RIMS Symposium (Open)

Harmonic Analysis and Nonlinear Partial Differential Equations

Date: 3 July – 5 July 2023

Location: Maskawa Hall, Maskawa Building for Education and Research
Kyoto University, Kyoto 606-8502

Organisers: Yutaka Terasawa (Nagoya University)
Neal Bez (Saitama University)

Program

Monday 3 July

13:15〜14:15 Masaki Kawamoto (Ehime University)
Modified scattering for nonlinear Schrödinger equations with long-range potentials

14:30〜15:30 Naoto Shida (Nagoya University)
Boundedness of bilinear pseudo-differential operators with symbols in the bilinear Hörmander class $S_{0,0}$ *on Besov spaces*

15:45〜16:45 Konstantin Merz (Osaka University)
Random Schrödinger operators with complex decaying potentials

Tuesday 4 July

9:00〜10:00 Kenta Oishi (Waseda University)
On the global well-posedness and decay of a free boundary problem of the Navier-Stokes equation in the two-dimensional half space

10:15〜11:15 Keisuke Takasao (Kyoto University)
Phase field method for mean curvature flow and vanishing of discrepancy measure

11:15〜13:15 Lunch

13:15〜14:15　Yasunori Maekawa (Kyoto University)
Optimal rate of convergence to nondegenerate asymptotic profiles for fast diffusion

14:30〜15:30　Po-Lam Yung (Australian National University)
Fourier decoupling via the high-low method

15:45〜16:45　Joshua Zahl (University of British Columbia)
Sticky Kakeya sets, and the sticky Kakeya conjecture

Wednesday 5 July

9:30〜10:30　Helmut Abels (Universität Regensburg)
Sharp interface limits of diffuse interface models I

10:45〜11:45　Helmut Abels (Universität Regensburg)
Sharp interface limits of diffuse interface models II

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Abstracts

Helmut Abels (Universität Regensburg)

Sharp interface limits of diffuse interface models I, II

Abstract: We consider the dynamics of two components, which are partly miscible on a small length scale proportional to some parameter $\varepsilon > 0$. This is described by an order parameter as e.g. a concentration difference, which is close to two distinct values (e.g. 1 and -1) in most of the domain, and varies smoothly, but with a large gradient, between these two values in a small interfacial region of thickness proportional to ε. Such models are called diffuse interface models and have various applications. We are interested in rigorous results on convergence to classical sharp interface models as ε tends to zero, where the components fill two disjoint regions is separated by a lower dimensional interface. We will start with the case of the scalar Allen-Cahn equation, discuss the method of formally matched asymptotics and show how to use this method in a rigorous proof of convergence. Afterwards we will consider results on a coupled Navier-Stokes/Allen-Cahn system, which describes a two-phase flow of macroscopically immiscible viscous Newtonian fluids. In this case the results depend essentially on the choice of a mobility coefficients in Allen-Cahn equation.

Masaki Kawamoto (Ehime University)

Modified scattering for nonlinear Schrödinger equations with long-range potentials

Abstract: We consider the scattering problem for the nonlinear Schrödinger equation with linear potentials. In particular, we mainly focus on the case where both of nonlinearities and potentials are of long-range type in the sense of scattering. In this talk, we introduce a concrete form of the asymptotic behavior of solutions of this equation and show the modified scattering for the associated final state problem, as well as the existence of modified wave operators. This talk is based on the joint work with Haruya Mizutani (Osaka University).

Yasunori Maekawa (Kyoto University)

Optimal rate of convergence to nondegenerate asymptotic profiles for fast diffusion

Abstract: In this talk we consider the (possibly sign-changing) solutions to the Cauchy-Dirichlet problem for the fast diffusion equation,

$$\begin{aligned} \partial_t B(u) &= \Delta u \quad \text{ in } \Omega \times (0,\infty), \\ u &= 0 \text{ on } \partial\Omega \times (0,\infty), \\ u &= u_0 \text{ on } \Omega \times \{0\}. \end{aligned} \tag{1}$$

Here Ω is a bounded Lipschitz domain in $\mathbb{R}^N$ $(N \geq 2)$, $B(u) = |u|^{q-2}u$, and $u_0 \in H_0^1(\Omega) \setminus \{0\}$. We focus on the case of the fast diffusion, that is,

$$2 < q < 2^* := \frac{2N}{(N-2)_+}. \tag{2}$$

Under this condition on q, the diffusion coefficient diverges on the boundary, due to the homogeneous Dirichlet boundary condition. As a result, it is well known that every weak solution $u = u(x,t)$ of (1) vanishes at a finite time $t_* = t_*(u_0)$, which is uniquely determined by the initial datum u_0. The rate of the extinction is also known to be estimated as

$$c_1(t_* - t)_+^{1/(q-2)} \leq \|u(\cdot,t)\|_{H_0^1(\Omega)} \leq c_2(t_* - t)_+^{1/(q-2)} \tag{3}$$

with $c_1, c_2 > 0$ for all $t \geq 0$, provided that $u_0 \not\equiv 0$. Then the transformation

$$v(x,s) = (t_* - t)^{-1/(q-2)}u(x,t) \quad \text{and} \quad s = \log(t_*/(t_* - t)) \tag{4}$$

is well-defined, which has been used to study the asymptotic behavior of the solution near the extinction time. We show that, if the transformed (possibly sign-changing) solution $v = v(x, s)$ converges strongly in $H^1_0(\Omega)$ to a nondegenerate (possibly sign-changing) profile $\phi = \phi(x)$ as $s \to +\infty$, then the rate of the convergence is estimated as $O(e^{-\lambda_0 s})$, where $\lambda_0 = \mu_{k_0} - \frac{q-1}{q-2}$. Here $\{\frac{1}{\mu_k}\}_{k\geq 1}$, $0 < \mu_1 < \mu_2 < \ldots$, are the eigenvalues of the compact self-adjoint operator $(-\Delta)^{-1}B'_\phi$ with $B'_\phi = (q-1)|\phi|^{q-2}$ in $H^1_0(\Omega)$, and k_0 is the number such that $\mu_{k_0} - \frac{q-1}{q-2}$ firstly excesses zero. This convergence rate has been known for positive nondegenerate profiles in smooth bounded domains, while our result is new for possibly sign-changing nondegenerate profiles in bounded Lipschitz domains. We also discuss the optimality of this convergence rate. This talk is based on the joint work with Goro Akagi (Tohoku University).

Konstantin Merz (Osaka University)

Random Schrödinger operators with complex decaying potentials

Abstract: We estimate complex eigenvalues of continuum random Schrödinger operators of Anderson type. Our analysis relies on methods of J. Bourgain (Discrete Contin. Dyn. Syst., 2002, Lecture Notes in Math., 2003) related to almost sure scattering for random lattice Schrödinger operators, and allows us to consider potentials which decay almost twice as slowly as in the deterministic case. The talk is based on joint work with Jean-Claude Cuenin.

Kenta Oishi (Waseda University)

On the global well-posedness and decay of a free boundary problem of the Navier-Stokes equation in the two-dimensional half space

Abstract: We establish the global well-posedness and some decay properties for a free boundary problem of the incompressible Navier-Stokes equations in $\mathbb{R}^N_+$ with $N \geq 2$. For $N = 2$, the decay of the solution to the free boundary problem is too slow to control the nonlinear term on the boundary and that is why we could not handle the case $N = 2$ in our preceding work. We overcome this difficulty owing to the trace estimate $\|f(x', 0)\|_{L_q(\mathbb{R}^{N-1})} \leq \|f\|^{1-1/q}_{L_q(\mathbb{R}^N_+)} \|\nabla f\|^{1/q}_{L_q(\mathbb{R}^N_+)}$, where $x' = (x_1, \cdots, x_{N-1})$.
This work was supported by JSPS KAKENHI, Grant-in-Aid for Young Scientists, Grant Number 22K13945.

Naoto Shida (Nagoya University)

Boundedness of bilinear pseudo-differential operators with symbols in the bilinear Hörmander class $S_{0,0}$ on Besov spaces

Abstract: We consider bilinear pseudo-differential operators with symbols in the bilinear Hörmander class $S_{0,0}$. The boundedness of these operators on Lebesgue spaces has been established by Miyachi-Tomita (2013) and Kato-Miyachi-Tomita (2022). In this talk, we discuss the boundedness of these operators in the settings of Besov spaces.

Keisuke Takasao (Kyoto University)

Phase field method for mean curvature flow and vanishing of discrepancy measure

Abstract: In this talk, we consider the mean curvature flow with forcing term and the volume preserving mean curvature flow. To construct of the weak solutions for the flows, we use the Allen-Cahn equations with suitable additional terms. If the L^1-norm of the difference of the Dirichlet energy and the potential energy converges to 0 (the vanishing of the discrepancy measure), then the Allen-Cahn equation converges to the weak solution to the flow in the sense of varifolds. We will show the vanishing via the Cole-Hopf transformation and the monotonicity formula.

Po-Lam Yung (Australian National University)
Fourier decoupling via the high-low method
Abstract: We will explain, using examples, how Guth, Wang and Maldague proved decoupling for the parabola in $\mathbb{R}^2$, and how Guo, Li and myself adapted it to study some related decoupling estimates.

Joshua Zahl (University of British Columbia)
Sticky Kakeya sets, and the sticky Kakeya conjecture
Abstract: A Kakeya set is a compact subset of $\mathbb{R}^n$, that contains a unit line segment pointing in every direction. The Kakeya conjecture asserts that such sets must have dimension n. This conjecture is closely related to several open problems in harmonic analysis, and it sits at the base of a hierarchy of increasingly difficult questions about the behavior of the Fourier transform in Euclidean space.

There is a special class of Kakeya sets, called sticky Kakeya sets. Sticky Kakeya sets exhibit an approximate self-similarity at many scales, and sets of this type played an important role in Katz, Laba, and Tao's groundbreaking 1999 work on the Kakeya problem. In this talk, I will discuss a special case of the Kakeya conjecture, which asserts that sticky Kakeya sets must have dimension n. I will discuss the proof of this conjecture in dimension 3. This is joint work with Hong Wang.