Lecture Announcement
– A Geometric Construction of Algebras –

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The theme of lectures is the interplay of representation theory and geometry. I will explain a geometric construction of affine Lie algebras (or their q-analogue) and their representations. The spaces, which I will use, are moduli spaces of vector bundles over complex surfaces or their variant. These spaces have been studied intensively from a geometric point of view, but their relation to the representation theory is a new and hot topic.

Our method is an application of more general technique which has been used successfully in the representation theory during the last several decades. It is the construction of algebras by the *convolution product*, defined on homology groups (or their variants) of manifolds. For example, Weyl groups and affine Hecke algebras were constructed by convolutions on homology groups and equivariant K-homology groups of flag varieties (Springer, Borho-MacPherson, Lusztig, Ginzburg, Kazhdan-Lusztig, etc). Also upper triangular parts of quantum enveloping algebras and their canonical bases were constructed by convolutions using perverse sheaves on moduli spaces of representations of quivers (Lusztig).

More precise plan of the lectures is the following: I will first prepare geometric staff, i.e., the convolution product on homology groups. Then I will introduce Hilbert schemes of points on complex surfaces, and connect their homology groups with infinite dimensional Heisenberg algebras. Then I will introduce *quiver varieties*, and study their homology groups. As an application, we get a geometric construction of Kashiwara's crystal base. If I still have time, I will explain a construction of quantum affine algebras via the convolution on the equivariant K-groups of quiver varieties.

References.

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