DEPARTMENT OF MATHEMATICS AND STATISTICS
UNIVERSITY OF MASSACHUSETTS

MATH 233 **EXAM 2** **NOVEMBER 14, 2002**

NAME:________________________________

Section Number:____________________ Instructor's Name: ____________________

In problems that require reasoning, algebraic calculation, or the use of your graphing calculator, it is not sufficient just to write the answers. You must explain how you arrived at your answers, show your algebraic calculations, and indicate how you used your graphing calculator.

If approximate numerical answers are used, they should be rounded off to 5 significant figures.

$< x, y, z >$, $[x, y, z]$, $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ are all permissible notations for the vector $x\vec{i} + y\vec{j} + z\vec{k}$

1.	(20)	__________
2.	(20)	__________
3.	(20)	__________
4.	(20)	__________
5.	(20)	__________
Total		__________

Perfect Paper $\longrightarrow$ 100 Points.

There are six pages, including this one, in this exam and five problems. Make sure you have them all before you begin!

1. Let $f(x,y,z) = e^{xyz} + e^{x^2yz} + e^{xy^2z}$. (a) (3 points) Check that the point $Q = (1,-1,0)$ lies on the surface $f(x,y,z) = 3$.

(b) (6 points) Find an equation of the tangent plane to the surface at Q.

(c) (5 points) Find the rate of change of $f(x,y,z)$ at Q, and in the direction towards the origin.

(d) (6 points) Find the minimal directional rate of change of $f(x,y,z)$ at Q.

2. Consider a function $f(x,y)$ such that $f(x,y) = \frac{u}{v} + \ln\frac{u}{v}$ where $u = e^{x^2y^2-2xy} + \sin(x-y+1)$ and $v = g(x,y)$ is a function such that $g(1,2) = 1$, $g_x(1,2) = 4$, $g_y(1,2) = 1$.

(a) (10 points) Calculate $\left.\frac{\partial f}{\partial x}\right|_{x=1,\ y=2}$ and $\left.\frac{\partial f}{\partial y}\right|_{x=1,\ y=2}$, the partial derivatives of $f(x,y)$ with respect to x and y, when $x = 1$ and $y = 2$.

(b) (10 points) Calculate approximately $\left.f(x,y)\right|_{x=1.1,\ s=2.1}$ by using a linearization or a differential.

3. (20 points) Find the minimum and the maximum of the function $f(x, y) = 2x + 3y$ on the circle $x^2 + y^2 = 169$.

4. (a) (10 points) Find all local minima, local maxima and saddle points of

$$f(x,y) = xy(1-x-y).$$

(b) (10 points) Find the absolute maximum and minimum of the function $f(x,y)$ in the triangle with vertices at $(0,0)$, $(1,0)$ and $(0,1)$.

5. (a) (3 points) Show that the point $(2,1)$ is on the curve $\cos(x^3 + y^2 - 9) = y$.

(b) (5 points) Find an equation of the tangent line to the curve $\cos(x^3 + y^2 - 9) = y$ at the point $(2,1)$.

(c) (12 points) Find the cheapest design of a box of volume 2 such that the material used for the sides is twice as expansive as the material used for the top and bottom.