

Analyticity for the Navier-Stokes equations

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Abstract

We study the Cauchy problem for the incompressible Navier-Stokes equations

$$u_t - \Delta u + u \cdot \nabla u + \nabla p = 0, \quad \operatorname{div} u = 0, \quad u(0, x) = u_0.$$

We show the analyticity of the local solutions of the Navier-Stokes equation with any initial data in critical Besov spaces $\dot{B}_{p,q}^{n/p-1}(\mathbb{R}^n)$ with $1 < p < \infty$, $1 \leq q \leq \infty$ and the solution is global if u_0 is sufficiently small in $\dot{B}_{p,q}^{n/p-1}(\mathbb{R}^n)$. In the case $p = \infty$, the analyticity for the local solutions of the Navier-Stokes equation with any initial data in modulation space $M_{\infty,1}^{-1}(\mathbb{R}^n)$ is obtained. Similar results also hold for the generalized Navier-Stokes equation.