

# COMMENTS ON “THE ABSOLUTE ANABELIAN GEOMETRY OF HYPERBOLIC CURVES”

SHINICHI MOCHIZUKI

June 2016

(1.) The assumption of the existence of a *splitting over some open subgroup* in the discussion preceding Lemma 1.1.4 is, in fact, *unnecessary*, at least in the context of Lemma 1.1.4. Indeed, this assumption is never applied in the proof of Lemma 1.1.4, (i). In the proof of Lemma 1.1.4, (ii), this assumption is applied; on the other hand, the application of this assumption may be circumvented by applying instead the well-known fact that  $H^2(G, \widehat{\mathbb{Z}} \otimes_{\mathbb{Z}} \mathbb{Q}) = 0$ .

(2.) In the Appendix, the phrase “*dual graph with compact structure*” should read “*dual semi-graph with compact structure*”.

(3.) The final portion (beginning with the *third sentence*) of the *second paragraph* of the proof of Lemma 1.3.9 should be replaced by the following text:

Since  $r_i$  may be recovered group-theoretically, given any finite étale coverings

$$Z_i \rightarrow V_i \rightarrow X_i$$

such that  $Z_i$  is *cyclic* (hence *Galois*), of degree a *power of  $l$* , over  $V_i$ , one may determine group-theoretically whether or not  $Z_i \rightarrow V_i$  is *totally ramified* (i.e., at some point of  $Z_i$ ), since this condition is easily verified to be equivalent to the condition that the covering  $Z_i \rightarrow V_i$  admit a *factorization*  $Z_i \rightarrow W_i \rightarrow V_i$ , where  $W_i \rightarrow V_i$  is finite étale of degree  $l$ , and  $r_{W_i} < l \cdot r_{V_i}$ . Moreover, this group-theoreticity of the condition that a cyclic covering be *totally ramified* extends immediately to the case of *pro- $l$  cyclic coverings*  $Z_i \rightarrow V_i$ . Thus, by Lemma 1.3.7, we conclude that *the inertia groups of cusps in  $(\Delta_{X_i})^{(l)}$*  (i.e., the maximal pro- $l$  quotient of  $\Delta_{X_i}$ ) *may be characterized (group-theoretically!)* as the maximal subgroups of  $(\Delta_{X_i})^{(l)}$  that correspond to (profinite) coverings satisfying this condition.