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Modulational instability in realistic directional seas

Takuji Waseda, Wataru Fujimoto[+] and Yuki Kita

Graduate School of Frontier Sciences, the University of Tokyo
[+]MS&AD InterRisk Research & Consulting, Inc.

1 Introduction

Freak wave has been a hot topic in the wave community for the last two decades. In this opportunity celebrating Prof. Mitsuhiro Tanaka's retirement in March of 2019, I would like to present a brief overview of where everything started, and where we are after 20 years of extensive research on freak waves. In the course, I will highlight the contributions of Prof. Tanaka, which are not as well recognized in the community as they deserve to be. I would also like to convey a message that the significance of modulational instability in the formation of freak waves in the ocean is not just about the occurrence probability but should include analysis of the wave geometry and kinematics, as well as its association with meteorological conditions.

2 Freak wave research: where everything started

The wave community recognizes the paper by Trulsen and Dysthe (1997) presented at the 21st Symposium on Naval Hydrodynamics as the first presenting the possibility of modulational instability as a cause of freak wave generation in the ocean. They state that the "basic assumption is that these waves can be produced by nonlinear self-modulation of a slowly modulated wave train," and that "field observations have been presented, suggesting that the wave conditions leading to freak waves can be described as a narrow-banded weakly nonlinear wave trains." This concept is further elaborated in Dysthe and Trulsen (1999) in which they point out that analytical solution to the Non-Linear Schrodinger equation (NLS hereafter) such as the Ma breather and the Peregrine solutions may provide "useful and simple analytical models for 'freak' wave events." Indeed, the NLS became an apparatus to produce a wide variety of 'freak' looking waves that "appear from nowhere and disappear without a trace" (Akhmediev et al. 2009).

The expression presented by Akhmediev captures the most important characteristics of the freak waves, that is its isolation from the surrounding waves in both space and time. However, many studies have focused on its statistical properties rather than its appearance. Freak waves are statistically rare, as depicted by the variations of its name; rogue wave, mad dog wave, abnormal wave, and extreme wave. The question is how rare is this wave, and will nonlinearity enhance its occurrence probability? The role of the modulational instability or the quasi-resonance is to slightly modify the Gaussian statistics of random wave field and thereby enhance the tail of the distributions of wave height and crest height. The nonlinear modification of the higher-order moments of the surface elevation was derived by Yuen and Lake in 1982 based on the Zakharov's equation (Zakharov 1968). Janssen (2003) recognized its significance in explaining the freak wave occurrence and shown that as the spectral bandwidth narrows, the Kurtosis increases. The key was the balance of the nonlinearity and dispersion, and the parameter relating their contributions was coined the Benjamin-Feir Index (NFI) in recognition to the first work by Benjamin and Feir (1967) on the instability of the Stokes wave. Janssen (2003) had shown that Kurtosis increases with BFI (Fig. 1 left). Onorato et al. (2004) experimentally showed that the tail of the exceedance probability of a random unidirectional wave field indeed enhances with the BFI (Fig. 1 right). This is because the slight deviation of the surface elevation distribution from Gaussian modifies the probability density function in the sense of Gram-Charrier expansion (Mori and Janssen 2006). As a result of these researches, the evaluation and understanding of kurtosis of the random wave field became the central issue of freak wave research. In retrospect, this probably deterred the focus of the freak wave research from the understanding and detection of its unusual appearance as depicted by Akhmediev et al. (2009).

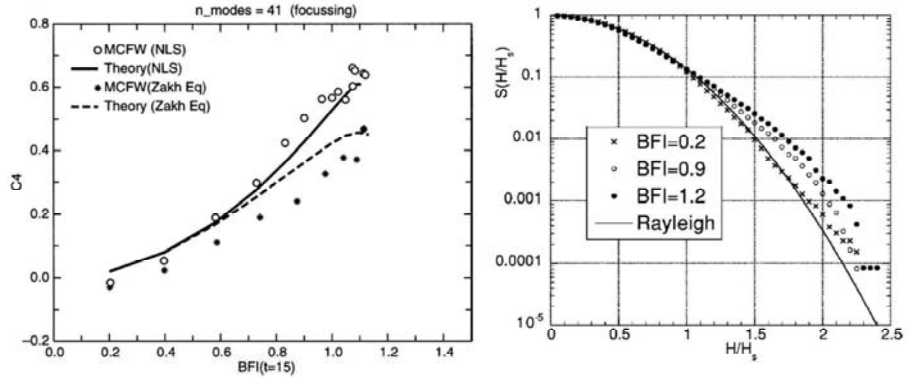

Figure 1: (left) Kurtosis plotted against the BFI (Benjamin-Feir Index) for random unidirectional wave field (reproduced from Janssen 2003); (right) Exceedance probability density of wave height for different BFIs (reproduced from Onorato et al. 2004).

3 Freak wave research: where we are

Notwithstanding the numerous researches demonstrating the increase of Kurtosis as the spectrum narrows, in the last several years, several papers were presented denying the relevance of modulational instability in the realistic ocean waves. Trulsen et al. (2015) analyzing the seas state during the Prestige accident, stated that “the possible nonlinear interaction between the two crossing wave system practically did not modify neither the kurtosis nor the largest crest elevation”, implying that modulational instabiltiy is absent in case of the mixed sea condition. The studies of the Andrea wave in the North Sea (Magnusson and Donelan 2015) showed that “the Benjamin-Feir instability may not have been a strong contributing factor to the development of rogue waves” (Dias et al. 2015) as well, and the “rogue waves are likely to be rare occurrences of weakly nonlinear random seas” (Fedele et al. 2016). Investigations of observed wave records also show that “random superposition of nonlinear waves is sufficient to explain the observations of individual rogue waves” (Gemmrich and Thomson 2017), and “the crest enhancement due to modulational instability has been shown – theoretically, numerically and observationally – to be minor” (Donelan and Magnusson 2017).

The common technique used is the combination of the third-generation wave model and the Higher-Order Spectral simulation (HOS, West et al. 1987). In the context of freak wave study, Toffoli et al. (2008, 2010) and Sergeeva and Slunyaev (2013) are recognized as the first studies to use HOS simulation to elucidate the phase-resolved statistics based on a given wave spectrum. In the aforementioned studies, known large wave events such as the Andrea storm, Prestige accident case, Draupner wave, and Killard wave, were investigated based on the “realistic” wave spectrum. The aim was to find out how large the kurtosis can be under realistic sea condition, and the results are that the Kurtosis remained small (Table 1). That is because the directional spreading suppresses the effect of quasi-resonance, discovered numerically and theoretically by controlled experiments systematically changing the directional spreading (Socquet-Juglard et al. 2005, Waseda 2006, Waseda et al. 2009, Onorato et al. 2009ab, Mori and Janssen 2011), Fig.2.

The work of Professor Tanaka on HOS simulation of the directional wave field is well known discovering that the Hasselman-type nonlinear transfer function can be attained only after a short period starting from the initially linear wave field (Tanaka 2001). However, his work on the freak wave using HOS is less known to the community. In his RIAM report (Tanaka 2006), he had conducted simulations of a wave field initialized with JONSWAP frequency spectrum and cosine to the power of n directional spreading. He recognized that while Kurtosis increases with the total energy (steepness), the occurrence probability of freak waves remain the same (Fig. 3). That is not surprising as his simulations were conducted with a broad directional spreading (n=2 and 4); the excess Kurtosis remained less than 0.1 while the uni-drectional case was around 0.5. It is interesting, however, that this work precedes the various works conducting a systematic study on the directional spreading to the kurtosis, and that he had already casted doubt on the validity of the parameterization of the occurrence probability of freak waves by the Kurtosis.

Table 1: Summary the estimated kurtosis from HOS simulation, and the depth condition of the historical freak waves.

Bitner-Gregersen et al. 2014	"Andrea storm"	k4 ~ 3	kpD=1.47
Trulsen et al. 2015	"Prestige accident"	k4 = 3.01	D > 1000 m
Dias et al. 2015	"Andrea storm"	k4 = 3.03	kpD=1.47
Fedele et al. 2016	"Andrea storm"	k4 = 3.04	kpD=1.47
	"Draupner"	k4 = 3.03	kpD=1.30
	"Killard"	k4 = 2.99	kpD=0.90
Fujimoto et al. 2018	"JKEO-Broad"	k4 = 3.03	D ~ 5400 m
	"JKEO-Narrow"	k4 = 3.04~3.05	D ~ 5400 m

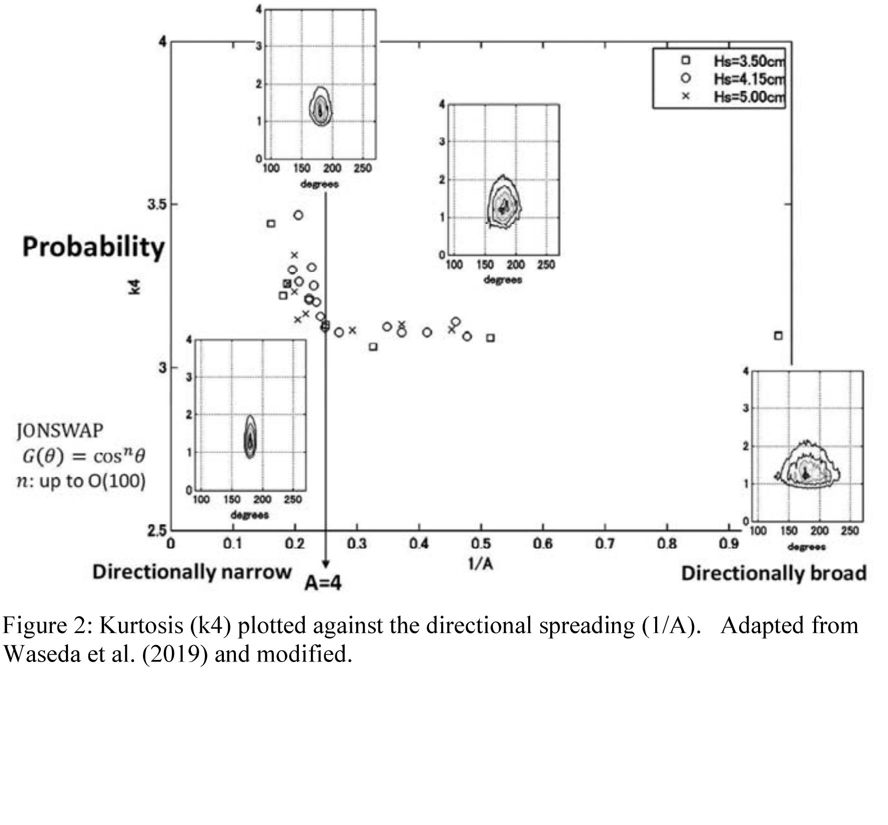

Figure 2: Kurtosis (k4) plotted against the directional spreading (1/A). Adapted from Waseda et al. (2019) and modified.

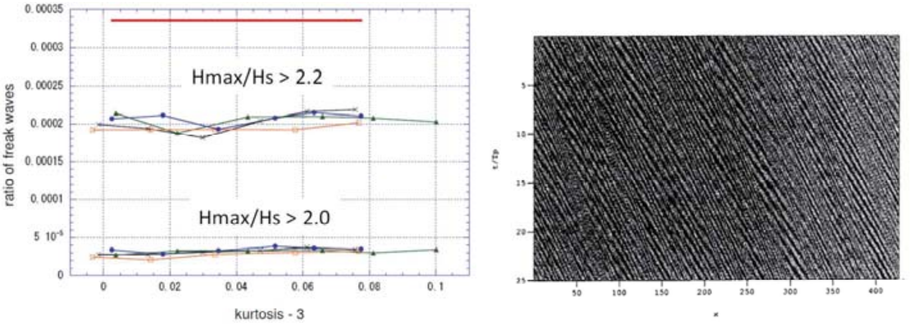

Figure 3: (left) Ratio freak waves plotted against excess kurtosis, adapted and altered from Tanaka (2006); (right) A snapshot of the surface elevation from the HOS simulation (reporoduced from Tanaka 2005).

Based on these numerical simulations, Professor Tanaka had investigated the relation between freak wave occurrence probability and nonlinearity, and conjectured that the generation of freak wave is associated with a formation of a wave group (Fig. 3 right). Wave group can linearly form when the spectrum is narrow, but when high waves are generated due to random superposition, the nonlinearity kicks in and enhance the development of the wave group. The nonlinear coherent wave groups will sustain much longer than a linearly superposed wave groups.

During the 21st symposium on Naval Hydrodynamics in 1997, when Trulsen and Dysthe first presented the idea of modulational instability as a cause of the freak wave generation, Stansberg raised a question: “One has reason to believe that increased spectral bandwidth as well as multidirectionality will reduce the growth of nonlinear self-modulations. This has, for example, been experimentally demonstrated in Stansberg (1995), where these effects have been systematically investigated. It may possibly be explained by the reduced “lifetime” of wave groups.” In retrospect, the foresight of this statement is rather fascinating. First, he had pointed out correctly that the directional spread will suppress the chances of modulational instability to occur (see Fig. 4). Second, he had related that to the reduction of the “lifetime” of the wave groups. This latter point will be elaborated in our study.

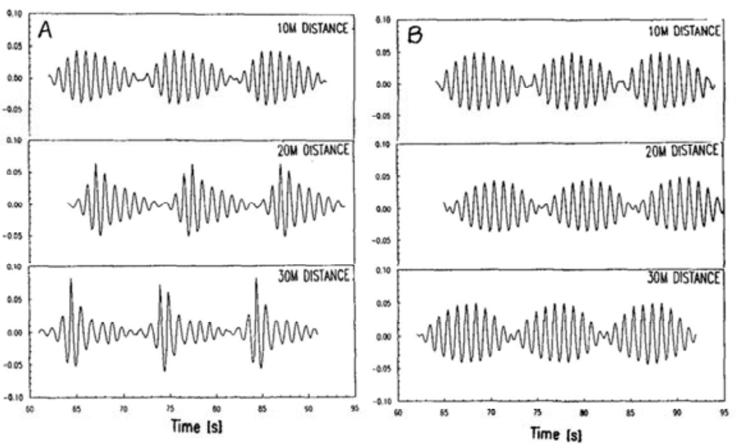

Fig. 1. Wave elevation from bichromatic test records.
A = long-crested. B = short-crested (bi-directional).

Figure 4: An evidence of the suppression of modulational instability by increased directional spread, reproduced from Stansberg in 1995.

4 Freak wave research: where we should head

The outstanding questions are:
Why wouldn't the kurtosis increase in directional seas?
Is there a better indicator representing the significance of nonlinearity?
How narrow can the directional spectrum be in the ocean?
In answer to these questions, we consider that the following points need to be considered:
Bound (Canonical) and Dynamic kurtosis;
Lifetime and the freak wave geometry;
Spectral evolution and its meteorological causes.

The question is whether the third order nonlinear wave interaction is relevant in enhancing the tail of the wave height and crest distributions or not. The estimated kurtosis from different studies summarized in Table 1 shows a marginal change from the Gaussian state. These values are considered too small to modify the distribution of the wave height,

$$P(H) = P_{Rayleigh}(H) \times \left(1 + C_4 f(H)\right), \tag{1}$$

where κ_{40} is the excess kurtosis or the fourth cumulant. The form of $f(H)$ are presented by Tayfun and Fedele (2007), Mori and Janssen (2006) and Mori and Yasuda (2002) and others in different forms. In the so-called Tayfun (1980) or Tayfun-Fedele (2007) distribution, the wave height should be corrected by the steepness. The fact that the C_4 is negligible means that the second order nonlinearity is sufficient to explain the probability distribution of ocean waves, therefore, the Tayfun distribution is suitable. This is the basis of the conjecture that the "modulational instability" is not playing a role in the generation of the freak waves.

We cast doubt on this conjecture. The total kurtosis C_4 can be separated into canonical and dynamic components, Janssen (2009),

$$C_4 = C_4^{canonical} + C_4^{dynamic}. \tag{2}$$

While $C_4^{canonical}$ is always positive, $C_4^{dynamic}$ can take a negative value. Therefore, a small value of C_4 does not necessarily imply that $C_4^{dynamic}$ is negligible. The canonical kurtosis accounts for the second order nonlinearity or the Stokes correction. The dynamic kurtosis is a consequence of the modulational instability and was derived by Janssen (2003). He had shown numerically that the kurtosis increases from the initial state due to modulational instability, and was confirmed experimentally by Onorato et al. (2004). Unless the spectrum largely changes, the $C_4^{canonical}$ remains unchanged while $C_4^{dynamic}$ changes at the dynamic time scale ($O(100)$ periods).

Two freak waves were observed at the deep water mooring station JKEO (JAMSTEC Kuroshio Extension Observatory) in 2009 (Waseda et al. 2012, 2015). Separated by only a day, the two events occur under completely different sea states, one unidirectional and narrow spectrum (JKEO-Narrow) and other bimodal and broad spectrum (JKEO-Broad). Based on both the observed directional spectra and the third-generation model estimates, the HOS simulations were conducted (Fujimoto et al. 2018). Following the usual procedure, the kurtosis was estimated from the 100 ensembles, totaling 5000 wave periods. In case of the JKEO-Narrow, the total kurtosis increased gradually in 50 wave periods, while the total kurtosis remained nearly unchanged in case of the JKEO-Broad (Fig. 5). To elucidate the bound contribution and the dynamic contribution of the nonlinearity, the HOS simulations were conducted with M=2 and M=3 settings where M represents the truncation order of the nonlinearity. The exact resonance of four waves occur at M=3 only.

Onorato et al. (2007) have shown that the Hamiltonian dynamical equation is equivalent to the HOS formulation. It was shown that by Fourier transforming the M=3 HOS equation, the third order evolution equation of the free surface elevation and the surface velocity potentials derived from the Hamiltonian formalism are re-derived. Obviously, the quadratic interaction of free waves occurs at the third order. Then, it is natural to consider that the M=2 HOS solution contains only the three wave interaction. However, the Hamiltonian dynamical equation without applying the canonical transformation contains interaction of free and bound waves, thereby allowing interaction of pseudo-four-waves at M=2. Consequently, the kurtosis obtained at M=2 does not only contain the canonical Kurtosis as derived by Janssen (2009) but includes contribution from the pseudo-four-wave interaction. The reduced gravity equation (e.g. Krasitskii 1994) expresses the interaction among four free waves excluding the contributions of such contamination by bound waves:

$$i\frac{\partial a_1}{\partial t} = \omega_1 a_1 + \int T_{1,2,3,4} a_2^* a_3 a_4 \delta_{1+2-3-4} d\boldsymbol{k}_{123}. \quad (3)$$

This is because the bound contributions are incorporated in the interaction coefficient:

$$T_{1,2,3,4} = W_{1234}^{(2)} + f\left(V^{(-)}, V^{(+)}\right). \quad (4)$$

The first term $W_{1234}^{(2)}$ is called the direct interaction while $f\left(V^{(-)}, V^{(+)}\right)$ is called the virtual-state interaction (Janssen 2009). The former appears only at M=3 HOS simulation, but the latter appears at M=2 HOS simulation. As such, the total kurtosis from the M=2 HOS simulation is contaminated by the virtual-state interaction. However, the difference of the M=3 and the M=2 kurtoses is the dynamical kurtosis ${C_4}^{dynamic}$.

The total kurtoses were evaluated from the HOS simulations of the JKEO-Narrow and the JKEO-Broad cases (Fujimoto et al. 2018). The C_4 tends to increase in the case of JKEO-Narrow but remains constant for the JKEO-Broad case (Fig. 5). By taking the difference of M=3 and M=2 cases, the ${C_4}^{dynamic}$ is evaluated. For both cases, the ${C_4}^{dynamic}$ is negative. The ${C_4}^{dynamic}$ tends to increase when the directional spread is narrow (JKEO-Narrow case) but remains unchanged otherwise. That means that the modulational instability is taking place when the directional spread is small. This fact can easily be dismissed if only the total kurtosis is studied. Of course, the value of the total kurtosis remains small and therefore the alteration of the probability density (1) will be minor. This is the reason why the presence of nonlinear interaction at the third order was dismissed in most other studies. The total kurtosis is not the best indicator to identify the relevance of the nonlinear interaction.

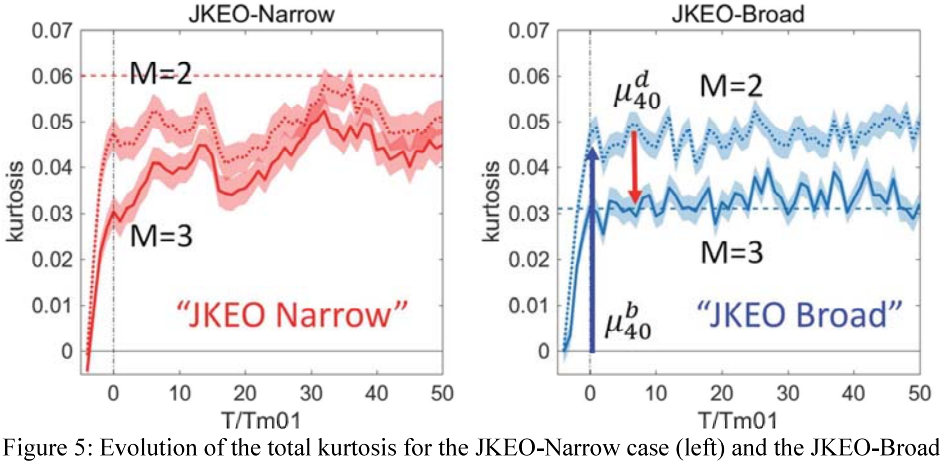

Figure 5: Evolution of the total kurtosis for the JKEO-Narrow case (left) and the JKEO-Broad case (right). The thin solid lines indicate the M=3 results and the dashed lines indicate the M=2 results. The shaded region indicates the standard deviation of the ensemble average. Adapted from Fujimoto et al. (2018) and annotated.

Two alternative indices are suggested in Fujimoto et al. (2018). First is the life time of the coherent wave groups. The wave groups that contain freak waves are identified in the HOS simulation. For the JKEO-Narrow case, relatively long-lived coherent wave groups are identified (Fig. 6 left). On the other hand, the life-time of the wave groups are much shorter in the case of JKEO-Broad and those groups are scattered in the space-time domain. Therefore, the probability density function of the life-time of the freak wave groups is much enhanced at the tail of the distribution when the directional spreading is narrow. Random superposition may explain the generation of the short-time wave groups in the case of broad spectrum, but when the directional spreading is narrow, the wave groups become more coherent and thereby the lifetime becomes longer. The presence of the wave group in a random directional wave field was noticed by Tanaka (2005) and Stansberg have pointed out that the life time of the wave group will reduce as the directional spread broadens. Indeed, the investigations of the two observed freak wave cases revealed that these hypotheses are correct.

These coherent wave groups are localized in the space-time domain. Although the wave spectrum is directionally spread, evolution of each individual wave group bares the characteristic of the solution of the NLS. Lo and Mei (1987) investigated the evolution of wave groups with Dysthe's equation (Dysthe 1979) and showed that the wave crest deforms and becomes asymmetric. The shape of the freak wave, therefore, is an indication that the modulational instability is taking place. The averaged shape of the freak waves of the JKEO-Narrow and JKEO-Broad are compared in Fig. 7 (Fujimoto et al. 2018). Apparently, the front trough is shallower than the rear trough in case of JKEO-Broad, while the JKEO-Broad case seems more symmetric. The front-rear asymmetry is one of the characteristics of the waves in a coherent nonlinear wave group (Clamond et al 2006). It is quite interesting to see how the findings from the uni-directional modulated wave train applies to the directional waves in the ocean. And what is more profound is that this asymmetry appears only when the directional spread was narrow. This fact is closely related to the evolution of waves described by the envelope equations (NLS, Dysthe etc.) and is pronounced when the directional spread is narrow. The asymmetry was not as strongly affected by the lifetime. As a final remark, the short lifetime freak waves possibly due to random superposition and the long lifetime freak waves possibly due to nonlinear focusing coexist and their distribution depends on the spectral broadness.

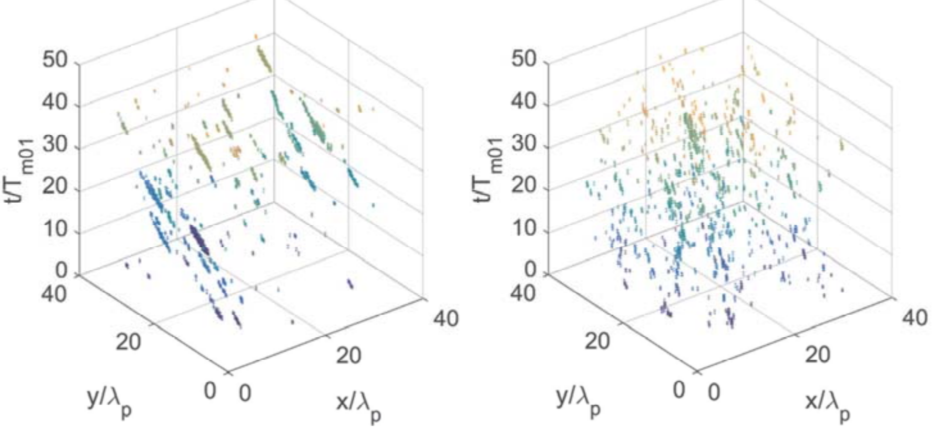

Figure 6: Freak wave groups identified in the HOS simulation for the JKEO-Narrow case (left) and the JKEO-Broad case (right). Color indicates indices assigned to each freak wave group. Reproduced from Fujimoto et al. (2018).

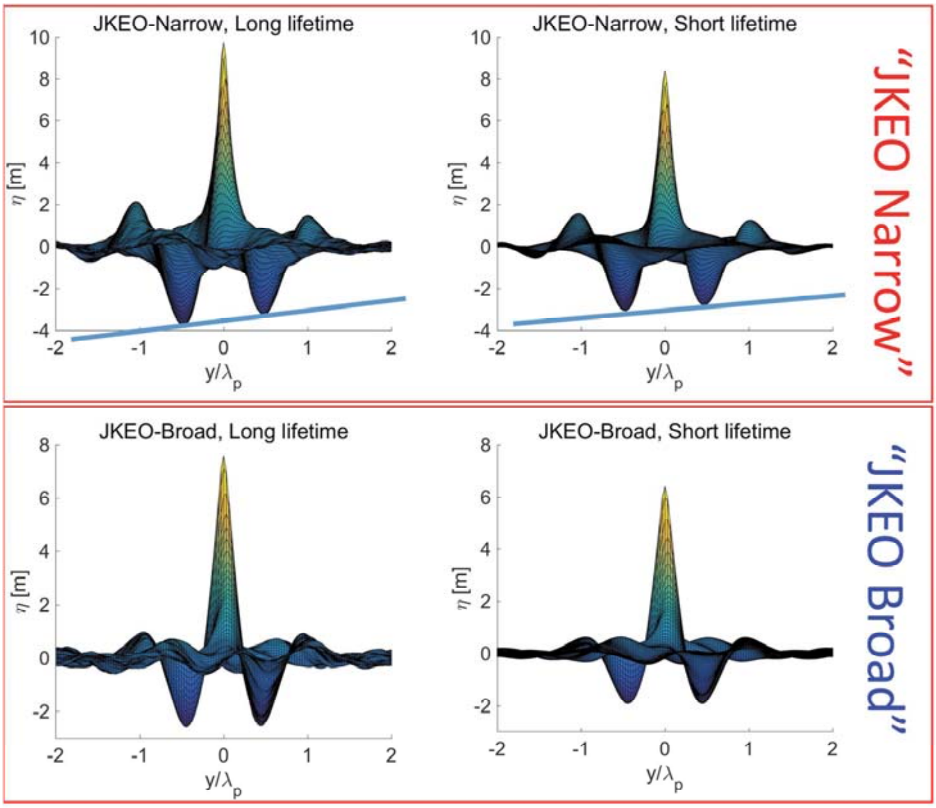

Figure 7: Side views of average freak wave shapes $\bar{\eta}(x)$ for JKEO-Narrow (upper) and JKEO-Broad (lower) ensemble HOS simulation results. The propagation directions are rotated such that the mean wave directions are from left to right. Left and right columns correspond to long-lifetime and short-lifetime freak waves, respectively. Figures are adapted from Fujimoto et al. (2018) and modified.

It is clear that modulational instability plays an important role when the directional distribution is narrow. Then the question is how often such directionally confined spectra are realized in the ocean. During the marine accident cases near Japan, hindcast analyses revealed that the directional spectrum narrowed preconditioned by different meteorological causes. One of the most dynamically interesting case was the nonlinear interaction of the swell system and windsea, such that the swell system grew at an expense of the energy of the windsea propagating at an angle (Tamura et al. 2009). Another case was related to the running fetch condition of the gale system (Waseda et al. 2012) and others were related to a variety of meteorological conditions (Waseda et al. 2014). A systematic study of an idealized typhoon was conducted by Mori (2012) and showed that certain quadrant of the typhoon affected sea resulted in a directionally narrow spectrum.

Here we give an example of another violent weather system called the Explosive Cyclone (EC) or the Bomb Cyclone. The EC is a midlatitude cyclone that grow rapidly due to excessive heat and moisture flux, typically at a rate higher than 24 hPa decrease within 24 hours. The 21 year hindcast simulation (TodaiWW3, Waseda et al. 2016, Webb et al. 2016) was analyzed identifying typical as well as composited wave systems under EC (Kita et al.

2018). The presence of fronts and the rapid translation speed distinguishes the wave development from that under influence of typhoon. Notably, a delay in the growth of the windsea with respect to the minimum pressure, and two isolated areas where the directional spectrum narrow. The spatial distributions of the significant wave height, wind speed, directional spreading and frequency bandwidth are depicted in Fig. 8 for one of the strongest ECs. The directionally narrow and steep wave systems are realized in two locations, Zone A and B as indicated in the inserted schematic. Zone A is located behind the cold front associated with the dry conveyor belt wind system. Whereas zone B is located along the warm front associated with the cold conveyer belt wind system. These two locations are robust as they commonly exist within a large number of ECs studied in Kita et al. (2018). It is of interest to see if the occurrence probability of the freak waves increase at these locations. An attempt was made comparing the observed wave records with the hindcasted wave field in the North Sea revealing that the occurrence probability enhances when the directional spectrum narrows (Waseda et al. 2012). Further study is warranted.

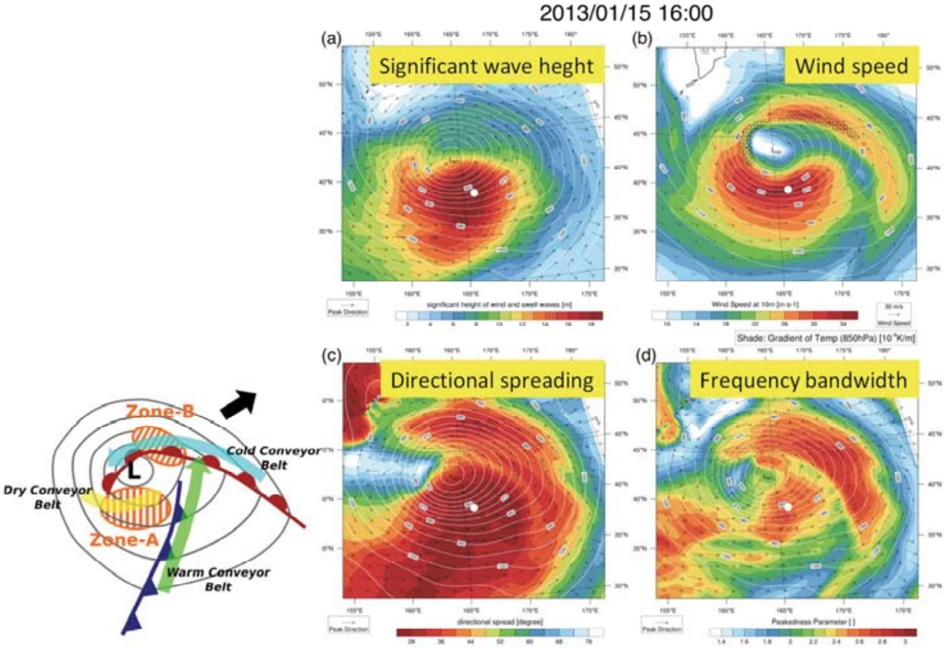

Figure 8: Snapshots of ocean wave and atmosphere fields at 6:00 UTC on January 15th, 2013. Color scales are used to display (a) H_s, (b) U_10, (c) σ_θ, (d) Q_p. White contours in all diagrams show SLP. The inserted schematic in the left illustrates ocean waves under EC conditions at most mature development stage. Nomenclature is provided in Kita et al. (2018) where these images are adapted from.

5 Concluding remark

In the last 20years, freak wave research involved researchers from the oceanography, engineering and physics communities and evolved into an interdisciplinary research subject. As an outcome, oil and gas industry has now incorporated the “freak wave” conditions in their design criterion and the classification society has now started to consider incorporating the “freak wave” condition as well. In the meantime, physical community has largely shifted their interest to the study of optical rogue wave that is a much cleaner and controlled media to study rogue wave generation mechanism in the context of NLS. So, what have we learned about freak waves in the ocean?

Unfortunately, the community is now largely split into two groups; one that considers linear superposition of random waves as the main or the only mechanism, and the other that considers modulational instability is important. This is a rather unfortunate situation as the issue is not a black or white question. Some freak waves are generated due to random focusing and some others are generated due to modulational instability. It seems that a study based on a selected observation cannot prove anything.

Our opinion is summarized below:

a) Relatively small occurrence probability of freak waves does not necessarily imply lack of nonlinear process in the formation of freak waves
b) For any sea state, freak waves due to modulational instability and linear focusing coexist
c) Spectral narrowing occurs under certain meteorological condition

It seems that there is still a considerable lack of observational evidence as to prove what mechanism is responsible for the freak wave generation in the real ocean. Study of an extensive wave records such as Christou and Evans (2014) should be revisited and the conditional analysis similar to the one conducted by Waseda et al. (2012) should be applied. The spatio-temporal measurements such as Benetazzo et al. (2015) should be revisited as well extending its capability to visualize a much broader area containing the wave group. And last but not least, the directional spectral estimate of both by the third generation wave model and the in-situ and satellite observations should be revisited as they both tend to show a broader spectral peak which is crucial in understanding the dynamics of freak wave generation.

Acknowledgement

The first author congratulates Prof. Tanaka for his distinguished accomplishments in water wave research. It was an honor to be invited to participate in this wonderful workshop organized by Prof. Murashige.

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Department of Ocean Technology Policy and Environment,
Graduate school of Frontier Sciences,
the University of Tokyo
Kashiwa 277-8563
JAPAN
E-mail address: waseda@k.u-tokyo.ac.jp