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Instanton Floer theory and the homology cobordism group

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In this article, we give a review of the homology cobordism invariants $\{r_s\}$ introduced by the authors in [NST19].

1 Backgrounds

1.1 Cobordism problems in low dimensional topology

The cobordism theory plays a central role in differential topology. For example, in dimension grater than 4, the orientation preserving diffeomorphism can be expressed by the homotopy cobordism under some conditions. In this paper, we focus on manifolds with dimension smaller than 5. The main contents of [NST19] are analyzing the 3-dimensional homology cobordism group and knot concordance group. First we give the definition of the homology cobordism group.

Definition 1.1 (The homology cobordism group)**.** Oriented homology 3-spheres Y_0 and Y_1 are *homology cobordant* (denoted $Y_0 \sim_{\mathbb{Z}H} Y_1$) if there exists a compact oriented 4-manifold W with $\partial W = Y_0 \amalg (-Y_1)$ such that the maps $H_*(Y_i;\mathbb{Z}) \to H_*(W;\mathbb{Z})$ induced by the inclusions $Y_i \to W$ $(i = 0, 1)$ are isomorphisms. Then, the quotient set

$$\Theta^3_{\mathbb{Z}} := \{\text{homology 3-spheres}\}/\sim_{\mathbb{Z}H}$$

with connected sum operation is an abelian group. We call $\Theta^3_{\mathbb{Z}}$ *the homology cobordism group.*

To detect the group structure of $\Theta^3_{\mathbb{Z}}$ is an open problem in low dimensional topology. There is the following preceding study related to $\Theta^3_{\mathbb{Z}}$;

- Any topological manifold M with dim ≥ 5 admits a triangulation $\iff$ $0 = \exists\delta(\Delta(M)) \in H^5(M, \mathrm{Ker}\,\mu)$, where $\mu : \Theta^3_{\mathbb{Z}} \to \mathbb{Z}_2$ is the Rokhlin homomorphism. ([GS80], [Mat78])

Next, we give the definition of the knot concordance group.

Definition 1.2 (The knot concordance group)**.** Oriented knots K_0 and K_1 in S^3 are *concordant* (denoted $K_0 \sim_c K_1$) if there exists an embedding $J\colon S^1 \times [0,1] \to S^3 \times [0,1]$ such that $J(S^1 \times \{i\}) = K_i \times \{i\}$ $(i = 0, 1)$. Then, the quotient set

$$\mathcal{C} := \{\text{oriented knots in } S^3\}/\sim_c$$

with connected sum operation is an abelian group. We call $\mathcal{C}$ *the knot concordance group.*

To detect the group structure of $\mathcal{C}$ is also open. Moreover, there are similar preceding studies as in the case of $\Theta^3_{\mathbb{Z}}$.

- The n-dimensional knot concordance group $\mathcal{C}^n$ is completely determined for $n \neq 1$. ([Lev69])
- The group $\Theta^3_{\mathbb{Q}}$ is defined by replacing $\mathbb{Z}$ with $\mathbb{Q}$ in the definition of $\Theta^3_{\mathbb{Z}}$. Taking the double branched cover gives a homomorphism
$$\Sigma : \mathcal{C} \to \Theta^3_{\mathbb{Q}}.$$

1.2 Yang-Mills gauge theory and $\Theta^3_{\mathbb{Z}}$

Here, we review preceding studies related to [NST19]. Here, $\Sigma(p,q,r)$ denotes the (p,q,r)-Brieskorn sphere.

- In 1983, Donaldson [Don83] showed Theorem A for simply connected negative definite 4-manifolds. Then Furuta [Fur87] generalized this theorem to the case of $H_1(X,\mathbb{Z}) = 0$. This implies that $\Sigma(2,3,5)$ is not a torsion in $\Theta^3_{\mathbb{Z}}$.
- In 1985, Fintushel-Stern [FS85] developed orbifold gauge theory and showed that $\Sigma(p,q,pqk-1)$ is not a torsion in $\Theta^3_{\mathbb{Z}}$ for any coprime pair (p,q) and positive integer k.
- In 1990, Furuta [Fur90] and Fintushel-Stern [FS90] developed gauge theory for orbifolds with cylindrical ends and showed that $\{\Sigma(p,q,pqk-1)\}_{k=1}^{\infty}$ are linearly independent in $\Theta^3_{\mathbb{Z}}$.

These results can be reproved by using the invariants $\{r_s(Y)\}$.

2 Main theorem

2.1 The invariants r_s

In [NST19], we gave a new family of homology cobordism invariants $\{r_s(Y)\}$ of homology 3-spheres.

Theorem 2.1. *For any $s \in \mathbb{R}_{\leq 0} \cup \{-\infty\}$ and oriented homology 3-sphere Y, we define $r_s(Y) \in \mathbb{R}_{>0} \cup \{\infty\}$ satisfying the following properties:*

1. *If $s \leq s'$, then $r_{s'}(Y) \leq r_s(Y)$.*
2. *The value $r_s(Y)$ is contained in the set of critical values of the Chern-Simons functional of Y.*
3. *Let Y_0 and Y_1 be homology 3-spheres and W a negative definite cobordism with $\partial W = Y_0 \amalg -Y_1$. Then $r_s(Y_1) \leq r_s(Y_0)$ holds for any s. Moreover, if $\pi_1(W) = 1$ and $r_s(Y_0) < \infty$, then $r_s(Y_1) < r_s(Y_0)$ holds.*

4. *The invariant* r_0 *satisfies*

$$r_0(Y_1 \# Y_2) \geq \min\{r_0(Y_1), r_0(Y_2)\}.$$

5. *The value* $r_{-\infty}(Y)$ *is finite if and only if* $h(Y) < 0$ *holds, where* $h\colon \Theta^3_{\mathbb{Z}} \to \mathbb{Z}$ *is the Frøyshov homomorphism [Frø02].*

2.2 Remark for r_s

- Recently, using instanton Floer theory, Daemi [Dae18] introduced a family of $(\mathbb{R}_{\geq 0} \cup \{\infty\})$-valued invariants of Y parametrized by $\mathbb{Z}$:

$$\cdots \leq \Gamma_Y(-1) \leq \Gamma_Y(0) \leq \Gamma_Y(1) \leq \cdots .$$

  Note that $\Gamma_Y(k)$ also satisfies the properties 2, 3 and 5 in Theorem 2.1 for any positive k. The invariants $\{r_s\}$ can be regarded as a one-parameter family converging to $\gamma_{-Y}(1)$. Precisely, the authors prove in [NST19, Section 4] that

$$r_0(Y) \leq \cdots \leq r_s(Y) \leq \cdots \leq r_{-\infty}(Y) = \Gamma_Y(1).$$

- There exists an example of Y such that $r_s(Y)$ is not constant with respect to s. Indeed, we can verify by combining the following computations results, Theorem 2.1 and the fact that $h(\Sigma(2,3,6k-1)) = 1$ for any positive k that $n\Sigma(2,3,5)\#(-\Sigma(2,3,6k+5))$ has non-constant r_s for any positive n and k.

3 Computations

Roughly speaking, $r_0(Y)$ is given by

$$\inf\left\{-\frac{1}{8\pi^2}\int_{Y\times\mathbb{R}} \mathrm{Tr}(F(A)\wedge F(A)) \;\middle|\; A \in \Omega^1_{Y\times\mathbb{R}} \otimes \mathfrak{su}(2) \text{ with } (*)\right\}$$
$$= \inf\left\{\mathrm{cs}(b) \;\middle|\; A \in \Omega^1_{Y\times\mathbb{R}} \otimes \mathfrak{su}(2) \text{ with } (*), b = \exists \lim_{t\to-\infty} A|_{Y\times\{t\}}\right\}$$

The condition $(*)$ is

- $0 = \exists \lim_{t\to\infty} A|_{Y\times\{t\}}$.
- $\exists$ Riemann metric g on Y such that the ASD-equation $\frac{1}{2}(1 + *_{g+dt^2})F(A) = 0$ is satisfied.
- The Fredholm index of the operator $d^+_A + d^*_A$ on $Y \times \mathbb{R}$ is 1.

Example 3.1. $r_s(S^3) = \infty$ for any s.

Theorem 3.2. *For any s, the equality*

$$r_s(-\Sigma(p,q,pqn-1)) = \frac{1}{4pq(pqn-1)}$$

holds.

More generally, we can see

$$\bigcup_s r_s(\Theta^3_S) \subset \mathbb{Q}_{>0} \cup \{\infty\},$$

where Θ^3_S is the subgroup of $\Theta^3_{\mathbb{Z}}$ generated by Seifert homology 3-spheres. We tried to compute r_s for the hyperbolic manifold $S^3_{1/2}(5^*_2)$, which is obtained by the 1/2-surgery along the mirror image of 5_2 in Rolfsen's table.

Theorem 3.3. *By the computer, for any s,*

$$r_s(S^3_{1/2}(5^*_2)) \approx 0.0017648904\ 7864885113\ 0739625897\ 0947779330\ 4925308209,$$

where its error is at most 10^{-50}.

Our computation is based on Kirk and Klassen's formula (to be explained later).

Conjecture 3.4. $r_s(S^3_{1/2}(5^*_2))$ is irrational.

If the conjecture is true, we can conclude that $\Theta^3_{\mathbb{Z}}/\Theta^3_S$ is non-trivial.

4 Applications

4.1 Useful lemmas

We first introduce several lemmas which are useful for applying $\{r_s(Y)\}$ to concrete problems. All of them directly follows from Theorem 2.1. (See [NST19, Section 5.1] for details.)

Lemma 4.1. *Let $\{Y_n\}_{n=1}^{\infty}$ be a sequence of oriented homology 3-spheres satisfying the following two conditions:*

- $r_0(Y_1) > r_0(Y_2) > \dots$ *and*
- $r_0(-Y_n) = \infty$ *for any n.*

Then the sequence $\{Y_n\}_{n=1}^{\infty}$ are linearly independent in both $\Theta^3_{\mathbb{Z}}$ and $\Theta^3_{\mathbb{Q}}$.

Lemma 4.2. *Let Y_0 and Y_1 be homology 3-spheres and W a negative definite cobordism with $\partial W = Y_0 \amalg -Y_1$. If $\pi_1(W) = 1$ and $r_0(Y_0) < \infty$, then $r_0(Y_1) < r_0(Y_0)$ holds.*

Lemma 4.3. *If Y bounds a negative definite 4-manifold, then $r_0(Y) = \infty$ holds.*

4.2 Three applications of $\{r_s\}$

Here we introduce three applications of $\{r_s\}$ to low-dimensional cobordism problems. First, we give an infinite family of homology 3-spheres with no definite bounding.

Theorem 4.4. *There exist infinitely many homology 3-spheres $\{Y_k\}$ such that Y_k does not admit any definite bounding.*

Proof. Set $Y_k := 2\Sigma(2,3,5)\#(-\Sigma(2,3,6k+5))$ $(k \geq 1)$. Then using connected sum formula, we have $r_0(Y_k) = \frac{1}{24(6k+5)} < \infty$. Moreover, the calculation $h(-Y_k) = -1$ implies that $r_0(-Y_k) < \infty$. □

Corollary 4.5. *$[Y_k]$ does not contain any Seifert homology 3-sphere and homology 3-sphere obtained by a surgery on a knot in S^3.*

Proof. It is known that all Seifert homology 3-spheres and homology 3-spheres obtained by surgeries on knots admit a definite bounding. □

Second, we give a sufficient condition for the linear independence of positive $1/n$-surgeries on a knot.

Theorem 4.6. *For any knot K in S^3 with $h(S^3_1(K)) < 0$, the sequence $\{S^3_{1/n}(K)\}_{n=1}^{\infty}$ are linearly independent in $\Theta^3_{\mathbb{Z}}$.*

Since $\{\Sigma(p,q,pqn-1)\}_{n=1}^{\infty}$ are the $1/n$-surgeries of the (p,q)-torus knot $T_{p,q}$, this theorem is a generalization of the result of Furuta [Fur90] and Fintushel-Stern [FS90]. Moreover, we can find an infinite family of hyperbolic knots and satellite knots respectively such that the Frøyshov invariants of their 1-surgeries are negative. (This fact is shown in [NST19, Section 5.3].)

Proof of Theorem 4.6. Set $Y_n := S^3_{1/n}(K)$. The fifth and third properties of r_0 imply $r_0(Y_1) < \infty$ and $r_0(-Y_n) = \infty$. Moreover, we can construct a simply connected positive definite cobordism W_n with $\partial(W_n) = -Y_n \amalg (Y_{n+1})$. (The construction is shown in [NST19, Section 5.3].) Therefore, the third property of r_0 implies that

$$r_0(Y_1) > r_0(Y_2) > \cdots .$$

This fact and Lemma 4.1 proves the theorem. □

Third, we prove the linear independence of an infinite family of Whitehead doubles in $\mathcal{C}$. (The concordance problem among Whitehead doubles are interesting because all Whitehead doubles are topologically slice.) Let $D_{p,q}$ denote the Whitehead double of $T_{p,q}$. Hedden-Kirk proved the following theorem.

Theorem 4.7 ([HK12])**.** *$\{D_{2,2^n-1})\}_{n=2}^{\infty}$ are linearly independent in $\mathcal{C}$.*

The invariants $\{r_s\}$ enables us to refine the above theorem as follows.

Theorem 4.8. *For any coprime integers $p, q > 0$, $\{D_{p,np+q}\}_{n=0}^{\infty}$ are linearly independent in $\mathcal{C}$. In particular, $\{D_{2,2n-1}\}_{n=2}^{\infty}$ are linearly independent in $\mathcal{C}$.*

Proof. Recall that taking double branched cover gives a homomorphism

$$\Sigma\colon \mathcal{C} \to \Theta^3_{\mathbb{Q}},$$

and hence it is sufficient to prove $\{\Sigma(D_{p,kp+q})\}_{k=1}^{\infty}$ are linearly independent in $\Theta^3_{\mathbb{Q}}$. Moreover, since $\Sigma(D_{p,q}) = S^3_{1/2}(T_{p,q}\# T_{p,q})$ are homology 3-spheres, we only need to prove

- $r_0(\Sigma(D_{p,q})) < \infty$, and
- $r_0(\Sigma(D_{p,q})) > r_0(\Sigma(D_{p,p+q}))$

for any coprime $p, q > 0$. To prove the above assertions, we construct

- negative definite cobordism with boundary $\Sigma(p, q, 2pq-1)) \amalg (-\Sigma(D_{p,q}))$, and
- simply connected negative definite cobordism with boundary $\Sigma(D_{p,q}) \amalg (-\Sigma(D_{p,p+q}))$.

Here we mention that both of the above cobordisms are obtained by the following lemma.

Lemma 4.9. *If a knot K_0 is deformed into a knot K_1 by finitely many crossing changes from positive crossings to negative crossing, then there exists a negative definite cobordism with boundary $S^3_{1/n}(K_1) \amalg (-S^3_{1/n}(K_0))$ for any $n \in \mathbb{Z}$.*

□

For more details, see [NST19, Section 5.4].

5 Construction of r_s

In this section, we give a rough construction of $\{r_s(Y)\}$.

Let Y be an oriented homology 3-sphere.

- In 1988, Floer [Flo88] introduced instanton homology $I_*(Y)$ with $* \in \mathbb{Z}/8\mathbb{Z}$.
- In 1992, Fintushel-Stern [FS92] introduced a filtered version of instanton homology $I_*^{[r,r+1]}(Y)$ with $* \in \mathbb{Z}$ for any $r \in \mathbb{R}$.
- In 2002, Donaldson [Don02] defined the obstruction class $[\theta_Y] \in I^1(Y)$. If Y admits a positive definite bounding with non-standard intersection form, then $0 \neq [\theta_Y] \in I^1(Y;\mathbb{Q})$.
- In 2019, the authors [NST19] defined a filtered instanton cohomology $I^*_{[s,r]}(Y)$ and the filtered version $[\theta_Y^{[s,r]}] \in I^*_{[s,r]}(Y;\mathbb{Q})$ of the obstruction class.

Then we can give a formal definition of $\{r_s\}$.

Definition 5.1. For an oriented homology 3-sphere Y, we set

$$r_s(Y) := \sup\{r \in \mathbb{R} \mid 0 = [\theta_Y^{[s,r]}] \in I^*_{[s,r]}(Y;\mathbb{Q})\}$$

for any $s \in \mathbb{R}_{\leq 0} \cup \{-\infty\}$.

6 Open problems

6.1 Critical values of Chern-Simons functionnal

Daemi's invariants $\{\Gamma(k)\}$ [Dae18] and the authors invariants $\{r_s\}$ give much information of $\Theta^3_{\mathbb{Z}}$. On the other hand, these invariants are lying in the critical values of the Chern-Simons functional cs. These facts give a new motivation to compute the critical values of cs. However, it is hard to determine the values in general. For instance, the following is a famous open problem for cs.

Problem 6.1. Is there a homology 3-sphere Y such that the set of critical values of cs contains an irrational value?

As a concrete example, it is known that the Poincaré sphere $\Sigma(2,3,5)$ has

$$\left\{ m - \frac{1}{120},\ m - \frac{49}{120} \,\middle|\, m \in \mathbb{Z} \right\}$$

as the set of the critical values of cs. By using this fact, we can compute

$$\Gamma_{\Sigma(2,3,5)}(1) = r^+(-\Sigma(2,3,5)) = \frac{1}{120} \text{ and } \Gamma_{\Sigma(2,3,5)}(2) = \frac{49}{120}.$$

Question 6.2. Denote by Θ^3_S the subgroup of $\Theta^3_{\mathbb{Z}}$ generated by Seifert homology 3-spheres. Then, is the quotient group $\Theta^3_{\mathbb{Z}}/\Theta^3_S$ non-trivial?

Here we mention that our invariant $r_0\colon \Theta^3_{\mathbb{Z}} \to \mathbb{R}_{\geq 0} \cup \{\infty\}$ is related to the above problem. In fact, the value $r_0(Y)$ is contained in the set of critical values of cs, and if Y is a linear combination of Seifert homology 3-spheres, then the set of critical values of cs is contained in $\mathbb{Q}$. These imply that if a homology 3-sphere Y has irrational r_0, then its homology cobordism class $[Y]$ is not contained in Θ^3_S. On the other hand, by Mathematica, the authors estimated the value $r_0(S^3_{1/2}(5^*_2))$ with an error of at most 10^{-50}. It is known that $S^3_{1/2}(5^*_2)$ is a hyperbolic manifold [BW01]. The result seems to imply that $r_0(S^3_{1/2}(5^*_2))$ is irrational. If the value $r_0(S^3_{1/2}(5^*_2))$ is truly irrational, then we can conclude that $[S^3_{1/2}(5^*_2)] \notin \Theta^3_S$.

Question 6.3. Is the value $r_0(S^3_{1/2}(5^*_2))$ irrational?

The method of our computation is based on Kirk and Klassen's formula of *cs* given by the integration along a path in the space of irreducible $SL(2,\mathbb{C})$-representations. To obtain the approximate value of r_0, we use a description of the space of $SL(2,\mathbb{C})$-representations of $\pi_1(S^3 \setminus 5_2)$ in terms of a *Riley polynomial* $\phi(t,u) \in \mathbb{Z}[t^{\pm 1}, u]$ with $\deg_u \phi = 3$. Then we can explicitly solve the equation $\phi(u,t) = 0$ with respect to u and use the solutions to compute r_0. However, Riley polynomials $\phi(t,u)$ of 2-bridge knots K might be of degree larger than 4. In this case, one cannot solve $\phi(t,u) = 0$ in general.

Problem 6.4. In the case $\deg_u \phi > 4$, give a method to compute an approximate value of $r_0(S^3_{1/n}(K))$.

6.2 Homology cobordism group of homology 3-spheres

The following problem is open for $\Theta^3_{\mathbb{Z}}$.

Problem 6.5. Is there a torsion in $\Theta^3_{\mathbb{Z}}$?

As examples which may be torsion elements, we see that the splice $S(K, -K^*)$ of any oriented knot K and its orientation reversed mirror $-K^*$ has at most order two in $\Theta^3_{\mathbb{Z}}$.

Problem 6.6. For any oriented knot K, is the splice $S(K, -K^*)$ trivial in $\Theta^3_{\mathbb{Z}}$?

The invariants $\{r_s\}$ and $\{\Gamma(k)\}$ are possibly non-trivial for $S(K, -K^*)$. In this sense, the following problem is meaningful.

Problem 6.7. Give formulas of r_s, $\Gamma(k)$ and cs for the splice of knots.

Next, we mention a problem for comparison among other Floer theories. Recently, by using the involutive Floer theory, Dai-Hom-Stoffregen-Truong [DHST18] show that $\Theta^3_{\mathbb{Z}}$ has a $\mathbb{Z}^\infty$-summand.

Problem 6.8. Is there an instanton theoretic proof of the result?

6.3 Problem related to instanton Floer theory

Although the group $I_*(Y)$ is the first example of Floer homology groups for 3-manifolds, even the following fundamental problem is still open.

Problem 6.9. Construct a well-defined equivariant instanton Floer homology for $SU(2)$-bundles on all 3-manifolds.

The main problems are to deal with the reducible solutions and the dependence of perturbations. For example, the dependence of perturbations made in [AB96] is still open. We also mention a problem related to Floer homotopy types introduced in [CJS95]. It is known that several Floer theoretical invariants of 3- or 4-manifolds are obtained as the singular homology of some topological objects, and the stable homotopy types of the topological objects themselves are invariants of 3- or 4-manifolds. Thus, the homotopy type is called *the Floer homotopy type* ([Man03, LS14]). For the group $I_*(Y)$, its Floer homotopy type has been unknown.

Problem 6.10. Construct a Floer homotopy type of $I_*(Y)$.

The main problems to define an instanton Floer homotopy type are related to the bubble phenomena and the existence of structures of manifolds with corners on the compactification of moduli spaces of trajectories and the framings. If the problem is solved, we can apply a generalized cohomology theory and obtain a family of invariants.

References

[AB96] David M. Austin and Peter J. Braam. Equivariant Floer theory and gluing Donaldson polynomials. *Topology*, 35(1):167–200, 1996.

[BW01] Mark Brittenham and Ying-Qing Wu. The classification of exceptional Dehn surgeries on 2-bridge knots. *Comm. Anal. Geom.*, 9(1):97–113, 2001.

[CJS95] R. L. Cohen, J. D. S. Jones, and G. B. Segal. Floer's infinite-dimensional Morse theory and homotopy theory. In *The Floer memorial volume*, volume 133 of *Progr. Math.*, pages 297–325. Birkhäuser, Basel, 1995.

[Dae18] Aliakbar Daemi. Chern-simons functional and the homology cobordism group, 2018, arXiv:1810.08176.

[DHST18] Irving Dai, Jennifer Hom, Matthew Stoffregen, and Linh Truong. An infinite-rank summand of the homology cobordism group, 2018, arXiv:1810.06145.

[Don83] S. K. Donaldson. An application of gauge theory to four-dimensional topology. *J. Differential Geom.*, 18(2):279–315, 1983.

[Don02] S. K. Donaldson. *Floer homology groups in Yang-Mills theory*, volume 147 of *Cambridge Tracts in Mathematics*. Cambridge University Press, Cambridge, 2002. With the assistance of M. Furuta and D. Kotschick.

[Flo88] Andreas Floer. An instanton-invariant for 3-manifolds. *Comm. Math. Phys.*, 118(2):215–240, 1988.

[Frø02] Kim A. Frøyshov. Equivariant aspects of Yang-Mills Floer theory. *Topology*, 41(3):525–552, 2002.

[FS85] Ronald Fintushel and Ronald J. Stern. Pseudofree orbifolds. *Ann. of Math. (2)*, 122(2):335–364, 1985.

[FS90] Ronald Fintushel and Ronald J. Stern. Instanton homology of Seifert fibred homology three spheres. *Proc. London Math. Soc. (3)*, 61(1):109–137, 1990.

[FS92] Ronald Fintushel and Ronald J. Stern. Integer graded instanton homology groups for homology three-spheres. *Topology*, 31(3):589–604, 1992.

[Fur87] Mikio Furuta. Perturbation of moduli spaces of self-dual connections. *J. Fac. Sci. Univ. Tokyo Sect. IA Math.*, 34(2):275–297, 1987.

[Fur90] Mikio Furuta. Homology cobordism group of homology 3-spheres. *Invent. Math.*, 100(2):339–355, 1990.

[GS80] David E. Galewski and Ronald J. Stern. Classification of simplicial triangulations of topological manifolds. *Ann. of Math. (2)*, 111(1):1–34, 1980.

[HK12] Matthew Hedden and Paul Kirk. Instantons, concordance, and Whitehead doubling. *J. Differential Geom.*, 91(2):281–319, 2012.

[Lev69] J. Levine. Invariants of knot cobordism. *Invent. Math. 8 (1969), 98–110; addendum, ibid.*, 8:355, 1969.

[LS14] Robert Lipshitz and Sucharit Sarkar. A Khovanov stable homotopy type. *J. Amer. Math. Soc.*, 27(4):983–1042, 2014.

[Man03] Ciprian Manolescu. Seiberg-Witten-Floer stable homotopy type of three-manifolds with $b_1 = 0$. *Geom. Topol.*, 7:889–932, 2003.

[Mat78] Takao Matumoto. Triangulation of manifolds. In *Algebraic and geometric topology (Proc. Sympos. Pure Math., Stanford Univ., Stanford, Calif., 1976), Part 2*, Proc. Sympos. Pure Math., XXXII, pages 3–6. Amer. Math. Soc., Providence, R.I., 1978.

[NST19] Yuta Nozaki, Kouki Sato, and Masaki Taniguchi. Filtered instanton floer homology and the homology cobordism group, 2019, arXiv:1905.04001.

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