Kyoto Operator Algebra Theory Workshop

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RIMS 420 and Zoom, 2022 January 13 & 14 (Time Zone: JST=UTC+9)
Organizer: Benoit Collins

January 13 (Thu)

15:30∼16:30 **Ryosuke Sato (Nagoya)**
Character theory of the BC-type infinite-dimensional quantum groups and Markov semigroups generated by characters of quantum group

17:30∼18:00 **Laura Mančinska (Copenhagen)**
Certification of quantum devices via operator-algebraic techniques

January 14 (Fri)

09:30∼10:30 **Mike Brannan (Waterloo)**
Crossed product equivalence of quantum automorphism groups of finite dimensional C*-algebras

11:00∼12:00 **Todd Kemp (San Diego)**
The PDE Approach to Brown Measure for Free Heat Flow and "Random Walks"

13:30∼14:30 **Sheng Yin (Kyoto)**
Atoms, zero divisors and Atiyah conjecture

Abstracts

R. Sato. Character theory of the BC-type infinite-dimensional quantum groups and Markov semigroups generated by characters of quantum group

In this talk, we will discuss the asymptotic representation theory for the inductive limits of compact quantum groups of type BC and mainly investigate the characters of these infinite-dimensional quantum groups. We give complete parameterizations of extreme characters of these quantum groups and some concrete examples of characters based on BC-analog Voiculescu functions. Moreover, we discuss Markov semigroups generated by characters of these quantum groups, which are BC-analog of our previous work on type A.

L. Mančinska. Certification of quantum devices via operator-algebraic techniques

In this talk I will introduce the concept of self-testing which aims to answer the fundamental question of how do we certify proper functioning of black-box quantum devices. We will see that operator-algebraic techniques can be applied to this area and that there is a close link between self-testing and stability of algebraic relations. We will leverage this link to propose a family of protocols capable of certifying quantum states and measurements of arbitrarily large dimension with just four binary-outcome measurements. One of our main proof ingredients is a certain algebraic analogue of Gowers-Hatami stability theorem for group representations.

This is a joint work with Chris Schafhauser and Jitendra Prakash.

M. Brannan. Crossed product equivalence of quantum automorphism groups of finite dimensional C*-algebras

Given a finite dimensional C*-algebra B equipped with a faithful state f, one can associate to the pair (B, f) a universal compact quantum group which acts on the C*-algebra B in a state-preserving way. This quantum group is called the quantum automorphism group of (B, f), and can be thought of as a quantum analogue of a permutation group. In this talk, I will describe some recent advances on the structure theory of the II_1-factors associated to these quantum groups in the case where the state f is chosen to be the canonical "Plancharel" trace. In this case we show that, up to crossed products by finite group actions, these II_1 factors only depend on the dimension of the C*-algebra B, and not the particular form of B as a direct sum of matrix algebras. Our results provide new examples of strongly 1-bounded II_1-factors, and also new examples of "linear" discrete quantum groups.

This is joint work with Floris Elzinga (Oslo), Samuel Harris (Texas A&M University), and Makoto Yamashita (Oslo).

T. Kemp. The PDE Approach to Brown Measure for Free Heat Flow and "Random Walks"

Brown measure, a substitute for spectral measure for non-normal operators in tracial von Neumann algebras, is notoriously difficult to compute. In 2019, in joint work with Driver and Hall, I introduced a new methodology to compute Brown measures of operators flowing under free analogs of the heat semigroup. The technique involves a regularized version of Brown

measure (which is robust under random matrix models), where the regularizing parameter then becomes an extrinsic variable in a Hamilton–Jacobi PDE. Standard PDE methods can solve these PDEs, but the question of removing the (now dependently flowing) regularization is quite delicate.

In this talk, I will discuss our original program which allowed for the complete calculation of the Brown measure of the free multiplicative Brownian motion: the large-N limit of the Brownian motion on the Lie group $\mathrm{GL}(N)$. I will then discuss recent progress using our methods by several groups of researchers (Ho, Zhong, Demni, Hamdi, Hall, Nowak, and others) to compute Brown measures in other additive and multiplicative free heat flow problems. Finally, I will describe my very recent joint work with Nemish to compute the Brown measure of a free multiplicative "random walk", verifying a conjecture from the Physics literature.

S. Yin. Atoms, zero divisors and Atiyah conjecture

For a discrete group G, it is conjectured that the group algebra $\mathbb{C}G$ is a domain (i.e., does not contain nontrivial zero divisor) if and only if the group G is torsion-free. This purely algebraic question is known as Kaplansky's zero divisor conjecture. It can be easily deduced from Atiyah conjecture on L^2 Betti numbers (of torsion free group). Moreover, for a discrete group that may have finite subgroups, Atiyah conjecture then predicts that its L^2 Betti numbers have to be rational numbers under some conditions. In this talk, we will rephrase these problems with an operator algebraic as well as a (free) probabilistic point of view with the help of notion of kernel, rank and atom. Actually, in free probability we are interested in the calculation of atoms for free random variables, which are intrinsically related to the above mentioned questions. Recently an algebraic method were developed to do such calculation. We will present our new calculation machinery along with the consequences that we can draw for Atiyah conjecture.

This talk is based on two recent joint-works: one with Tobias Mai and Roland Speicher and the other with Octavio Arizmendi, Guillaume Cébron and Roland Speicher.