小沢 登高 (OZAWA Narutaka)

A. 研究概要

2009 年度は離散群と関数解析についての研究を行った．群の Hilbert 空間上の表現について考えるとき, ユニタリ表現に関しては美しい理論があるが, ユニタリでない無限次元表現はまったくとらえがたい対象である．考えている群が従順ならば, 任意の一様連続な表現はユニタリ表現と相似（共役ともいう）になることが知られているが, その逆も成り立つかを問うのが Dixmier の相似問題 (1950) である．私は, スイス連邦工科大学の N. Monod 教授との共同研究 [8] においてこの問題に挑戦し, 部分的な解答を得た．特に, この問題の試金石とされていた自由 Burnside 群（従順でないことが知られている）にユニタリ化可能でない一様連続な表現が存在することを示した．従順性の対極に位置する群の性質として, Kazhdan の性質 (T) があり, それを強めたものに Burger–Monod の性質 (TT) がある．私は, 論文 [9] において, それをさらに強めた性質 (TTT) を導入し, $\mathrm{SL}(n \geq 3, \mathbf{R})$ とその格子がこの性質 (TTT) を持つことを示した．この定理の系として, これらの格子から従順群あるいは双曲群への擬順同型は有限の像を持つことが示される．性質 (TTT) には Ulam 型の問題に対する応用も見込まれる．

In the academic year 2009, N. Ozawa studied functional analytic aspects of discrete groups. Compared with unitary representations, there is barely any general theory for uniformly continuous representations on (infinite dimensional) Hilbert spaces. It is known that an amenable group is unitarizable in the sense that every uniformly continuous representation of it is similar (i.e., conjugate) to a unitary representation. Dixmier's similarity problem asks whether the converse also holds true: Does unitarizability imply amenability? N. Ozawa, in collaboration with N. Monod (EPFL), tackled this problem and obtained a partial solution ([8]). In particular, it was proved that certain Burnside groups are not unitarizable. Burnside groups had been known as tests for the similarity problem. On the opposite side of amenability are Kazhdan's property (T), and its stronger sibling, property (TT) of Burger and Monod. Adding more rigidity to (TT), N. Ozawa introduced property (TTT) and proved that $\mathrm{SL}(n \geq 3, \mathbf{R})$ and their lattices have that property ([9]). As a corollary, it was proved that every quasi-homomorphism from such a lattice into an amenable group or a hyperbolic group has finite image. This generalizes a well-known fact for homomorphisms. It is expected that property (TTT) is also useful in study of the Ulam type problems.

B. 発表論文

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5. N. Ozawa and S. Popa; "On a class of II_1 factors with at most one Cartan subalgebra," Ann. of Math. (2), accepted.
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7. N. Ozawa and S. Popa; "On a class of II_1 factors with at most one Cartan subalgebra II," Amer. J. Math., accepted.
8. N. Monod and N. Ozawa; "The Dixmier problem, lamplighters and Burnside groups," J. Funct. Anal., **258** (2010), 255–259.
9. N. Ozawa; "Quasi-homomorphism rigidity with noncommutative targets," J. Reine Angew. Math., accepted.

C. 口頭発表

1. *On a class of* II_1 *factors with at most one Cartan subalgebra*; (1) Topics in von Neumann Algebras, BIRS, March 08. (2) 東大作用素環セミナー, April 08. (3) Operator Algebras, Dynamics, and Classification, Texas A&M University, August 08. (4) Non-commutative Harmonic Analysis with Applications to Probability, Będlewo, August 08. (5) Analytic Properties of Infinite Groups, Genève, August 08. (6) von Neumann Algebras and Ergodic Theory of Group Actions, Oberwolfach, October 08. (7) Harmonic analysis, operator algebras and representations, CIRM, November 08.

2. *von Neumann algebras and ergodic theory* (Minicourse); (1) von Neumann algebras, Ergodic theory and Geometric Group theory, IMSc (Chennai), February 09. (2) Ergodic Theory of Group Actions, Göttingen, August 09.

3. *Dixmier's Similarity Problem*; (1) 東北大学情報数理談話会, April, 09. (2) Noncommutative L_p spaces, operator spaces and applications, CIRM, June 09. (3) 東大作用素環セミナー, July 09. (4) Geometry and Rigidity of Groups, Münster, August 09. (5) Operators and Operator Algebras, Edinburgh, December 09. (6) 東大数理談話会, December 09.

4. *Hyperlinearity, sofic groups and applications to group theory* (Mini Course); Operator Spaces and Approximation Properties of Discrete Groups, Texas A&M University, August 09.

5. *Quasi-homomorphism rigidity with non-commutative targets*; Rigidity in cohomology, K-theory, geometry and ergodic theory, HIM (Bonn), November 09.

D. 講義

1. 解析学 IV（理数・3 年）・解析学特別演習 I: 測度論と Lebesgue 積分論. 演習つき.

2. 数学 IA（理 I・1 年）：微積分学.

3. 数理科学 V（理系・2 年）：微積分学の復習, 距離空間論初歩, Fourier 展開初歩.

4. 数学 II（社会科学）：線形代数.

E. 修士・博士論文

1. (博士) 水田 有一 (MIZUTA Naokazu): Weak amenability for a group acting on a finite dimensional CAT(0) cube complex.

2. (修士) 麗β 欽龍 (LI Qinlong): Nuclearity of free product C*-algebras.

G. 受賞

1. ICM 招待講演 (Operator Algebras and Functional Analysis), 2006 年 8 月.

2. 春季賞 (日本数学会), 2009 年 4 月.

3. 日本学術振興会賞, 2010 年 3 月.