ANOMALOUS SCALING REGIME FOR ONE-DIMENSIONAL MOTT VARIABLE-RANGE HOPPING

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Abstract. The Mott variable range model is a random walk on a random point process, where jumps occur at rate one and the conductance between two points decays exponentially in their distance. We discuss the regime where the density of points is low, resulting in large gaps that act as a blocking mechanism for the random walk. We identify the scaling limit under the appropriate subdiffusive rescaling. Joint work with David Croydon and Ryoki Fukushima.

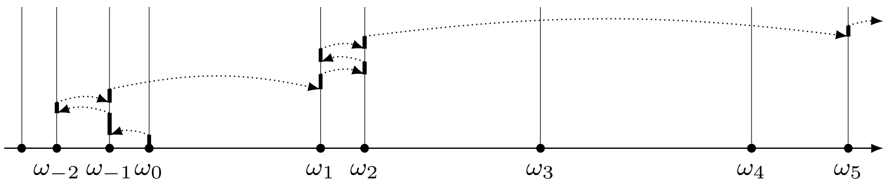
Mott variable range hopping was introduced to model electron transport in a disordered medium. Let $((\omega_i)_{i\in\mathbb{Z}}, \mathbf{P})$ denote the atoms of a one-dimensional Poisson process of intensity $\rho > 0$, conditioned on $\omega_0 = 0$. Define conductances

$$c(\omega_i, \omega_j) := e^{-|\omega_i - \omega_j|}$$

between the sites, and let (X, P_ω) denote the Markov process with generator

$$(Lf)(\omega_i) := \sum_{j\neq i} \frac{c(\omega_i, \omega_j)}{c(\omega_i)}(f(\omega_j) - f(\omega_i)),$$

where $c(\omega_i) := \sum_{j\neq i} c(\omega_i, \omega_j)$. That is, X is a reversible random walk on ω with constant speed and unbounded jump range.



The process is known to satisfy an invariance principle if the point process ω has sufficiently high density.

Theorem 1 (Caputo, Faggionato [1]). *For $\rho > 1$, there exists $\sigma(\rho) \in (0, \infty)$ such that, under P_ω for almost all ω,*

$$(n^{-1}X_{tn^2})_{t\geq 0} \xrightarrow[n\to\infty]{d} (\sigma(\rho)B_t)_{t\geq 0},$$

where B is Brownian motion.

In contrast, in the low density regime $\rho < 1$ the inhomogeneity of the environment persists asymptotically, leading to an anomalous, sub-diffusive scaling limit. To state the result we introduce the annealed law $\mathbb{P}$ of the process,

$$\mathbb{P}(\cdot) := \int P_\omega(\cdot)\mathbf{P}(\mathrm{d}\omega).$$

Theorem 2 (Croydon, Fukushima, J. [2]). *For $\rho < 1$, there exists a stochastic process Z such that, under the annealed law $\mathbb{P}$,*

$$(n^{-1}X_{tn^{1+1/\rho}})_{t\geq 0} \xrightarrow[n\to\infty]{d} (Z_t)_{t\geq 0}.$$

We prove the result for a more general version of the above dynamics which allows a small (vanishing) drift and where each atom of ω is associated with a random energy mark. The limiting process Z has an explicit representation in terms of a Brownian motion and an independent ρ-stable subordinator. We obtain some path-properties of Z and we show that $\mathbb{P}$ cannot be replaced by the quenched law.

The scaling limit Z can be regarded as something of a dual to the Fontes-Isopi-Newman (FIN) diffusion from [3]. Motivated by this observation we discuss a variation of the process where the holding time at each atom of ω is chosen randomly according to a heavy-tailed distribution. We prove that there exists a limit process which has both the "blocking" behavior observed in Theorem 2 and the "trapping" behavior from the FIN diffusion.

References

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[3] L. R. G. Fontes, M. Isopi, and C. M. Newman. Random walks with strongly inhomogeneous rates and singular diffusions: convergence, localization and aging in one dimension. *Ann. Probab.*, 30(2):579–604, 2002.

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