– Abstracts –

Strong instability of standing waves for nonlinear Schrödinger equations with a partial confinement

Masahito Ohta (Tokyo University of Science)

We consider the nonlinear Schrödinger equation with a one-dimensional harmonic potential

$$i\partial_t u = -\Delta u + x_N^2 u - |u|^{p-1}u, \quad (t,x) \in \mathbf{R} \times \mathbf{R}^N, \tag{1}$$

where $N \geq 2$, x_N is the N-th component of $x = (x_1, ..., x_N) \in \mathbf{R}^N$, Δ is the Laplacian in x, and $1 < p < 1 + 4/(N-2)$.

For $\omega \in (-1, \infty)$, let $\phi_\omega(x)$ be a ground state for the stationary problem

$$-\Delta\phi + x_N^2\phi + \omega\phi - |\phi|^{p-1}\phi = 0, \quad x \in \mathbf{R}^N.$$

We prove that if $1 + 4/(N-1) \leq p < 1 + 4/(N-2)$, then for any $\omega \in (-1, \infty)$, the standing wave solution $e^{i\omega t}\phi_\omega(x)$ of (1) is strongly unstable by blowup. Notice that Bellazzini, Boussaïd, Jeanjean and Visciglia (2017) recently constructed orbitally stable standing wave solutions of (1) for the case $1 + 4/N < p < 1 + 4/(N-1)$.

Modified scattering for the Klein-Gordon equation with the critical nonlinearity in two and three dimensions

Jun-ichi Segata (Tohoku University)

We consider the long time behavior of solution to the Klein-Gordon equation with the gauge invariant critical nonlinearity in two and three dimensions. Notice that the scattering for the Klein-Gordon equation with the gauge invariant critical nonlinearity in higher dimensions was out of scope in the previous works due to the lack of smoothness of the nonlinear term.

In this talk, we show that for a given asymptotic profile, there exists a solution to the nonlinear Klein-Gordon equation which converges to given asymptotic profile as t to infinity. Here the asymptotic profile is given by the leading term of the solution to the linear Klein-Gordon equation with a logarithmic phase correction. For the two dimensional case, construction of a suitable approximate solution is based on Fourier series expansion of the nonlinearity. For the three dimensional case, construction of an approximate solution is based on the combination of Fourier series expansion for the nonlinearity and smooth modification of phase correction by Ginibre-Ozawa. My talk is based on the joint work with Satoshi Masaki (Osaka University).

On the effects of spatial expansion and contraction on the semilinear Klein-Gordon equation

Makoto Nakamura (Yamagata University)

The semilinear Klein-Gordon equation is considered and its nonrelativistic limit are considered in a uniform and isotropic space. The scale-function of the space is constructed based on the Einstein equation. The Cauchy problem is considered, and global and blow-up solutions are shown in Sobolev spaces. The effects of spatial variance on the problem are studied, and some dissipative and antidissipative properties are remarked.

Lifespan of periodic solutions to nonlinear Schrödinger equations

Tohru Ozawa (Waseda University)

An explicit lifespan estimate is presented for nonlinear Schrödinger equations on one-dimensional torus. This talk is based on my recent joint-work with Kazumasa Fujiwara (JSPS fellow, PD).

[1] (with K. Fujiwara) Lifespan of strong solutions to the periodic nonlinear Schrödinger equation without gauge invariance, J. Evol. Equ., in press.

[2] (with K. Fujiwara) Lifespan of strong solutions to the periodic derivative nonlinear Schrödinger equation, preprint (2017).

[3] T. Oh, A blowup result for the periodic NLS without gauge invariance, C. R. Math. Acad. Sci. Paris, 350, 389–392 (2012).

[4] (with Y. Yamazaki) Life-span of smooth solutions to the complex Ginzburg-Landau type equation on a torus, Nonlinearity, 16, 2029–2034 (2003).

A rigidity result for the Camassa-Holm equation

Luc Molinet (Université François-Rabelais, Tours)

The Camassa-Holm equation possesses peaked solitary waves called peakons. We prove a Liouville property for uniformly almost localized (up to translations) H^1-global solutions of the Camassa-Holm equation with a momentum density that is a non negative finite measure. More precisely, we show that such solution has to be a peakon. As a consequence, we prove that peakons are asymptotically stable in the class of H^1-functions with a momentum density that is a non negative finite measure.

Ill-posedness of derivative nonlinear Schrödinger equations on the torus

Kotaro Tsugawa (Nagoya University)

We consider the Cauchy problem of derivative nonlinear Schrödinger equations on the torus. Since the dispersive effect of the linear Schrödinger equation on the torus is weak, we can not treat some nonlinear terms as perturbations when they depend on the first derivative of the unknown function. In fact, some ill-posedness results are known for some nonlinear terms. Our aim is to give a necessary and sufficient condition on nonlinear terms to ensure the local well-posedness when the initial data is sufficiently smooth. The key idea of the proof of the ill-posedness is to derive a parabolic smoothing effect from nonlinear terms. Since the proof of well-posedness follows from the standard argument of the energy method with the gauge transform, we mainly focus on the proof of the ill-posedness.

The Ginzburg-Landau functional with vanishing magnetic field
(after K. Attar, B. Helffer-A. Kachmar, Kachmar-Nasrallah, ..)

Bernard Helffer (Université de Nantes, Nantes)

We study the infimum of the Ginzburg-Landau functional in the case with a vanishing external magnetic field in a two dimensional simply connected domain. We obtain an energy asymptotics which is valid when the Ginzburg-Landau parameter is large and the strength of the external field is comparable with the third critical field. Compared with the known results when the external magnetic field does not vanish, we show in this regime a concentration of the energy near the zero set of the external magnetic field. Our results complete former results obtained by K. Attar and X.-B. Pan–K.-H. Kwek.

Long term dynamics of dispersive evolution equations

Wilhelm Schlag (University of Chicago, Chicago)

We will describe some of the recent work on the soliton resolution problem for nonlinear dispersive equations. Ideas and methods from dynamical systems, such as invariant manifolds, are used in this theory. In the presence of dissipation, dynamical systems arguments are of essential importance.

Energy transfer model and large periodic boundary value problem for the quintic NLS

Hideo Takaoka (Kobe University)

In this talk, we consider a dynamics of mass density $|\widehat{u}(t,\xi)|$ and energy exchanges between a linear oscillator and a nonlinear interaction for the following defocusing quintic NLS equation:

$$i\partial_t u + \partial_x^2 u = |u|^4 u, \quad t \in \mathbb{R},\ x \in \mathbb{T}_L = [-L, L],$$

where $u = u(t,x) : \mathbb{R} \times \mathbb{T}_L \to \mathbb{C}$ is a complex-valued function and the spatial domain $\mathbb{T}_L$ is taken to be a torus of length $L > 0$, i.e., we assume the periodic boundary condition. At least, the NLS satisfies the mass conservation, which imposes the constraints on the dynamics of mass density. Grébert and Thomann (2012) proved that there exists solutions with initial data built on four Fourier modes, which involves periodic energy exchanges between the modes initially excited. We shall discuss a smooth global solutions with initially excited in multi-frequency modes. More precisely, we exhibit the conservative energy exchange of solutions between the modes initially excited. The argument relies on obtaining the dynamics in a toy model (finite dimensional approximation) of NLS along with error estimates between finite dimensional model and the full infinite dimensional model.

On derivative nonlinear Schrödinger systems with multiple masses

Hideaki Sunagawa (Osaka University)

We consider the initial value problem for systems of cubic derivative nonlinear Schrödinger equations in one space dimension with the masses satisfying a suitable resonance relation. We give structural conditions on the nonlinearity under which the small data solution gains an additional logarithmic decay as $t \to \infty$ compared with the corresponding free evolution. We also discuss a non-resonance counterpart of them. This talk is based on joint works ([1], [2]) with Chunhua Li (Yanbian University).

References

[1] C. Li and H. Sunagawa, Nonlinearity 29 (2016), 1537–1563.

[2] C. Li and H. Sunagawa, arXiv:1603.04966.

Global dynamics of the nonlinear Schrödinger equation with a potential

Kenji Nakanishi (Osaka University)

Consider the nonlinear Schrödinger equation (NLS) with the focusing cubic power and a slightly attractive potential supporting a single bound state in three space dimensions. This is one of the simplest settings with the four typical solutions of nonlinear dispersive equations: scattering, blow-up, stable solitons and unstable solitons. We can classify all the solutions into nine sets by their global behavior under the restrictions by small mass and an energy upper bound slightly above the first excited states. The analysis reveals not only the asymptotic behavior of solutions, but also intermediate and transient dynamics between different types of evolution. More precisely, the set of solutions staying close to spatial translation of the first excited states for large time form a center-stable manifold with codimension one in the energy space, separating the blow-up solutions from the scattering solutions. The manifold intersects its time inversion transversely, thereby decomposing the phase space into the nine sets. The main difficulty is that the scattering solutions have a ground state component, whose long-time behavior is poorly controlled.