RIMS/SYMMETRIES AND CORRESPONDENCES WORKSHOP: INTER-UNIVERSAL TEICHMÜLLER THEORY SUMMIT 2016

Organizers: Ivan Fesenko (The University of Nottingham)
Shinichi Mochizuki (RIMS, Kyoto University)
Yuichiro Taguchi (Tokyo Institute of Technology)

Dates: July 18 – 27, 2016

Place: RIMS, Kyoto University, Room 420 (July 18 – 22),
Room 111 (July 25 – 27)

July 18 (Monday)

09:30 – 09:40 Opening address

Part I: Preparatory talks

09:45 – 10:45 Koichiro Sawada (RIMS, Kyoto University)
Uchida's theorem for one-dimensional function fields over finite fields

11:00 – 12:00 Kazumi Higashiyama (RIMS, Kyoto University)
Mono-anabelian geometry I: Reconstruction of function fields via Belyi cuspidalization

12:15 – 13:15 Arata Minamide (RIMS, Kyoto University)
Mono-anabelian geometry II: Mono-anabelian geometry over mixed characteristic local fields

14:45 – 15:45 Ippei Nagamachi (The University of Tokyo)
Log-shell, log-volume, and log-link I

16:00 – 17:00 Ippei Nagamachi (The University of Tokyo)
Log-shell, log-volume, and log-link II

17:15 – 18:15 Weronika Czerniawska (The University of Nottingham)
Frobenioids 1

July 19 (Tuesday)

09:30 – 10:30 Weronika Czerniawska (The University of Nottingham)
Frobenioids 2

10:45 – 11:45 Arata Minamide (RIMS, Kyoto University)
Étale theta functions and mono-theta environments I

12:00 – 13:00 Arata Minamide (RIMS, Kyoto University)
Étale theta functions and mono-theta environments II

14:30 – 15:30 Seidai Yasuda (Osaka University)
Étale theta functions, mono-theta environments, and [IUTchI] §1 – §3, I

15:45 – 16:45 Seidai Yasuda (Osaka University)
Étale theta functions, mono-theta environments, and [IUTchI] §1 – §3, II

Part II: Overall survey

17:00 – 18:00 Shinichi Mochizuki (RIMS, Kyoto University)
The mathematics of mutually alien copies: from Gaussian integrals to inter-universal Teichmüller theory I

July 20 (Wednesday)

09:30 – 10:30 Shinichi Mochizuki (RIMS, Kyoto University)
The mathematics of mutually alien copies: from Gaussian integrals to inter-universal Teichmüller theory II

10:45 – 11:45 Shinichi Mochizuki (RIMS, Kyoto University)
The mathematics of mutually alien copies: from Gaussian integrals to inter-universal Teichmüller theory III

12:00 – 13:00 Shinichi Mochizuki (RIMS, Kyoto University)
The mathematics of mutually alien copies: from Gaussian integrals to inter-universal Teichmüller theory IV

14:30 – 15:30 Shinichi Mochizuki (RIMS, Kyoto University)
The mathematics of mutually alien copies: from Gaussian integrals to inter-universal Teichmüller theory V

15:45 – 16:45 Shinichi Mochizuki (RIMS, Kyoto University)
The mathematics of mutually alien copies: from Gaussian integrals to inter-universal Teichmüller theory VI

Part III: [IUTchI-II]

17:00 – 18:00 Seidai Yasuda (Osaka University)
Étale theta functions, mono-theta environments, and [IUTchI] §1 – §3, III

July 21 (Thursday)

Part II: Overall survey (continued)

09:30 – 10:30 Shinichi Mochizuki (RIMS, Kyoto University)
The mathematics of mutually alien copies: from Gaussian integrals to inter-universal Teichmüller theory VII

10:45 – 11:45 Shinichi Mochizuki (RIMS, Kyoto University)
The mathematics of mutually alien copies: from Gaussian integrals to inter-universal Teichmüller theory VIII

Part III: [IUTchI-II] (continued)

12:00 – 13:00 Fucheng Tan (Shanghai Center for Mathematical Sciences and Shanghai Jiao Tong University)
IUT-I: Hodge theaters and label classes of cusps 1

14:30 – 15:30 Fucheng Tan (Shanghai Center for Mathematical Sciences and Shanghai Jiao Tong University)
IUT-I: Hodge theaters and label classes of cusps 2

15:45 – 16:45 Taylor Dupuy (Hebrew University and the University of Vermont)
Functors to and from mono-theta environments

17:00 – 18:00 Emmanuel Lepage (University Paris 6)
Hodge-Arakelov evaluation 1

19:00 – 21:00 Buffet style dinner

July 22 (Friday)

09:30 – 10:30 Taylor Dupuy (Hebrew University and the University of Vermont)
Multiradiality

10:45 – 11:45 Emmanuel Lepage (University Paris 6)
Hodge-Arakelov evaluation 2

Part IV: [IUTchIII-IV] from the point of view of mono-anabelian transport

12:00 – 13:00 Yuichiro Hoshi (RIMS, Kyoto University)
[IUTchIII-IV] from the point of view of mono-anabelian transport I

14:30 – 15:30 Yuichiro Hoshi (RIMS, Kyoto University)
[IUTchIII-IV] from the point of view of mono-anabelian transport II

15:45 – 16:45 Yuichiro Hoshi (RIMS, Kyoto University)
[IUTchIII-IV] from the point of view of mono-anabelian transport III

17:00 – 18:00 Yuichiro Hoshi (RIMS, Kyoto University)
[IUTchIII-IV] from the point of view of mono-anabelian transport IV

July 25 (Monday)

Part V: Related topics

09:30 – 10:30 Yuki Wada (RIMS, Kyoto University)
Near miss *abc*-triples in compactly bounded subsets

10:45 – 11:45 Vesselin Dimitrov (Yale University)
Notes on the epsilon term in the *abc* conjecture

12:00 – 13:00 Kobi Kremnitzer (The University of Oxford)
Milnor-Wood inequality and bounded cohomology

14:30 – 15:30 Ivan Fesenko (The University of Nottingham)
Reciprocity and IUT

15:45 – 16:45 Boris Zilber (The University of Oxford)
Model theory of anabelian geometry

17:00 – 18:00 Free discussion

July 26 (Tuesday)

Part VI: [IUTchIII-IV] with remarks on the function-theoretic roots of the theory

09:30 – 10:30 Go Yamashita (RIMS, Kyoto University)
[IUTchIII-IV] with remarks on the function-theoretic roots of the theory I

10:45 – 11:45 Go Yamashita (RIMS, Kyoto University)
[IUTchIII-IV] with remarks on the function-theoretic roots of the theory II

12:00 – 13:00 Go Yamashita (RIMS, Kyoto University)
[IUTchIII-IV] with remarks on the function-theoretic roots of the theory III

14:30 – 15:30 Go Yamashita (RIMS, Kyoto University)
[IUTchIII-IV] with remarks on the function-theoretic roots of the theory IV

15:45 – 16:45 Free discussion

17:00 – 18:00 Free discussion

July 27 (Wednesday)

Part VII: Overall discussion and impromptu talks

09:30 – 10:10 Adam Topaz (University of California, Berkeley)
Kummer theory and anabelian geometry over algebraically closed fields

10:25 – 10:45 Dinesh Thakur (Rochester University)
Diophantine approximation and deformation hierarchies in finite characteristic

11:00 – 12:00 Paul Vojta (University of California, Berkeley)
On changes to the Thue-Siegel method

12:15 – 13:00 Free discussion

14:30 – 15:30 Free discussion

15:45 – 16:45 Free discussion

17:00 – 18:00 Free discussion

18:00 – 18:15 Closing remarks

Abstracts

Part I: Preparatory talks

Koichiro Sawada (RIMS, Kyoto University)
Uchida's theorem for one-dimensional function fields over finite fields
Abstract:
Uchida proved that a one-dimensional function field over a finite field can be reconstructed from the absolute Galois group. In this talk, I will explain this theorem.

Kazumi Higashiyama (RIMS, Kyoto University)
Mono-anabelian geometry I: Reconstruction of function fields via Belyi cuspidalization
Abstract:
In this talk, we study mono-anabelian geometry. In more concrete terms, we prove the following assertion: Let k_0 be a number field, X_0/k_0 a hyperbolic curve which is isogenous to a hyperbolic curve of genus 0, $k_0 \subseteq k$, where k is sub-p-adic. Then we reconstruct group-theoretically the function field $\overline{k}_0(X_0)$ of $X_0 \times_{k_0} \overline{k}_0$ from

$$1 \to \pi_1(X_0 \times_{k_0} \overline{k}) \to \pi_1(X_0 \times_{k_0} k) \to \mathrm{Gal}_k \to 1$$

(regarded as an exact sequence of abstract profinite groups), via the technique of Belyi cuspidalization.

Arata Minamide (RIMS, Kyoto University)
Mono-anabelian geometry II: Mono-anabelian geometry over mixed characteristic local fields
Abstract:
Let k be a mixed characteristic local field, X a hyperbolic curve of strictly Belyi type over k. In the present talk, I will discuss:
(i) group-theoretic algorithms for reconstructing various invariants, groups, and monoids related to k from the absolute Galois group of k;
(ii) cyclotomic rigidity via local class field theory;
(iii) a group-theoretic algorithm for reconstructing the function field of X from the étale fundamental group of X.

Ippei Nagamachi (The University of Tokyo)
Log-shell, log-volume, and log-link I - II
Abstract:
In this talk, I will explain some construction algorithms and notions which are main ingredients of "log-link" in IUT. First, I will give an algorithm for constructing a new field $k^{\sim}$ from an algebraic closure $\overline{k}$ of a mixed characteristic local field k by perfection of its unit group and p-adic logarithm. Then I define "log-volume" (notion corresponding to Haar measure) and "log-shell" (the subgroup of $k^{\sim}$ nomalizing log-volume). These notions can be mono-analytically reconstructed. More precisely, we can define the (étale-like) log-shell and log-volume for the reconstructed $(k^{\sim})^{\times}$ from a profinite group which is isomorphic to some absolute Galois group of a mixed characteristic local field, despite we can not reconstruct the field structure of $k^{\sim}$. A key diagram in my talk is the following one

$$\mathcal{O}_{\overline{k}}^{\rhd} \quad \supseteq \quad \mathcal{O}_{\overline{k}}^{\times} \quad \overset{\text{perfection}}{\longrightarrow} \quad (\mathcal{O}_{\overline{k}}^{\times})^{\mathrm{pf}} \quad =: \quad k^{\sim}.$$

Here, $\mathcal{O}_{\overline{k}}^{\triangleright}$ is the multiplicative monoid $\mathcal{O}_{\overline{k}} \setminus \{0\}$, which plays a crucial role for yielding cyclotomic rigidity.

Weronika Czerniawska (The University of Nottingham)
Frobenioids 1-2
Abstract:
Frobenioids are a categorical generalization of the line bundles or monoids of divisors over a Galois category. Almost all of frobenioids occuring in the IUT papers are model frobenioids, among them crucial ones are associated to submonoids of familiar abelian groups in in local and global class field theories. In my talks I will discuss their basic properties including a torsor-theoretic approach. I will then present various results from the theory of Frobenioids which are important from the point of view of studying IUT.

Arata Minamide (RIMS, Kyoto University)
Étale theta functions and mono-theta environments I
Abstract:
In this talk, first, I will review basic properties of a theta function on a tempered covering of a once-punctured elliptic curve over a p-adic local field. By means of the Kummer classes associated to theta functions, i.e., étale theta functions, we can construct mono-theta environments, which serve as a sort of bridge between tempered fundamental groups and tempered frobenioids. Finally, I will discuss various anabelian properties related to étale theta functions and mono-theta environments.

Étale theta functions and mono-theta environments II
Abstract:
In this talk, first, I will explain three rigidities — cyclotomic rigidity, constant multiple rigidity, and discrete rigidity — of a mono-theta environment. Next, I will introduce the notion of a multiradial functorial algorithm and discuss a multiradial interpretation of étale theta functions.

Seidai Yasuda (Osaka University)
Étale theta functions, mono-theta environments, and [IUTchI] §1 – §3, I - II
Abstract:
In the first talk, I will explain the notion of initial Θ-data that provides the initial setting where the IU Teichmüller theory is developed. Several types of Frobenioids are associated with the initial Θ-data. Among those the Frobenioids $\underline{\underline{\mathcal{F}}}$ at the "bad" primes are of especially strong importance. I will recall the construction in a local situation, i.e, starting from a Tate elliptic curve over a p-adic field K. The construction of $\underline{\underline{\mathcal{F}}}$ involves a lot of interesting knowledge of E. In the first and the second talk we will see that such background knowledge is weaved into the purely categorical structure of $\underline{\underline{\mathcal{F}}}$. Precise statements are mostly given as solutions to categorical reconstruction problems, i.e., problems of recovering once forgotten information on objects around E only from the categorical structure of $\underline{\underline{\mathcal{F}}}$. Especially we will see that how canonical isomorphisms between cyclotomes and the mono-theta environments are recovered. In the second talk, the other kinds of Frobenioids associated to a given initial Θ-data will be briefly explained.

Part II: Overall survey

Shinichi Mochizuki (RIMS, Kyoto University)

The mathematics of mutually alien copies: from Gaussian integrals to inter-universal Teichmüller theory I - VIII

Abstract:

Inter-universal Teichmüller theory may be described as a construction of certain

canonical deformations of the *ring structure* of a *number field*

equipped with certain auxiliary data, which includes an *elliptic curve* over the number field and a *prime number* ≥ 5. In the present talk, we survey this theory by focusing on the *rich analogies* between this theory and the classical computation of the *Gaussian integral*. The main *common features* that underlie these analogies may be summarized as follows:

- the introduction of *two mutually alien copies* of the object of interest;
- the computation of the effect — i.e., on the two mutually alien copies of the object of interest — of *two-dimensional changes of coordinates* by considering the effect on *infinitesimals*;
- the passage from *planar cartesian* to *polar coordinates* and the resulting *splitting*, or *decoupling*, into *radial* — i.e., in more abstract valuation-theoretic terminology, *"value group"* — and *angular* — i.e., in more abstract valuation-theoretic terminology, *"unit group"* — portions;
- the straightforward evaluation of the *radial portion* by applying the *quadraticity* of the exponent of the Gaussian distribution;
- the straightforward evaluation of the *angular portion* by considering the *metric geometry* of the *group of units* determined by a suitable version of the natural *logarithm* function.

[Here, the intended sense of the descriptive *"alien"* is that of its original Latin root, i.e., a sense of *abstract, tautological "otherness"*.] After reviewing the classical computation of the Gaussian integral, we give a detailed survey of inter-universal Teichmüller theory by concentrating on the *common features* listed above. If time allows, we will also briefly discuss various *historical aspects* of the mathematics that appears in inter-universal Teichmüller theory.

Part III: [IUTchI-II]

Seidai Yasuda (Osaka University)

Étale theta functions, mono-theta environments, and [IUTchI] §1 – §3, III

Abstract:

In the third talk, we discuss the notion of Θ-Hodge theater. In the final part of the third talk, I will explain some technical results in Section 2 of [IUTchI] concerning subgroups of tempered fundamental groups which are conjugate in profinite fundamental groups.

Fucheng Tan (Shanghai Center for Mathematical Sciences and Shanghai Jiao Tong University)

IUT-I: Hodge theaters and label classes of cusps 1-2

Abstract:

In these two talks, I will explain the construction of the various prime-strips associated to the initial theta-data. They carry two different kinds of label classes of cusps with multiplicative or additive symmetry, which are of arithmetic or geometric nature, respectively. Between these prime-strips, there are bridges arising

from the restrictions from (curves over) global fields to local fields and from the evaluations sections at cusps. The prime-strips and the bridges among them will form the Hodge theaters. The natural maps from a Hodge theater to the underlying prime-strips and bridges will carry symmetries related to the symmetry on the corresponding label classes of cusps.

Taylor Dupuy (Hebrew University and the University of Vermont)
Functors to and from mono-theta environments
Abstract:
I will review in a functorial way how mono-theta environments can be constructed from both groups and frobenioids, and compare the two constructions. I will also discuss some auxiliary constructions needed in Emmanuel's discussions on the evaluation of the theta classes and conjugacy synchronization.

Emmanuel Lepage (University Paris 6)
Hodge-Arakelov evaluation 1
Abstract:
I will discuss evaluation of theta monoids at bad places. These evaluation maps are obtained by pulling back the cohomology classes defining the theta monoids along Galois sections coming from some torsion points of our elliptic curves (that can be recovered group theoretically through elliptic cuspidalization) and will give rise to the Gaussian monoids. The main technicality will be to avoid indeterminacies coming from the fact that the different decomposition groups are only defined up to conjugacy: we will study how profinite conjugacy acts on tempered decomposition groups and how the conjugacies on the various decomposition groups can be synchronized through symmetrizing isomorphisms.

Taylor Dupuy (Hebrew University and the University of Vermont)
Multiradiality
Abstract:
I am going to introduce the notion of multiradiality of a functor. I will explain how and where this notion is used at various points in IUT.

Emmanuel Lepage (University Paris 6)
Hodge-Arakelov evaluation 2
Abstract:
We will keep on studying Gaussian Frobenioids and will quickly define Gaussian frobenioids at good places. Finally, I will define global realified Gaussian frobenioids, to get modified versions of the theta-link defined in IUT-I.

Part IV: [IUTchIII-IV] from the point of view of mono-anabelian transport

Yuichiro Hoshi (RIMS, Kyoto University)
[IUTchIII-IV] from the point of view of mono-anabelian transport I - IV
Abstract:
The main purpose of the present talk is to give an explanation of multiradial representations — i.e., descriptions in terms that make sense from the point of view of an alien arithmetic holomorphic structure — of log-shells, theta values, and number fields from the point of view of mono-anabelian transport. We also discuss, in the present talk, a diophantine inequality which may be regarded as a main application of the multiradial representations.

Part V: Related topics

Yuki Wada (RIMS, Kyoto University)
Near miss *abc*-triples in compactly bounded subsets
Abstract:
In this talk, we study the existence of near miss *abc*-triples in compactly bounded subsets. In more concrete terms, we prove that there exist infinitely many *abc*-triples such that: (1) the absolute value of *abc* exceeds a certain quantity determined by the product of the distinct prime numbers of *abc*, and, moreover, (2) a certain value determined by a, b, and c, which corresponds to the quantity "λ" in the Legendre form of an elliptic curve, lies in a given compactly bounded subset.

Vesselin Dimitrov (Yale University)
Notes on the epsilon term in the *abc* conjecture
Abstract:
I will explain why Theorem 1.10 of IUT-IV implies an effective *abc* inequality. The resulting bound is however much too large to be computationally feasible, until a more efficient way of producing Belyi maps is found. I will also formulate the *abc* conjecture on the modular curve $Y(2)$ in a way that (a) manifests the Granville-Stark relationship to the Siegel zero problem for quadratic imaginary fields (odd Dirichlet characters), and (b) suggests an extension of the *abc* conjecture in a different framework than the Vojta conjectures. As it turns out, if IUT theory could be extended to give the same "$\Theta - q$" bound in Corollary 3.12 with the Archimedean places included, this would forbid a Siegel zero for the odd Dirichlet characters.

Kobi Kremnitzer (The University of Oxford)
Milnor-Wood inequality and bounded cohomology
Abstract:
I will explain the construction of the various prime-strips associated to the initial theta-data. They carry two different kinds of label classes of cusps with multiplicative or additive symmetry,I will explain the Milnor-Wood inequality in term of bounded cohomology and its relation to the *abc* conjecture in the function field case. I will then discuss possible applications of bounded cohomology in the number field setting.

Ivan Fesenko (The University of Nottingham)
Reciprocity and IUT
Abstract:
I will discuss links between IUT and class field theory, which is not used in IUT, as well as two-dimensional class field theory. I will also discuss similarities between IUT and two two-dimensional adelic structures on proper models of elliptic curves over number fields and the computation of its two-dimensional zeta integral.

Boris Zilber (The University of Oxford)
Model theory of anabelian geometry
Abstract:
We define a category M of structures (in the model-theoretic sense) which is equivalent to the category of profinite groups. The morphisms of the category M are interpretations and the functor realising the equivalence takes the structure to its automorphism group. The étale fundamental group of a variety X over k can be

identified as an automorphism group of a certain object (structure) $\mathrm{Cov}(X)$ of the category M. We then study the structure $\mathrm{Cov}(X)$ and identify a model-theoretic property of $\mathrm{Cov}(X)$ which is sufficient (and probably necessary) for the section conjecture to hold for X.

Part VI: [IUTchIII-IV] with remarks on the function-theoretic roots of the theory

Go Yamashita (RIMS, Kyoto University)
[IUTchIII-IV] with remarks on the function-theoretic roots of the theory I - IV
Abstract:
We will explain IUTch-III and IUT-IV with a motivation of Θ-link from Hodge-Arakelov theory.

Part VII: Overall discussion and impromptu talks

Adam Topaz (University of California, Berkeley)
Kummer theory and anabelian geometry over algebraically closed fields
Abstract:
This talk will describe how Kummer theory and divisor-like groups are used in anabelian geometry over algebraically closed fields, specifically in the context of Bogomolov's program in birational anabelian geometry. Along the way, I will highlight some similarities and differences between this context and the anabelian geometry used in IUT.

Dinesh Thakur (Rochester University)
Diophantine approximation and deformation hierarchies in finite characteristic

Paul Vojta (University of California, Berkeley)
On changes to the Thue-Siegel method
Abstract:
I will describe a potential change to the Thue-Siegel method.