COMMENTS ON "THE GEOMETRY OF FROBENIOIDS II"

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(1.) In the second line of the final display of Example 1.1, (ii), the notation "$\mathrm{ord}(K^\times) \subseteq \Phi(K)$" should read "$\mathrm{ord}(K^\times) \subseteq \Phi(K)^{\mathrm{gp}}$".

(2.) In the proof of Theorem 2.4, (ii), the notation "$\mathcal{B}(G_i, G_i^\circ)$" should read "$\mathcal{B}^{\mathrm{temp}}(G_i, G_i^\circ)$".

(3.) In the display following the phrase "the assignment" in Example 3.3, (i), the notation "$\mathrm{ord}(K) \cong \mathbb{R}_{>0} \cong \mathbb{R}_{\geq 0}$" should read

$$\text{"}\mathrm{ord}(\mathcal{O}_K^\triangleright) \cong \mathbb{R}_{\geq 0}\text{".}$$

Also, in the explanation following this display, the phrase "[where the isomorphism $\mathbb{R}_{>0} \xrightarrow{\sim} \mathbb{R}_{\geq 0}$ is given by the natural logarithm], then" should read

> "[where $\mathcal{O}_K^\triangleright \subseteq K^\times$ denotes the multiplicative submonoid of elements of norm ≤ 1, and the isomorphism $\mathrm{ord}(\mathcal{O}_K^\triangleright) \cong \mathbb{R}_{\geq 0}$ is given by *minus* the natural logarithm], then".

(4.) The observation "Observe that all *real* objects of $\mathcal{N}_0$ are *isomorphic*." at the beginning of Example 3.3, (iv), is correct as stated, but may be replaced by the *stronger* observation

> "Observe that all *real* objects of $\mathcal{N}_0$ are *isomorphic*, and all morphisms between such objects are *isomorphisms*.".

(5.) The following sentence should be added to the end of Definition 3.3, (iv):

> Finally, we shall refer to as the *angular region* of an object of $\mathcal{C}$, $\mathcal{A}$, $\mathcal{N}$, or $\mathcal{R}$ the angular region of the object obtained by projecting the given object to $\mathcal{C}_0$.

(6.) In the proof of Proposition 3.5, (iii), the notation "${}_C\mathcal{F}$" should read "${}_C\mathcal{G}[\mathbb{C}]$".

(7.) In the first displayed diagram of the proof of Theorem 5.5, the notation "$\Psi^{\square}$" should read "$(\Psi^{\mathrm{pf}})^{\mathrm{birat}}$".

(8.) In Example 5.6, (iii), (b), the phrase "for some group-like functor $\mathcal{D}_v \to \mathfrak{Mon}$" should read "for some group-like functor $\Phi_v^{\mathrm{cnst}} : \mathcal{D}_v \to \mathfrak{Mon}$".

(9.) In Example 1.1, (i), (ii), it is to be understood that $\Phi_0^{\mathbb{Z}} \stackrel{\mathrm{def}}{=} \Phi_0$, $\mathbb{B}_0^{\mathbb{Z}} \stackrel{\mathrm{def}}{=} \mathbb{B}_0$.

(10.) In the two lines of the final display of Example 1.1, (ii), the notation "$\Phi(K)$" should be replaced by "$\Phi(A)$"; the phrase "for every $\mathrm{Spec}(K) \in \mathrm{Ob}(\mathcal{D}_0)$, then" should be replaced by the phrase "for every $A \in \mathrm{Ob}(\mathcal{D})$, where we denote the image of A in $\mathcal{D}_0$ by $\mathrm{Spec}(K) \in \mathrm{Ob}(\mathcal{D}_0)$, then".

(11.) In the notation "$\mathrm{Aut}_{\mathcal{F}_A}(B)$" that appears in Definition 3.1, (v), the "B" is to be understood as the object of $\mathcal{F}_A$ determined by the morphism $B \to A$ of $\mathcal{F}$.

(12.) All tempered groups [hence also profinite groups that are regarded as tempered groups] (respectively, all [quasi-]temperoids) that appear in the present paper should be assumed to be equipped with a topology that admits a *countable basis* (respectively, assumed to be connected [quasi-]temperoids associated to such tempered groups). This assumption is necessary in order to ensure that the index sets of *"universal covering pro-objects"* implicit in the definition of the *tempered fundamental group* associated to a connected temperoid [cf. [Mzk2], Remark 3.2.1] may to be taken to be *countable.* This countability of the index sets involved implies that the various objects that constitute such a universal covering pro-object admit a *compatible system of basepoints*, i.e., that the *obstruction* to the existence of such a compatible system — which may be thought of as an element of a sort of "non-abelian $\mathbb{R}^1 \varprojlim$" — *vanishes.* In order to define the tempered fundamental group in an intrinsically meaningful fashion, it is necessary to know the existence of such a compatible system of basepoints.

(13.) In Remark 3.5.1, the phrase "since, for instance in the case of" should read "since, for instance, in the case of".

(14.) The following [essentially formal] modifications should be made to the proof of Proposition 3.4, (viii):

(i) In the fourth paragraph of this proof: "On the other hand, β" should read "On the other hand, if β is *not* an *isomorphism*, then β"; "we conclude that ϕ" should read "we conclude that if ϕ is *not* an *isomorphism*, then ϕ".

(ii) In the fifth paragraph of this proof: "of FSMI-morphisms $\phi_1, \ldots, \phi_n$ such that the domain of ϕ is equal to A" should read "of a morphism ϕ_1 whose domain is equal to A with FSMI-morphisms $\phi_2, \ldots, \phi_n$"; "If ϕ_j projects" should read "If, for $j \geq 2$, ϕ_j projects".

(iii) In the sixth paragraph of this proof: all instances of the term "FSMI-morphisms" should be replaced by the phrase "FSMI-morphisms/isomorphisms [i.e., morphisms which are either FSMI-morphisms or isomorphisms]".

(15.) The following [essentially formal] modifications should be made to Defnition 5.3, (v); Proposition 5.4; the statement and proof of Theorem 5.5, (iv):

(i) In Definition 5.3, (v), the text "[where "pf" is defined whenever $\mathfrak{C}$ is of *Frobenius-isotropic* type]" should read as follows: "[where "pf" is defined whenever $\mathfrak{C}$ is of *Frobenius-isotropic* type; "birat" is defined whenever $\mathfrak{C}$ is of *birationally Frobenius-normalized* type]".

(ii) In the statement and proof of Proposition 5.4, the term "*Frobenius-normalized*" should be replaced by the term "*birationally Frobenius-normalized*" (2 instances).

(iii) In the statement of Theorem 5.5, (iv), the phrase "*of* **poly-non-group-like** *type*" should read "*of* **poly-non-group-like** *and* **poly-birationally Frobenius-normalized** *type*". In the proof of Theorem 5.5, the text "of *standard*, *perfect*" should read "of *standard*, *birationally Frobenius-normalized* [cf. [Mzk5], Proposition 3.2, (ii)], *perfect*".

Bibliography

[FrdI] S. Mochizuki, The Geometry of Frobenioids I: The General Theory, *Kyushu J. Math.* **62** (2008), pp. 293-400.