

COMMENTS ON “ON THE COMBINATORIAL  
CUSPIDALIZATION OF HYPERBOLIC CURVES”

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(1.) In lines 4–5 of the third paragraph of the discussion entitled “*The Étale Fundamental Group of a Log Scheme*” in §0, the phrase “*unramified over  $R_\circ$* ” should read “*unramified over  $U_{S_\circ}$* ”.

(2.) In Fig. 1, the subscripts “*b*”, “*c*” in the notation “ $F_b$ ”, “ $F_c$ ” should be *reversed*.

(3.) In the statement of Proposition 1.3, (i), the phrase “the set of divisors  $\mathcal{D}_n^{\text{ver}}$  which” should read “the set of divisors which”; the notation “ $\delta \in \mathcal{D}_n^{\text{hor}}$ ” should read “ $\delta \in \mathcal{D}_n^{\text{ver}}$ ”.

(4.) In the statement of Proposition 1.3, (iv), the notation “ $\mathcal{D}_n^{\text{hor}}$ ” should read “ $\mathcal{D}_n^{\text{ver}}$ ”.

(5.) In the first paragraph of the proof of Corollary 1.14, the text “First, let us . . . Now let” should read as follows:

First, let us observe that relative to the *natural isomorphism*  $X_n^{\text{log}} \xrightarrow{\sim} (\overline{\mathcal{M}}_{0,n+3}^{\text{log}})_k$  [cf. Definition 1.1, (vi)], the divisors of  $X_n$  that belong to  $\mathcal{D}_n$  are precisely the *divisors at infinity* of  $(\overline{\mathcal{M}}_{0,n+3}^{\text{log}})_k$ . [Indeed, this follows immediately from the well-known geometry of  $(\overline{\mathcal{M}}_{0,n+3}^{\text{log}})_k$ .] In particular, the automorphisms of  $(\overline{\mathcal{M}}_{0,n+3}^{\text{log}})_k$  arising from the permutations of the ordering of the cusps *permute* the divisors that belong to  $\mathcal{D}_n$ . Thus, we conclude that the *outer modular symmetries*  $\in \text{Out}(\Pi_n)$  *normalize*  $\text{Out}^{\text{QS}}(\Pi_n) = \text{Out}^{\text{FC}}(\Pi_n)^{\text{cusp}}$  [cf. Proposition 1.3, (vi), (vii)]. Now let

(6.) In Definition 5.2, (ii), the phrase “of  $x$  in  $\mathcal{U}$ ” should read “of  $x$  in  $\mathcal{N}$ ”.