The existence and non-existence of positive solutions of the nonlinear Schrodinger equations for one dimentional case

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We consider the following Schrodinger equations:

$$-\Delta u + (1 + b(x))u = f(u) \quad \text{in } \mathbf{R}^N, \quad u \in H^1(\mathbf{R}^N), \tag{$*$}$$

where $b(x)$ satisfies $1 + b(x) \geq 0$, $\lim_{|x|\to\infty} b(x) = 0$ and $\limsup_{|x|\to\infty} e^{\beta|x|} b(x) \leq 0$ for some $\beta > 2$ and a typical example of our $f(u)$ is u^p. In this talk, we mainly consider about one dimentional case $N = 1$.

By the concentration compactness arguments, we see that $(*)$ has at least a positive solution for the case $b < b_0$, here b is a mountain pass value of the functional corresponding to $(*)$ and b_0 is a mountain pass value corresponding to the limiting problem $-\Delta u + u = f(u)$ in $\mathbf{R}^N$, $u \in H^1(\mathbf{R}^N)$. In this talk, we also consider the case $b = b_0$. When $N \geq 2$ and $b = b_0$, we can also show the existence of the positive solution of $(*)$ by the Bahri-Li's minimax procedure. When $N = 1$ and $b = b_0$, depending on the $b(x)$, $(*)$ has at least a positive solution or no non-trivial solutions.