REPORT PROBLEM 02

Problem 3. *Let T be a self-adjoint operator on a Hilbert space $\mathcal{H}$ such that $\|T\| = 1$, and let E denote the corresponding projection-valued measure:*

$$T = \int_{-1}^{1} t \, dE(t).$$

Assume that $1 \in \operatorname{supp}(E)$ but $E(\{1\}) = 0$. Prove that there is unit vector $\xi \in \mathcal{H}$ such that the probability measure $E_\xi(\,\cdot\,) := \langle E(\,\cdot\,)\xi, \xi\rangle$ satisfies $1 \in \operatorname{supp}(E_\xi)$ but $E_\xi(\{1\}) = 0$.

Problem 4. *Let $b\colon G \to \mathcal{H}$ be a μ-harmonic cocycle. Prove that*

$$\int_G \|b(x)\|^2 \, d\mu^{*n}(x) = n \int_G \|b(x)\|^2 \, d\mu(x)$$

for every $n \in \mathbb{N}$.