Dimension groups for self-similar maps and associated C^*-algebras

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Abstract. This is a joint work with Tsuyoshi Kajiwara.

Dimension groups for topological Markov shifts were introduced and studied by Krieger motivated by the K-groups for the AF-subalgebras of the associated Cuntz-Krieger algebras. The dimension group is an ordered abelian group with a canonical automorphism. The dimension group for topological Markov shifts is very important, because it is a complete invariant up to shift equivalence. Matsumoto introduced a class of C^*-algebras associated with general subshifts and studied dimension groups for subshifts . We introduce dimension groups for self-similar maps as the K_0-groups of the cores of C^*-algebras associated with self-similar maps, where the core is the fixed point subalgebra under the gauge action of the C^*-algebra associated with a self-similar map. But canonical automorphisms on dimension goups can be generalized as only endomorphisms in the case of self-similar maps. The key step for the computation is an explicit description of the cores as the inductive limit using matrix representations over the coefficient algebra, which heavily depends on the structure of branched points.

Self-similar maps on compact metric spaces produce many interesting self-similar sets like Sierpinski Gasket. We constructed C^*-algebras for self-similar maps using C^*-correspondence and Pimsner construction. C^*-algebras associated with self-similar maps are simple, purely infinite and in UCT class, and are classifiable by their K-groups. The algebra of the continuous functions on the self-similar set is shown to be maximal abelian.

We express the K groups of the cores of C^*-algebras associated with tent map and Sierpinski Gasket respectively as inductive limit. We compute that the dimension group for the tent map is isomorphic to the countably generated free abelian group $\mathbb{Z}^\infty \cong \mathbb{Z}[t]$ together with the unilatral shift, i.e. the multiplication map by t as an abstract group. Thus the canonical endomorphisms on the K_0-groups are not automorphisms in geneal. This is a different point compared with dimension groups for topological Markov shifts. We can count the singularity structure in the dimension groups.