

小澤 登高 (OZAWA, Narutaka)

#### A. 研究概要

2011 年度は Banach 環, 距離幾何学 (純代数的) 環論等の雑多な研究を行った.

1. 「任意の可縮な Banach 環は有限次元半単純なものに限るか?」という有名未解決問題に取り組み, 適当な仮定のもとでそれが成り立つことを示した. 即ち, 一様近似性を持つ Banach 空間の上に忠実な作用を持つような可縮な Banach 環は有限次元半単純に限る. 一般の場合にはワイルドな反例が存在すると考えられている.

2. (粗い) 距離空間の漸近次元に関して, 有限次元ならばその距離空間は従順という Higson–Roe による良く知られた定理がある. 本年度の研究では, Dranishnikov & Sapir の問題に答えて, より一般に漸近次元増大度が劣指数関数的でも同じ結論が成り立つことを示した. この結果は, 「Thompson 群が従順か否か?」という有名未解決問題とゆるく関係している.

3. (単位元を持つとは限らない) 環  $R$  は  $R = \text{span } R^2$  を満たすときベキ等であると言われる. 単位元を持つ環はもちろんベキ等である. 私は N. Monod (EPFL) 及び A. Thom (Leipzig) との共同研究で, 「有限生成ベキ等環  $R$  は常にイデアルとして単元生成か?」という問題に取り組み, 適当な仮定のもとでそれが成り立つことを示した. 特に, 可換環 (Kaplansky の定理), 有限環, 半群環に対して成り立つことを確認した. 一般の場合は正しくないと考えられている. この問題は群論における未解決問題である Wiegold 問題と密接に関係している.

In the academic year 2011, N. Ozawa tried to solve longstanding problems in various subjects. (1): When B.E. Johnson initiated the systematic study of homological aspects of Banach algebras, he asked whether every *con-*

*tractible* Banach algebra is necessarily finite-dimensional. I proved it is the case if the contractible Banach algebra  $A$  admits a faithful action on a Banach space  $V$  with the uniform approximation property. This result generalizes a theorem of Paulsen and Smith who proved the same in the case  $V$  being a Hilbert space. (2): It is a well-known result of Higson and Roe that a coarse metric space  $X$  with finite *asymptotic dimension* is coarsely amenable. I generalized this by proving that the same conclusion still holds if  $X$  has subexponential asymptotic dimension growth. This answers a problem raised by Dranishnikov and Sapir, and may have something to do with the famous problem whether Thompson’s group  $F$  is amenable. (3): A *rng* is a ring which may not have a unit. A *rng*  $R$  is said to be an *irng* if it is *idempotent*, i.e.,  $R = \text{span } R^2$ . In a joint work with N. Monod (EPFL) and A. Thom (Leipzig), I studied the problem whether every finitely generated *irng* is singly generated as an ideal. It is proved to be the case for commutative *rngs* (Kaplansky’s theorem), finite *rngs* and semi-group *rngs*. This problem has a close connection to the Wiegold problem in group theory.

#### B. 発表論文

1. N. P. Brown and N. Ozawa; “ $C^*$ -algebras and finite-dimensional approximations,” Graduate Studies in Mathematics, 88. American Mathematical Society, Providence, RI, 2008. xvi+509 pp.
2. N. Ozawa and S. Popa; “On a class of  $\text{II}_1$  factors with at most one Cartan subalgebra,” Ann. of Math. (2), **172** (2010), 713–749.

3. N. Ozawa and S. Popa; “On a class of  $\text{II}_1$  factors with at most one Cartan subalgebra II,” Amer. J. Math., **132** (2010), 841–866.
  4. N. Monod and N. Ozawa; “The Dixmier problem, lamplighters and Burnside groups,” J. Funct. Anal., **258** (2010), 255–259.
  5. N. Ozawa; “Quasi-homomorphism rigidity with noncommutative targets,” J. reine angew. Math., **655** (2011), 89–104.
  6. M. Burger, N. Ozawa and A. Thom; “On Ulam stability,” Israel J. Math., accepted.
  7. N. Ozawa; “Examples of groups which are not weakly amenable,” Kyoto J. Math., **52** (2012), 333–344.
  8. N. Ozawa; “Metric spaces with subexponential asymptotic dimension growth,” Internat. J. Algebra Comput., accepted.
  9. N. Ozawa; “A remark on contractible Banach algebras,” Kyushu J. Math., accepted.
  10. N. Monod, N. Ozawa and A. Thom, “Is an irng singly generated as an ideal?” Internat. J. Algebra Comput., accepted.
- C. 口頭発表
1. *Quasi-homomorphism rigidity with noncommutative targets*; (1) Rigidity in cohomology,  $K$ -theory, geometry and ergodic theory, HIM (Bonn), November 09. (2) Colloquium at University of Hawaii, March 10. (3) Danish-Norwegian Operator Algebra Seminar, Copenhagen, April 10. (4) Geometry and Ergodic Theory Seminar, EPFL, April 10. (5) Recent Developments in Operator Algebras, 東京大学, June 10. (6) Satellite Conference on Operator Algebras to ICM 2010, IMSc (Chennai), August 10. (7) Séminaire d’Algèbres d’Opérateurs, Institut de Mathématiques de Jussieu, September 10. (8) AMS Fall Western Section Meeting, UCLA, October 10.
  2. *Survey on weak amenability for groups*; (1) Japanese Western Operator Algebra Seminar, Awarayunomachi, December 10. (2) Finite Dimensional Approximations of Discrete Groups, Oberwolfach, May 11. (3)  $\text{II}_1$  factors: rigidity, symmetries and classification, IHP (Paris), May 11. (4) Workshop in Analysis and Probability, Texas A&M University, July 11. (5) Banach Algebras 2011, Waterloo, August 11. (6)  $C^*$ -Algebras and Related Topics, RIMS, September 11. (7) von Neumann Algebras and Related Topics, RIMS, January 12.
  3. *Weak amenability for groups and its applications (Mini Course)*; (1) Von Neumann algebras and ergodic theory of group actions, IHP (Paris), May 11. (2) Winter School on Operator Algebras, RIMS, December 11.

#### D. 講義

##### 基礎数学からの展開 B (理数・2年)

自然数全体の集合  $\mathbb{N}$  を Stone–Čech コンパクト化して得られるコンパクト左半位相半群における算術を扱うことにより, 自然数と加法に関する幾つかの定理を導いた.

## F. 対外研究サービス

1. 学術雑誌「Groups, Geometry, and Dynamics」の editor.
2. 数理解析研究所年間プロジェクト研究 : 「作用素環とその応用」の co-organizer

## G. 受賞

1. ICM 招待講演 (Operator Algebras and Functional Analysis), 2006 年 8 月.
2. 春季賞 (日本数学会), 2009 年 4 月.
3. 日本学術振興会賞, 2010 年 3 月.