review of works related to it. We we then go on to the main problem, to outline a simple modeling approach and show some preliminary computational results related hereto.

2 History of mathematical models of the flapping flag

Lord Rayleigh was, in a paper from 1879 [1], apparently the first to give a mathematical analysis related to flapping flags and sails. The main topic of the paper is instability of cylindrical jets, with the shear layer (between the moving and the stationary fluid) modeled by a cylindrical vortex sheet. Flapping flags and sails were mentioned as applications of the special case of a zero-curvature (two-dimensional) vortex sheet. But as pointed out in a recent review paper by Shelley and Zhang, Rayleigh's analysis is more a model of the vortex wake shed from the trailing edge of a flapping flag than a model of the flag itself. Rayleigh's study [1] is elaborated and discussed also in Lamb's 'Hydrodynamics' [3].

Following the initial study of Rayleigh, ninety years passed by before the appearance of the next serious, scientific study of flapping flags, the purely experimental study of Taneda [4]. Taneda considered flags made of various types of fabrics, placed in a vertical wind tunnel with a $35\,\mathrm{cm} \times 35\,\mathrm{cm}$ working section. He found that for sufficiently low flow speeds the flags do not flutter and that the trailing edge of the flags begin to flutter at higher flow speeds. It was also found that, after onset of flutter, the flutter frequency increases with the flow speed; furthermore, the smaller the flag length and the lighter the flag the higher the flutter frequency.

More than thirty years later, experiments were reconsidered by Zhang et al. [5], who utilized flowing soap films to realize purely two-dimensional motions. The use of soap film also made very detailed flow visualizations possible.

Up to this time, to the best of our knowledge, since Rayleigh's 1879 paper, no other paper devoted to a further-going theoretical study of the problem had been published. On the other hand, starting with the birth of aeronautics, a large body of literature on plates in flow was developed (e.g. [6, 7]). Fitt and Pope [8] were probably the first to apply these studies to the flapping flag problem and to analyze the dynamics of a two-dimensional flag with bending stiffness. A potential flow model was employed. The effect of the flag on the flow was modeled by a vortex sheet (similarly to Rayleigh's approach [1]) which amounts to a non-local potential function. A Kutta condition was satisfied at the trailing edge, but vortex shedding therefrom was ignored.

These two papers, the experimental paper by Zhang et al. [5] and theoretical one by Fitt and Pope [8], seem to have initiated a revival of the interest in the problem, because a veritable boom in related papers appeared in the following years.

Argentina and Mahadevan [9] reconsidered the problem as defined by Pope and Fitt [8], but made use of a more elaborate aerodynamic model, based on the theory of Theodorsen [10]. This theory makes use of two potential functions, a non-circulatory velocity potential representing the 'structure' (flag or plate) and a circulatory velocity potential representing the wake, due to vortex shedding. While the non-circulatory flow is represented by a non-local potential function in Theodorsen's original theory, Argentina and Mahadevan simplified this part by employing a local formulation, originally due to Milne-Thomson [11]. The stability analysis, that is, the solution of the coupled fluid-structure boundary value problem, was carried out in a non-discretized fashion, using the path-following program AUTO.

Manela and Howe [12] employed the aerodynamic model developed by Argentina and Mahadevan [9] but made use of a standard Galerkin discretization approach. They investigated the effect of the upstream boundary conditions (pinned or clamped) and also computed the sound generated by the flapping flag. In a subsequent paper [13] they investigated the effect of a flagpole, and in particular the vortex street developed downstream from it, on the flag stability and dynamics.

Returning to the work of Argentina and Mahadevan [9], these authors also presented estimates regarding three-dimensional flow effects which are ubiquitous by finite-sized flags. They found that the three-dimensionality of the flow reduces the added mass and thus makes the vibrations more stable, that is, the flapping is initiated at higher flow speed that by a two-dimensional flow model. Eloy et al. [14] went a step further and developed a full three-dimensional potential flow model for a finite sized flag, which however still was assumed to perform only two-dimensional vibrations. A major result was the finding that flags of finite span are more stable than flags of infinite span (two-dimensional flags). This is in agreement with the just mentioned findings of Argentina and Mahadevan [9].

Papers on the dynamics of flapping flags continue to appear regularly. Most of the recent ones are however based on CFD (computational fluid dynamics) approaches. Such studies will not be considered here.

3 A simple flag model for a group of interacting flags

As discussed in section 1, the main purpose of this work is to study the interaction between a large number of flags and their wakes. This is a problem where hopes of analytical progress are not high. To get an understanding of the problem and its solution it is, at first instant, desirable to consider the simplest possible model. This appears to be a model where the flags are represented by articulated plates and where the vortex-dominated flow is represented by a discrete vortex method. Such an approach is described in the following.

3.1 Point vortex model

In terms of complex variables, the velocity potential at $z = x + \mathrm{i}y$ for a point vortex of strength Γ_n located at $z_n = x_n + \mathrm{i}y_n$ is given by [15]

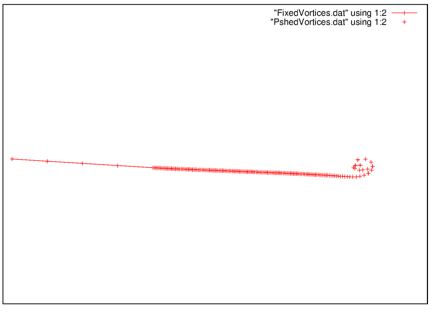
$$\phi_n = \frac{\Gamma_n}{2\pi \mathrm{i}} \log(z - z_n). \tag{1}$$

Katz and Plotkin [16] discuss how the vorticity distribution on a flat plate (in the following termed a panel), from a far field point of view, can be represented by a point vortex, as specified by (1), placed at the center of pressure, which is at the quarter-chord point (i. e. at the point $\ell/4$ measured from the upstream edge, if the panel has length ℓ.) It is shown in [16] that if a collocation point, at which the normal flow velocity is zero, is placed at the three-quarter-chord point $3\ell/4$, then the Kutta condition will be satisfied at the trailing edge of the panel.

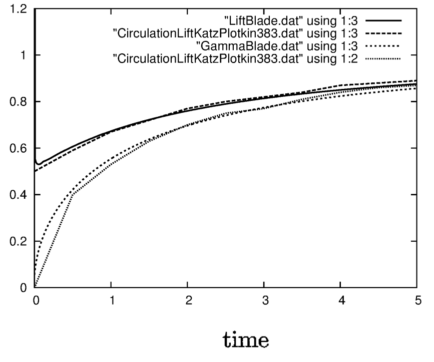
The flag model is here discretized into a number, N say, of such lumped vortex panels. In the simulation of the unsteady flow a point vortex is shed from the trailing edge of the flag (that is, from the trailing edge of the most downstream panel at each time step. The strength of each shed point vortex is determined from Kelvin's theorem, which says that the total strength of all point vortices (bound and shed/free) must remain constant [15].

Figure 1(a) shows a 'stiff flag' (a plate) discretized into four lumped vortex panels, and the distribution of shed point vortices after the plate has been impulsively started (to move) from rest; or equivalently, the flow has been impulsively 'turned on'.

Total circulation and lift results for this case are shown in Fig. 1(b). The upper two curves depict the lift. The full curve shows the present numerical result, while dashed curve is the analytical result based on Theodorsen's model [10] (this result is taken from ref. [16]). The lower curves represent the circulation around the plate. Here the dashed curve is for the present numerical result, while the finely dotted curve is again representing the Theodorsen solution. The agreement between both lift and circulation curves is seen to be very good.



(a)



(b)

Figure 1: (a) A plate impulsively started from rest. (b) Comparison between discrete vortex results and Theodorsens solution.

3.2 A simple structural model

We consider here a very simple structural model of the flag. Employing the same discretization into panels as described in the previous section, a concentrated mass is placed in the center of each panel. Concentrated lift forces are assumed to act also in the panel center points. The panels are interconnected via springs. Let m_q be the point mass attached to panel no. q and let k_q and k_{q+1} be the stiffness of the spring at upstream and downstream end, respectively. The damping constants which represent dissipation in these springs are c_q and c_{q+1}, respectively. Let the flag be discretized into N panels, each of length ℓ. Then its kinetic energy is given by

$$T = \sum_{q=1}^{N} \frac{1}{2} m_q \ell^2 \left\{ \left(\sum_{p=1}^{q} \epsilon_{pq} \sin\theta_p \, \dot{\theta}_p \right)^2 + \left(\sum_{p=1}^{q} \epsilon_{pq} \cos\theta_p \, \dot{\theta}_p \right)^2 \right\}, \tag{2}$$

where θ_p is the angle of panel no. p with vertical, and

$$\epsilon_{pq} = \frac{1}{2}\left(2 - \delta_{pq}\right) = \begin{cases} 1, & \text{for} \quad p \neq q \\ \frac{1}{2}, & \text{for} \quad p = q \end{cases} . \tag{3}$$

A dot denotes differentiation with respect to the time t. It is remarked that $\theta_p = 0$ for $p < 1$ and for $p > N$. The potential energy of the flag is given by

$$V = \sum_{q=1}^{N} \frac{1}{2} k_p \left(\theta_p - \theta_{p-1}\right)^2 . \tag{4}$$

The Rayleigh dissipation function is given by

$$D = \sum_{q=1}^{N} \frac{1}{2} c_p \left(\dot{\theta}_p - \dot{\theta}_{p-1}\right)^2 . \tag{5}$$

Let $L_q = \rho\, u_{tq} \Gamma_q$ be the lift force on panel no. q, where ρ is the fluid density, u_{tq} is the tangential velocity, and Γ_q is again the vortex strength. Then the generalized force on panel no. p is

$$Q_p = \ell \sum_{q=p}^{N} L_q \epsilon_{pq} \cos\left(\theta_p - \theta_q\right). \tag{6}$$

Employing Lagrange's equations [17]

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}_p}\right) + \frac{\partial D}{\partial \dot{\theta}_p} - \frac{\partial T}{\partial \theta_p} + \frac{\partial V}{\partial \theta_p} = Q_p, \quad p = 1, 2, \ldots, N, \tag{7}$$

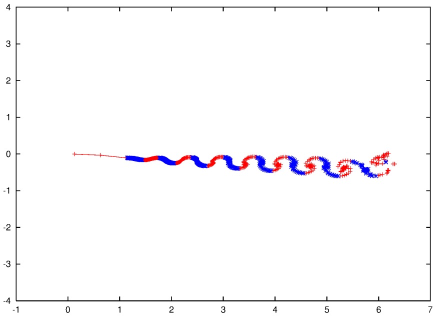
and *linearizing*, we obtain the linear equations of motion

$$\begin{aligned} &\sum_{q=p}^{N} \ell^2 m_q \epsilon_{pq} \sum_{n=1}^{q} \epsilon_{nq} \ddot{\theta}_n - c_p \dot{\theta}_{p-1} + \left(c_p + c_{p+1}\right) \dot{\theta}_p - c_{p+1} \dot{\theta}_{p+1} \\ &\qquad -k_p \theta_{p-1} + \left(k_p + k_{p+1}\right) \theta_p - k_{p+1} \theta_{p+1} = \ell \sum_{q=p}^{N} L_q \epsilon_{pq}, \end{aligned} \tag{8}$$

for $p = 1, 2, \cdots, N$, again with $\theta_p = 0$ for $p < 1$ and $p > N$. As an example, for $N = 2$ we have the system

$$\begin{aligned} &\ell^2 \begin{bmatrix} \frac{1}{4} m_1 + m_2 & \frac{1}{2} m_2 \\ \frac{1}{2} m_2 & \frac{1}{4} m_2 \end{bmatrix} \begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{Bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} \begin{Bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{Bmatrix} \\ &\qquad + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} = \ell \begin{Bmatrix} \frac{1}{2} L_1 + L_2 \\ \frac{1}{2} L_2 \end{Bmatrix}. \end{aligned} \tag{9}$$

In Fig. 2, examples of simulation results based on (9) are shown. The example in part (a) is for a low flow speed, much below the flutter threshold. Here the wake develops into a von Kármán-like vortex street. In part (b) the flow speed corresponds to the threshold of flutter. Here the wake pattern is govern mostly by the flapping motion of the flag. These results correspond qualitatively well to the experimental (soap film) results of Zhang et al. [5].



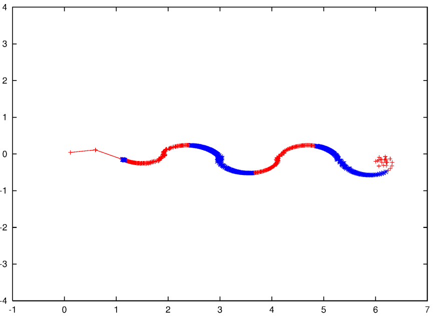


Figure 2: (a) Vibrating flag (plate) at a low flow speed. (b) The flag at the threshold of flutter. (In color (pdf/online), red points represent counter-clockwise rotating vortices, and blue clockwise rotating vortices.)

4 Concluding remarks

The present paper has, firstly, reviewed works related to the classical problem of the flapping flag; and, secondly, outlined a simple but efficient method the model the fluid-structure interaction of many flags. The model should be developed further, to represent a large collection of flags and to include a model of piezoelectric elements; this work is ongoing. At the same time, a number of open problems remain by fundamental flapping flag problem (effects of structural damping, non-local velocity potential, etc.) that are very interesting and attractive.

Acknowledgment

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