Casson 不変量に関わる 3 次元多様体の不変量 (Invariants of 3-manifolds related to the Casson invariant)
2017.1.25-27, RIMS, Kyoto university

Abstracts

望月厚志 (Atsushi Mochizuki)
Title: On the Casson-Walker invariant and a quantum representation of the mapping class group through the LMO invariant for genus one open books

Abstract: We construct a representation of the mapping class group through the degree one part of the LMO invariant and calculate the Casson-Walker invariant as the trace of the representation of monodromies, especially for 3-manifolds admitting a genus one open book decomposition.

辻 俊輔 (Shunsuke Tsuji)
Title: Construction of an invariant for integral homology spheres via Kauffman bracket skein algebras and its application

Abstract: Using an explicit formula for the action of the Dehn twist along a simple closed curve on the completed Kauffman bracket skein module of the surface, we introduce an embedding of the Torelli group into the completed skein algebra. This embedding and a Heegaard splitting enable us to construct an invariant for an integral homology sphere which is an element of $\mathbb{Q}[[A+1]]$. This invariant induces a finite type invariant of order $n+1$ which is an element of $\mathbb{Q}[[A+1]]/((A+1)^n)$. In this lecture, using this construction, we give a formula of Casson invariant in some situation.

森田 茂之 (Shigeyuki Morita)
Title: Casson invariant and structure of the mapping class group

Abstract: We begin by recalling how the Casson invariant was related to the structure of the mapping class group of surfaces in two ways. One is the interpretation as a secondary invariant associated with the fact that the first Mumford-Morita-Miller class vanishes on the Torelli group. The other is the appearance as a difference between the Johnson filtration of the mapping class group and the lower central filtration of the Torelli group.

Then we survey further works along these lines due to Garoufalidis- Levine and others, more precisely studies of relation between the structure of the mapping class group and finite type invariants of homology 3-spheres due to Ohtsuki. We also mention important open problems.

Finally we discuss our project to extend the above picture by enlarging both the mapping class group and the characteristic class in a wider context. The last part is based on joint work with Takuya Sakasai and Masaaki Suzuki.

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Delphine Moussard
Title: Finite type invariants of rational homology 3-spheres and their knots

Abstract: We consider finite type invariants of rational homology spheres with respect to Lagrangian-preserving surgeries. For this theory, we describe the graded space of finite type invariants by identifying it with the dual of a graded space of diagrams. If time permits, we will discuss a similar issue for null-homologous knots in rational homology spheres.

中村 信裕 (Nobuhiro Nakamura)
Title: Recent development of Seiberg-Witten Floer theory

Abstract: Recently, several kinds of homology cobordism invariants for homology 3-she res are constructed by using Seiberg-Witten Floer theory, (1) Froyshov invariant, (2) Manolescu' s invariants α, β, γ, (3) K-theoretic invariants due to Manolescu, Furuta-T.J.Li, J.Lin. All of these invariants can be defined by applying various equivariant cohomology theories to Seiberg-Witten-Floer homotopy type. In this talk, I will explain about their construction and applications to intersection forms of 4-manifolds with boundary, the disproof of the triangulation conjecture, and the homology cobordism group of integral homology 3-spheres.

市原 一裕 (Kazuhiro Ichihara)
Title: Generalizations of the Casson invariant and their applications to the cosmetic surgery conjecture

Abstract: I will talk about two generalizations of the Casson invariant, and their applications to the cosmetic surgery conjecture on knots. One is the SL(2,C) version of the Casson invariant originally introduced by Curtis, and the other is the degree 2 part of the Kontsevich-Kuperberg-Thurston universal finite type invariant studied by Lescop. By using their surgery formulae, several results can be obtained about cosmetic surgeries on knots in the 3-sphere. The former half of this talk is based on a joint work with Toshio Saito, and the latter is based on a joint work with Zhongtao Wu.

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北野 晃朗 (Teruaki Kitano)

Title: A polynomial invariant of a homology 3-sphere defined by Reidemeister torsion

Abstract: In the end of 1980s Dennis Johnson studied Reidemeister torsion for a homology 3-sphere from the view point of Casson invariant. He wrote unpublished lecture notes including followings, which never were published.

Let M be a homology 3-sphere with a fixed Heegaard splitting. Johnson gave volume forms on the spaces of conjugacy classes of SU(2)-irreducible representations for the closed surface and handlebodies. Here we assume the set of conjugacy classes of representations are finite and transversal for M. Under this assumption, we can consider a weight for any transversal intersection point. He proved this weight is equal to Reidemeister torsion of M for the corresponding irreducible representation composed with the adjoint representation. He called the sum of weights the geometric form of Casson invariant. Further he proposed to study polynomials whose zeros are the values of Reidemeister torsion of M. For algebraic simplicity, he consider SL(2;C)-reprensentations and Reidemeister torsion for such a representation. He computed explicitly this polynomial for homology 3-spheres obtained by 1/n-surgery along the torefoil knot.

In this talk, we would like to explain Johnson theory and show some formulas of the above polynomials for Brieskorn homology spheres and surgeried manifolds along the figure-eight knot.

This is a partially joint work with Anh Tran.

中島 啓 (Hiraku Nakajima)

Title: Coulomb branches of 3d gauge theories

Abstract: Given a compact Lie group G and its quaternionic representation M, physicists associate a SUSY 3d gauge theory. It is expected that a topological twist gives a TQFT, which includes the Casson invariant as an example with $G = SU(2)$, $M = 0$. For general G, M, almost nothing is known, but the Hilbert space for S^2 is a commutative ring whose spectrum is what physicists call the Coulomb branch. I will explain my recent rigorous construction of the Coulomb branch, when M is of the form $N \oplus N^*$. It is based on a joint work with Braverman and Finkelberg.

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渡邉 忠之 (Tadayuki Watanabe)

Title: Garoufalidis-Levine's finite type invariants for $\mathbb{Z}\pi$-homology equivalences of 3-manifolds

Abstract: Garoufalidis and Levine defined a filtration for 3-manifolds equipped with some degree 1 map ($\mathbb{Z}\pi$-homology equivalence) to a fixed 3-manifold N and showed that there is a natural surjection from a module of $\pi = \pi_1 N$-decorated graphs to the graded quotient of the filtration over $\mathbb{Z}[\frac{1}{2}]$. We show that in the case of $N = T^3$ the surjection of Garoufalidis–Levine is actually an isomorphism over $\mathbb{Q}$. For the proof, we construct a perturbative invariant by applying Fukaya's Morse homotopy theoretic construction to a local coefficient system of the quotient field of $\mathbb{Q}\pi$. The first invariant is an extension of the Casson invariant to $\mathbb{Z}\pi$-homology equivalences to the 3-torus.