Abstracts

Numerically reflective involutions of Enriques surfaces

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A (holomorphic) automorphism of an Enriques surface S is *numerically reflective* (resp. *numerically trivial*) if it acts on the $\mathbb{Q}$-cohomology group $H^2(S, \mathbb{Q})(\simeq \mathbb{Q}^{10})$ by reflection (resp. trivially). For K3 surfaces we have

- a numerically trivial automorphism is trivial, and
- no automorphisms are numerically reflective.

But these are no more true for Enriques surfaces. In my talk I summarized the classification and gave a very rough sketch of the proof. The details will be published elsewhere.

1. Numerically trivial involutions

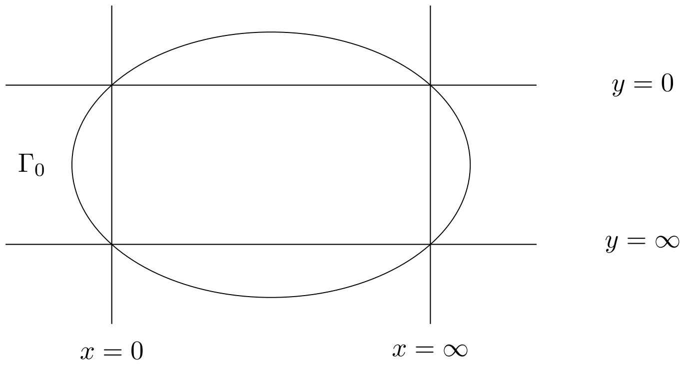
Let X_{BP} be the minimal model of the function field

$$\mathbb{C}\left(x, y, \sqrt{a(x+\frac{1}{x}) + b(y+\frac{1}{y}) + 2c}\right) \tag{1}$$

of two variables, where $a, b \in \mathbb{C}^\times$ and $c \in \mathbb{C}$ are constants. X_{BP} is the mimimal resolution of the double $\mathbb{P}^1 \times \mathbb{P}^1$ with branch the union of the coordinate quadrilateral and the curve

$$\Gamma_0 : a(x^2+1)y + bx(y^2+1) + 2cxy = 0 \tag{2}$$

of bidegree $(2, 2)$.



Assume further that $a \pm b \pm c \neq 0$. Then the involution $\varepsilon : (x, y, \sqrt{\ }) \mapsto (1/x, 1/y, -\sqrt{\ })$ has no fixed points on X_{BP}. Hence the quotient $S_{BP} = X_{BP}/\varepsilon$ is an Enriques surface. Let σ_{BP} be the involution of S_{BP} induced from the covering involution $\sqrt{\ } \mapsto -\sqrt{\ }$ of X_{BP}. Then σ_{BP} is *homologically trivial*, that is, its acts on the $\mathbb{Z}$-homology group $H_2(S_{BP}, \mathbb{Z})$ trivially ([1, (4.8)], [4, Exmaple 2]).

Theorem 1 *Every homologically trivial automorphism of an Enriques surface is either trivial or the above involution $\sigma_{BP} \curvearrowright S_{BP}$.*

Theorem 2 ([2]) *Let σ be a numerically trivial involution of an Enriques surface, and assume that σ is neither trivial nor σ_{BP}. Then the universal cover is a Kummer surface $Km(E_1 \times E_2)$ of product type and σ is either of Liberman type (*[4, Exmaple 1]*) or Kondo-Mukai type (*[4, Exmaple 2]*, see also* §*2).*

2. Numerically reflective involutions

Let X_{GBP} be the minimal model of the field

$$\mathbb{C}\left(x, y, \sqrt{a(x+\frac{1}{x}) + b(y+\frac{1}{y}) + c(\frac{x}{y}+\frac{y}{x}) + 2d}\right), \tag{3}$$

where $a, b, c \in \mathbb{C}^\times$ and $d \in \mathbb{C}$ are constants. X_{GBP} is the mimimal resolution of the double $\mathbb{P}^2$ with branch the union of the coordinate triangle and the cubic curve

$$\Gamma_1 : a(x^2+1)y + bx(y^2+1) + c(x^2+y^2) + 2dxyz = 0. \tag{4}$$

Assume further that

$$(a+b+c+d)(a+b-c-d)(a-b+c-d)(a-b-c+d) \neq 0. \tag{5}$$

Then the involution $\varepsilon : (x, y, \sqrt{\ }) \mapsto (1/x, 1/y, -\sqrt{\ })$ has no fixed points and we obtain the Enriques quotient $S_{GBP} := X_{GBP}/\varepsilon$. The involution σ_{GBP} of S_{GBP} induced from the covering involution is numerically reflective if (4) is irreducible and numerically trivial otherwise. By [2, Remark 9], σ_{GBP} is equivalent to [2, Example 2] in the latter case.

Let C be a curve of genus 2 and G a Göpel subgroup of the 2-torsion group of its Jacobian $J(C)$. For a non-bielliptic pair (C, G), we constructed an Enriques surface $Km(C)/\varepsilon_G$ and a numerically reflective involution $\sigma_G \curvearrowright Km(C)/\varepsilon_G$ in [3], where $Km(C)$ is the Kummer surface of $J(C)$.

Theorem 3 *Let σ be a numerically reflective involution of an Enriques surface S. Then either*

(1) *σ is isomorphic to the involution σ_{GBP}, or*
(2) *the universal cover of S is isomorphic to the Jacobian Kummer surface $Km(C)$ and (S, σ) is isomorphic to $(Km(C)/\varepsilon_G, \sigma_G)$ for a curve C of genus 2 and a Göpel subgroup G.*

References

[1] Barth, W. and Peters, C.: Automorphism of Enriques surfaces, Invent. math., **73**(1983), 383–411.
[2] Mukai, S.: Numerically trivial involutions of Enriques surfaces, RIMS preprint #1544, May 2006.
[3] Mukai, S.: Kummer's quartics and numerically reflective involutions of Enriques surfaces, RIMS preprint #1633, June 2008 (http://www.kurims.kyoto-u.ac.jp/preprint/file/RIMS1633.pdf).
[4] Mukai, S. and Namikawa, Y.: Automorphisms of Enriques surfaces which act trivially on the cohomology groups, Invent. math., **77**(1984), 383–397.

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