SMART Assessment in Geometry using DGS and LMS

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1 Introduction

This short report is about the recent progress of the Cinderella software system for Interactive Geometry, in particular its JavaScript-version CindyJS [6], with respect to the logging of user events for the implementation of automatics scoring of exercises and formative assessment. It is summarizing the presentation I gave at the RIMS conference and is not meant to be authorative, but as a contribution to the future development necessary when using dynamic geometry software (DGS) in learning management systems (LMS) like moodle. It is in line with previous presentations at RIMS workshops:

2003-11-22 (Teaching Mathematics with シンデレラ). At this conference about teaching geometric algebra using interactive tools, the first Japanese version of the Interactive Geometry Software Cinderella [17] was shown, as well as a preview of the next version Cinderella.2, which was released three years later in 2006. Also, it included a demonstration of using Cinderella on mobile devices, for example the SHARP Zaurus SL-5500 and SL-C700. The full documentation is available in [4].

2014-09-03 (Future Developments in 3D Input for Dynamic Geometry or Physics Simulation). This presentation focussed on the use of sensors, actuators and mobile devices with Cinderella to use real data and control real devices for realistic mathematics teaching.

2016-09-28 (Parallelisms ー Accessible High Performance Dynamic Mathematics on the Desktop and on Mobile Devices). This contribution, as documented in [8], introduced CindyGL, an extension to the JavaScript-Version CindyJS [6] of Cinderella that uses WebGL for parallel computation and real-time processing of image data.

For more than 20 years, the Cinderella project evolved to use the current technology, while maintaining the strong mathematical foundation [11]. However, some of the initial features, in particular interactive exercises with automated theorem checking, are no longer as accesible as they were in the first Java versions that could run in a web browser [14]. This is contradictory to the widespread adoption of learning management systems like *moodle*, which allow for integrated assessment of students. In this report, I will share thoughts on possible developments during the next few years.

図 1: German adaptation of the SMART project (machine translation to Japanese)

2 Context: The SMART project

SMART is an acronym for *Specific Mathematics Assessments that Reveal Thinking*, an Australian project by Kaye Stacey et al. [20]. According to the description on the website `www.smartvic.com`:

> A 'smart test' is a specific mathematics assessment that reveals thinking. These innovative tests, which are accessed through an intelligent on-line environment, provide teachers with an informative diagnosis of their students' conceptual understanding of many of the topics in upper primary and junior secondary school mathematics. Within a few seconds of a student completing a test, the research-backed diagnosis is ready for the teacher. We intend that this information will be concise enough to be readily useable by teachers, deep enough to make a real difference to lesson content, and linked to appropriate teaching resources. Tests can be used by students anywhere there is a computer with an internet connection.

The system has been licensed for Germany by the University of Duisburg-Essen (UDE), and the group lead by Bärbel Barzel at UDE aims to supply German teachers with localized versions of the original 130 SMART tests [7]. This involves not only literal translations, but also content-specific adaptions to the tests regarding the German culture, wording and curriculum. To this behalf, a complete rewrite of the underlying software framework was initiated, shown in Fig. 1.

The German SMART project is situated within the larger DZLM network. The

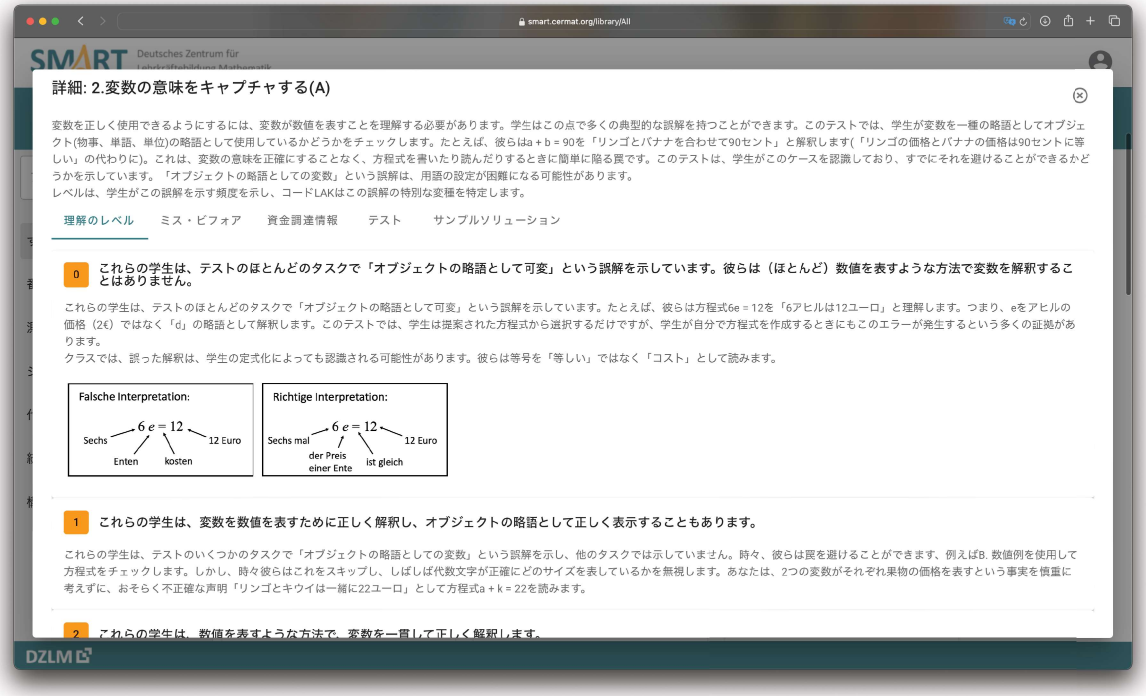

図 2: Hints for teachers in SMART (machine translation to Japanese)

German Center for Teacher Education in Mathematics (DZLM, short for Deutsches Zentrum für Lehrkräftebildung Mathematik) was founded in 2010 and aims at improving teachers' professional development across the German federal education system. The local group at the University of Potsdam within the DZLM network is working on the adaption of the Geometry part of the SMART project. As a consequence, there is a need to integrate geometry tools into an environment which is able to support the analysis and feedback needed to *reveal thinking*.

2.1 Assessing Products vs. Assessing Processes

When assessing Geometry tasks, we can identify several specialities as compared to arithmetic or algebra tasks:

- Usually, we ask for *more than one solution* to a problem. While there are routine or standard constructions for geometric problems, it is typical that there are several ways to solve a task, and it is desirable to encourage students to know more than one way to solve a problem.
- Failures – or wrong solutions – are usually interesting in Geometry. As in other mathematical tasks, a wrong solution can reveal ways of thinking, but in the same way that there are usually several correct solutions, the number of *interesting*

図 3: A geometry problem posed in the classroom

wrong solutions can exceed the number of interesting wrong solutions in other mathematical areas.

- Interactive Geometry software allows for visual manipulation, which can be used to *demonstrate* answers, as opposed to giving just a multiple choice selection.
- Various ways of *dragging* can be identified [1, 2] that should be distinguished for better diagnosis of students' answers.

In particular, we can distinguish *soft constructions* and *robust constructions* – the first, soft constructions, need targeted dragging to maintain certain properties, the second, robust constructions, maintain them automatically due to the properties and relations of the objects.

In Fig. 3 a problem is shown that is useful for a demonstration of robust and soft constructions: Given an equilateral triangle ABC, we ask for the possible placement of the point E that is created by constructing another equilateral ADE where D is on BC. A robust construction in a DGS will show that E will lie on a segment starting at C, which has the same length as BC and forms another equilateral triangle with A. The soft construction allows for free movement of D, and it is easily seen with this construction that the triangle ADE will rotate point D by 60° around A onto point E. This results in an easy argument for the observation in the robust construction: E will lie on the rotated segment BC. This kind of argumentation can be supported by using

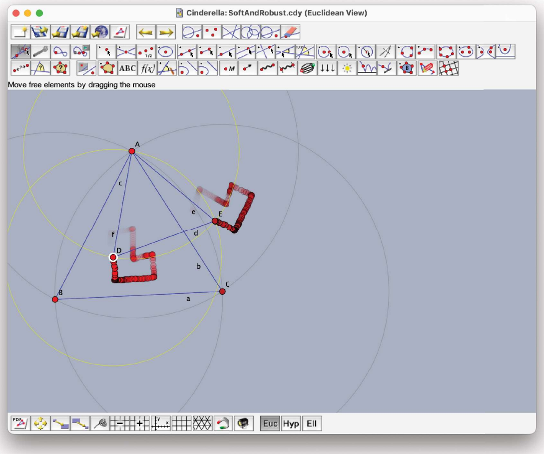

図 4: Demonstrating a rotation by 60° using a soft construction

geometry software (Fig. 4, however, the automatic detection of such an action is not straightforward.

The final result of solving a geometry problem can be a static figure, however, it is extremely difficult to extract the complete information about the solution process from the final picture, as can be seen in Fig. 5. This implies that we should try to record the process of creating a solution and not only the final solution. While this is probably true for all areas of mathematics, it is more important for geometry due to the interactive tools that are used for solving tasks and documenting solutions. Also, the additional process data can enable us to compare *thinking processes*, as shown in other contributions at this conference.

3 Event logging

In the previous section it was argued that it is necessary to record and analyze processes instead of (only) final products, in particular if we want to reveal thinking in geometric problem solving. Technically, this calls for recording a log of events. In [16] we describe an approach to intelligent computer-aided assessment in the classrom which already used the logging of events. This has been refined in [5, 19, 3] with the goal of clustering student activity in to units that can either be analyzed automatically or need a manual review. By this clustering, we can reduce the manual interpretation to special (and potentially interesting) cases that need further discussion in the classroom. In the context of the SMART project, we could aim at clustering student answers into answers that either reveal a certain kind of thinking, or are irrelevant for the analysis, with only a few cases remaining that might require an interpretation by the teacher.

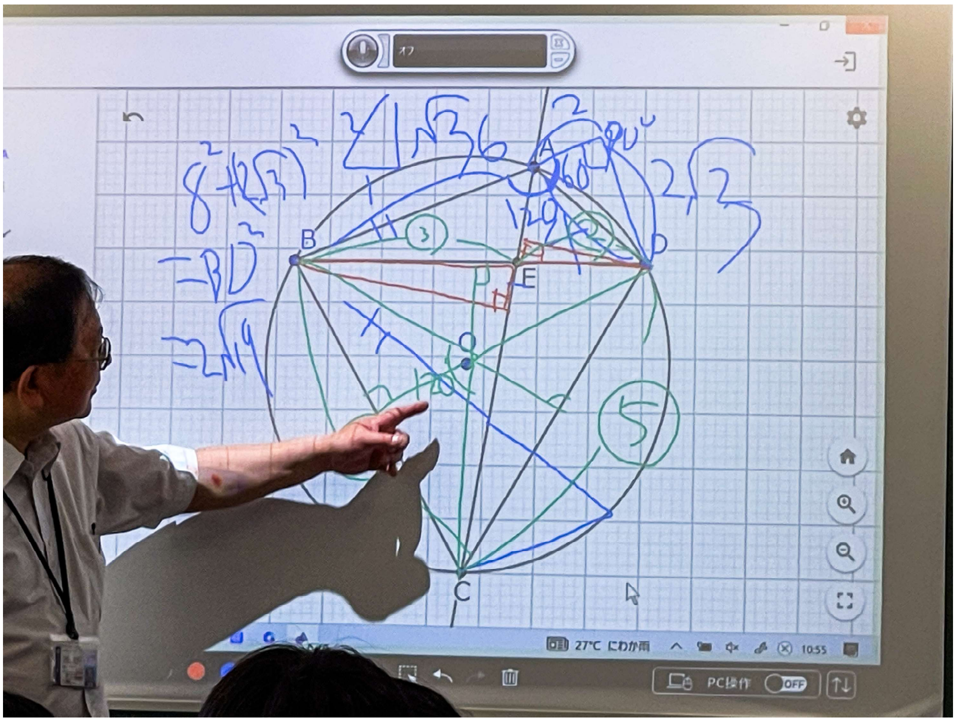

図 5: The final product of solving a geometry problem

3.1 Event hierarchies

In prior work between 2005 and 2012 we already identified two fruitful approaches to event logging: A general approach that captures mouse and keyboard activity and can play back these system events to either analyze the process or just recreate the activity, and a subject-specific approach where higher-level events that are special to geometry software are recorded, available as an extension to Cinderella, CINErella [10]. Both approaches were combined to support analyzing, guiding and assessment of processes in geometry software [18, 19].

Based on this prior research, four levels of event logging are evident.

Raw level On this level, the mouse and touch events as delivered by JavaScript or the underlying operating system are recorded.

This is the most general approach and it is not specific to the learning environment used, that is, it can be used with any software that allows for user input. Clicking on the screen or typing a key will be logged. However, the analysis of such events does not reveal any information beyond statistical information. We can find our, for example, how often a student interacts with a certain learning component, but not what he or she is doing – clueless clicking is indistinguishable from (guided) exploring. Nevertheless, the psychometric data might be of interest in some research setups.

Low-level On this level, mouse and touch events are recorded in logical coordinates and input is recorded structurally.

Here, *logical coordinates* are the mathematical coordinates that are on screen at a given (x, y) position. As they are not device dependend, they can be used for recording, playing and streaming interaction with the software, where playback may use the larger (or smaller) screen on another device. One example was the synchronization of Sharp ZAURUS constructions with the desktop and electronic whiteboards, which all three offer different screen resolutions and sizes, but show the same (Euclidean) plane [9, 12]. Again, it is possible to generate the information about these coordinates generically, without special programming of the construction author.

Semantic level On this level, events describe actions like moving elements, adding elements, pushing buttons, etc.

Another kind of elements that can be created automatically are semantic events, which correspond to certain user actions in a DGS. They can be used to collect and evaluate information about the kind of interaction with a geometric figure – how often has a certain point been moved, in which order did the student work, etc. For example, if a student should move the vertices of a general quadrilateral such that it becomes a rectangle, one strategy could be to adjust the second, third and fourth vertex one after the other. Other students might just move the second and fourth point, or they might just adjust all the points over and over again. It is possible to categorize the interaction of the students *a posteriori*.

Assessment level Finally, the assement level events describe whether certain tasks (or sub-tasks) have been completed successfully or failed.

The decision to log such an event usually is complex and needs further programming by the educational designer. For example, deciding whether the midpoint of a segment has been constructed could be logged, but only if this information is needed at all. The decision whether the goal has been achieved needs special design, but can be supported by automatic theorem proving and scripting [15, 13].

3.2 Recording Events in LMS – xAPI Logging

Our former attempts for the analysis of student processes always used desktop software that had access to the filesystem, so it was easy to write a log. However, nowadays most learning activities come as HTML and are usually embedded into an LMS such as Moodle, but could also be in any other system or just plain web pages. This means we have to find ways of writing the log data to a server-side database.

Also, it might be necessary to have information about the student and its former performance in other tasks. If we want to store data, we have to take care of data

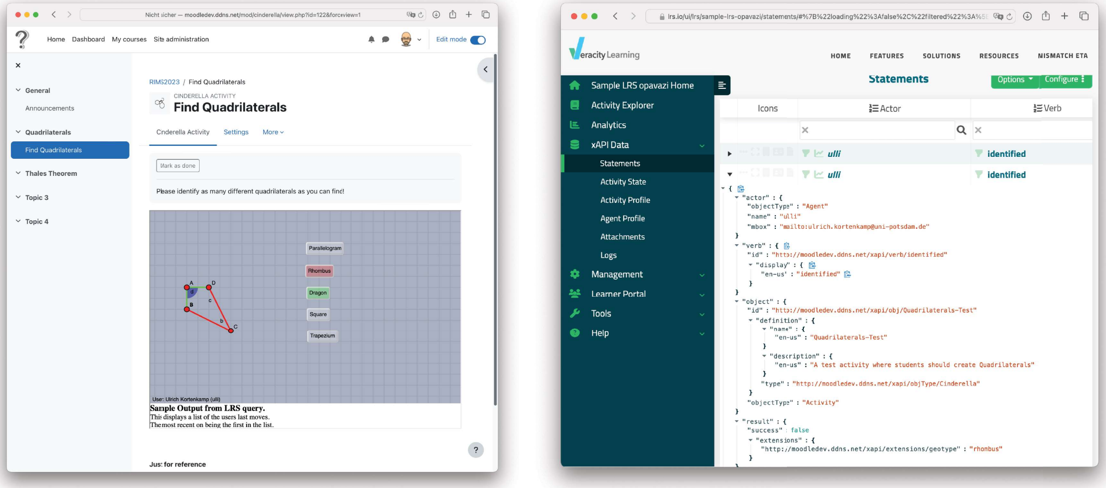

図 6: Communication between a DGS running in Moodle and a Learning Record Store

protection regulations and data security. Given all these considerations, there are many ways to solve this.

Here I want to present just one solution, using the eXperience API (xAPI or Tin Can API), which has been created by Rustici Software for the Advanced Distributed Learning (ADL) Initiative (U. S. DoD). xAPI is meant to succeed SCORM, which is a standard for packaging eLearning content, that is no flexible enough for today's use cases. xAPI is able to track learning progress across different software products in a Learning Record Store (LRS). Actually, recent Moodle versions come with an included LRS, but there are also stand-alone-versions of LRS that an moodle activity can authenticate to and then send structured events using pre-defined JSON objects to it. Later, these events can be retrieved and come with certain metadata, for example the user who created this event. Fig. 6 shows an example implementation of an activity integrated into a Moodle course that sends events to a stand-alone LRS.

However, this prototype implementation showed that there is no straightforward way to use xAPI to capture the hierarchy of events as discussed in the previous section. It is more suitable for final achievements (like *user A has acquired a driving license*), and not for in-depth analysis of problem solving processes.

4 Conclusion and Future Work

Our primary goal is to use assessment for learning, that is, all learning activities should achieve is an improvement in compentencies of the students. Grading of students or handing out certificates is not the main consideration. Therefore, we have to look at *processes* instead of products, and these processes can be described by a hierarchy of events: raw events, low-level events, semantic events and assessment level events. This corresponds to the levels in activity theory (activity oriented by motive, action directed at goals, operations as routine processes), with an additional level of abstraction.

As opposed to the final outcome, which does not tell the whole story, in particular in the case of misconceptions, these events capture enough data to give meaningful feedback to the students and the teachers.

One way to store learners' data that can be used across various LMS could be xAPI, but it turns out to be too slow and not granular enough for full insight into the processes. Nevertheless, this is just a technical detail that can be solved using other logging mechanisms. It will be a future effort to define a standard that can be used reliably as a stand-alone and as an integrated method of event logging for geometry.

参考文献

[1] Ferdinando Arzarello et al. "A Cognitive Analysis of Dragging Practises in Cabri Environments". In: *Zentralblatt für Didaktik der Mathematik* 34.3 (June 2002), pp. 66–72. ISSN: 1615-679X. DOI: 10.1007/BF02655708. (Visited on 08/30/2023).

[2] Anna Baccaglini-Frank and Maria Alessandra Mariotti. "Generating Conjectures in Dynamic Geometry: The Maintaining Dragging Model". In: *International Journal of Computers for Mathematical Learning* 15.3 (Oct. 2010), pp. 225–253. ISSN: 1382-3892, 1573-1766. DOI: 10.1007/s10758-010-9169-3. (Visited on 06/16/2023).

[3] Christine Bescherer et al. "E-learning Tools with Intelligent Assessment and Feedback". In: *Intelligent and Adaptive Learning Systems: Technology Enhanced Support for Learners and Teachers*. Ed. by Sabine Graf et al. IGI Global, 2012.

[4] Enno Brehm and Ulrich Kortenkamp. "Advanced Teaching of Geometry with Interactive Tools". In: *Proceedings of RIMS-Workshop ITMga 2003*. Ed. by Ryosuke Nagaoka, Hideyuki Ishi, and Eckhard Hitzer. RIMS Kôkyûroku 5. Kyoto: RIMS, 2004, pp. 105–120. URL: http://www.kurims.kyoto-u.ac.jp/~kyodo/kokyuroku/contents/pdf/1378-9.pdf.

[5] Andreas Fest and Ulrich Kortenkamp. "Process Orientated Learning Environments For Interactive Geometry Lessons". In: *Proceedings of the 9th International Conference on Technology in Mathematics Teachingh International Conference on Technology in Mathematics Teaching*. Ed. by C. Bardini et al. Metz: ICTMT-9, 2009.

[6] Martin von Gagern et al. "CindyJS. Mathematical visualization on modern devices". In: *Mathematical Software – ICMS 2016: 5th International Conference* (Berlin). Ed. by Gert-Martin Greuel et al. Lecture Notes in Computer Science, vol 9725. Cham: Springer International Publishing, 2016, pp. 319–326. ISBN: 978-3-319-42432-3. DOI: 10.1007/978-3-319-42432-3_39.

[7] Katrin Klingbeil et al. "SMART_A – Online-Diagnostic to Reveal Students' Algebraic Thinking and Enhance Teachers' Diagnostic Competencies". In: *Proceedings of the 15th International Conference on Technology in Mathematics Teaching*. Ed. by Uffe Thomas Jankvist et al. AU Library Scholarly Publishing Services, May 2022, pp. 290–297. ISBN: 978-87-7507-525-6. DOI: 10.7146/aul.452.

[8] Ulrich Kortenkamp. "Developments in Interactive Visualization and Physics Simulation with Cinderella". In: *Study of Mathematical Software and Its Effective Use for Mathematics Education*. Ed. by Yasuyuki Nakamura. Vol. 2017. RIMS Kôkyûroku 4. Kyoto: RIMS, 2017, pp. 17–28. URL: http://www.kurims.kyoto-u.ac.jp/~kyodo/kokyuroku/contents/pdf/2022-03.pdf.

[9] Ulrich Kortenkamp. "Dokumentation, Diskussion und Protokolle: Wie kommuniziert man Geometrie im Internetzeitalter?" German. In: *Strukturieren – Modellieren – Kommunizieren. Leitbilder mathematischer und informatischer Aktivitäten. Festschrift für Karl-Dieter Klose, Siegfried Krauter, Herbert Löthe und Heinrich Wölpert.* Ed. by Joachim Engel, Rose Vogel, and Silvia Wessolowski. Hildesheim: Franzbecker, 2005, pp. 141–150.

[10] Ulrich Kortenkamp. "Experimentieren und Publizieren". In: *WWW und Mathematik - Lehren und Lernen im Internet, Tagungsband der 21. Arbeitstagung des Arbeitskreis Mathematikunterricht und Informatik in Dillingen/Donau*. Ed. by Peter Bender et al. Hildesheim: Franzbecker, 2005, pp. 120–126.

[11] Ulrich Kortenkamp. "Foundations of Dynamic Geometry". Dissertation. Zürich: ETH Zürich, Nov. 1999. DOI: 10.3929/ethz-a-003876663. URL: https://www.research-collection.ethz.ch/handle/20.500.11850/144481.

[12] Ulrich Kortenkamp. "Kommunizieren und Dokumentieren von Geometrie". German. In: *Beiträge zum Mathematikunterricht. Vorträge auf der 38. Tagung für Didaktik der Mathematik in Augsburg*. Gesellschaft für Didaktik der Mathematik. Hildesheim: Franzbecker, 2004.

[13] Ulrich Kortenkamp and Christian Dohrmann. "User Interface Design for Dynamic Geometry Software". In: *Acta Didactica Napocensia* 3.2 (2010), pp. 59–66. URL: http://dppd.ubbcluj.ro/adn/article_3_2_6.pdf.

[14] Ulrich Kortenkamp and Dirk Materlik. "Geometry teaching in wireless classroom environments using Java and J2ME". In: *Science of Computer Programming* 53.1 (Oct. 2004), pp. 71–85. DOI: 10.1016/j.scico.2004.02.006.

[15] Ulrich Kortenkamp and Jürgen Richter-Gebert. "Using Automatic Theorem Proving to Improve the Usability of Geometry Software". In: *Proceedings of MathUI 2004*. Ed. by Paul Libbrecht. 2004.

[16] Wolfgang Müller et al. "Intelligent Computer-aided Assessment in the Math Classroom: State-of-the-art and Perspectives". In: *Proceedings of the IFIP WG 3.1, 3.3, & 3.5 Joint Conference Imagining the future for ICT and Education, Ålesund, Norway* (Alesund). Ed. by Deryn Watson and Sindre Rosvik. Ålesund: IFIP, 2006.

[17] Jürgen Richter-Gebert and Ulrich H. Kortenkamp. **シンデレラ**. **幾何学のためのグラフィックス**. Japanese. Tokyo: Springer-Verlag, 2003.

[18] Christian Spannagel and Ulrich Kortenkamp. "CleverPHL – ein Werkzeug zum flexiblen Umgang mit Konstruktionsprozessen in DGS". In: *Beiträge zum Mathematikunterricht. Vorträge auf der 41. Tagung für Didaktik der Mathematik in Berlin*. Hildesheim: Franzbecker, 2007. URL: http://www.mathematik.tu-dortmund.de/ieem/cms/media/BzMU/BzMU2007/Spannagel_%20Kortenkamp.pdf.

[19] Christian Spannagel and Ulrich Kortenkamp. "Demonstrating, Guiding, and Analyzing Processes in Dynamic Geometry Systems". In: *Proceedings of the 9th International Conference on Technology in Mathematics Teaching*. Ed. by C. Bardini et al. Metz: ICTMT-9, 2009.

[20] Kaye Stacey et al. "Specific Mathematics Assessments that Reveal Thinking: An Online Tool to Build Teachers' Diagnostic Competence and Support Teaching". In: *Diagnostic Competence of Mathematics Teachers: Unpacking a Complex Construct in Teacher Education and Teacher Practice*. Ed. by Timo Leuders, Kathleen Philipp, and Juliane Leuders. Cham: Springer International Publishing, 2018, pp. 241–261. ISBN: 978-3-319-66327-2. DOI: 10.1007/978-3-319-66327-2_13. URL: https://doi.org/10.1007/978-3-319-66327-2_13.