Topological invariants in low dimensional topology

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Abstracts

Tadayuki Watanabe

Title: An equivariant perturbative invariant of 3-manifolds with $b_1 = 1$

Abstract: In this talk, I will explain that a topological invariant for 3-manifold M with $b_1(M) = 1$ can be constructed by applying Fukaya's Morse homotopy theoretic approach for Chern–Simons perturbation theory to a local coefficient system on M of rational functions associated to the maximal free abelian covering of M. The invariant takes values in Garoufalidis–Rozansky's space of Jacobi diagrams whose edges are colored by rational functions. It is expected that the invariant gives a lot of nontrivial finite type invariants of 3-manifolds. The definition of the invariant was strongly inspired by (and partially equivalent to) the equivariant invariant of Lescop.

Tatsuro Shimizu

Title: On the Bott-Cattaneo's Chern-Simons perturbation theory

Abstract: The Chern-Simons perturbation theory was established by Kontsevich, Axelrod and Singer in 1991. This gives an invariant of 3-manifold with a local system on the given manifold. Bott and Cattaneo gave a purely topological construction of Chern-Simons perturbation theory. In this talk we revisit their construction and define a kind of obstruction class. If this obstrcution class does not vanish, the original construction of Bott-Cattaneo should be modified. We show the modified construction. This construction is joint work with Alberto Cattaneo.

Andrew Kricker

Title: What is the l^2-Alexander invariant and how can we approximate it?

Abstract: The classical Alexander polynomial marks the beginning of mathematical knot theory. The l^2-Alexander invariant is a recent variation due to Li-Zhang (2006) and Dubois-Friedl-Luck (2014) where you twist the classical construction by the infinite-dimensional representation consisting of square-summable series of elements of the fundamental group. This invariant has remarkable properties - such as the fact that it determines the volume of a hyperbolic knot, determines the Thurston norm, and detects infinitely many knots. In this talk I'll introduce this invariant and survey what is known and unknown about it. It is a very strong invariant, but also is almost impossible to calculate. I'll also talk about some ongoing projects investigating its approximation.

Sakie Suzuki

Title: The universal quantum invariant and colored ideal triangulations

Abstract: The Drinfeld double of a finite dimensional Hopf algebra is a quasi-triangular Hopf algebra with the canonical element as the universal R-matrix, and one can obtain a ribbon Hopf algebra by adding the ribbon element. The universal quantum invariant of framed links is constructed using a ribbon Hopf algebra. In that construction, a copy of the universal R-matrix is attached to each crossing, and invariance under the Reidemeister III move is shown by the quantum Yang-Baxter equation of the universal R-matrix. On the other hand, the Heisenberg double of a finite dimensional Hopf algebra has the canonical element (the S-tensor) satisfying the pentagon relation. In this talk we reconstruct the universal quantum invariant using the Heisenberg double, and extend it to an invariant of equivalence classes of colored ideal triangulations of 3-manifolds up to colored moves. In this construction, a copy of the S-tensor is attached to each tetrahedron, and invariance under the colored Pachner (2,3) moves is shown by the pentagon relation of the S-tensor.

Christine Lescop

Title: On the cube of the Blanchfield pairing and on its generalization as a universal equivariant finite type knot invariant

Abstract: In 1999, Greg Kuperberg and Dylan Thurston proved that the Casson invariant of an integer homology 3-sphere M can be seen as the integral of the cube of a differential 2-form, which represents the linking form in a two-point configuration space of the manifold M. We will define an invariant of knots in such manifolds as a similar "cube" of the equivariant linking form, which is defined on the infinite cyclic cover of the knot complement. The obtained invariant will be defined as an equivariant algebraic intersection number in an equivariant configuration space. It is the degree one part of a more general equivariant invariant Z_e, which shares many properties -including a fundamental universality property- with the Kricker lift of the Kontsevich integral. This Kricker lift contains all the Vassiliev knot invariants and it provides the space of these finite type invariants with a strong additional structure. We will introduce Z_e and discuss its main properties.

Delphine Moussard

Title: A-ribbon 2-knots and factorized Alexander polynomial

Abstract: For classical knots, the Alexander polynomial is known to be symmetric and to have a good factorization property on ribbon knots. In contrast, any Laurent polynomial with integer coefficients, equal to 1 when evaluated at 1, can be realized as the Alexander polynomial of a ribbon 2-knot. We give a topological condition on 2-knots which implies the factorization of the Alexander polynomial. To this end, we introduce an alternative notion of ribbon 2-knots, the A-ribbon 2-knots, which are 2-knots that bound an immersed 3-ball with ribbon singularities homeomorphic to annuli. This is a joint work with Emmanuel Wagner.

Atsushi Mochizuki

Title: On the Casson-Walker invariant of 3-manifolds admitting genus one open book decompositions

Abstract: In this talk, I compute the Casson-Walker invariant of genus one open books in two ways; one is based on the combinatorial calculation of the degree one part of the LMO invariant, and the other is based on the action of the LMO invariant of mapping cylinders on the space of Jacobi diagrams on two intervals. Especially, in the latter case, I construct a representation of a central extension of the mapping class group of the genus one compact surface with one boundary.

Yuta Nozaki

Ttle: Homology cobordisms over a surface of genus one

Abstract: Morimoto proved that some lens spaces do not admit any open book decomposition whose page is a surface $\Sigma_{1,1}$. In contrast, we show that every lens space is obtained as the closure of a homology cobordism over $\Sigma_{1,1}$. The proof is based on the Chebotarev density theorem and binary quadratic forms in number theory.

Masatoshi Sato

Title: On the homology group of the genus three handlebody mapping class group

Abstract: We compute low-dimensional rational homology of the genus three handlebody mapping class group. The handlebody mapping class group acts on a simplicial complex called the meridian disk complex, and McCullough showed that it is contractible. The stabilizer of a vertex is the mapping class group of a genus two handlebody with two marked disks, and the stabilizer of one of edges are the mapping class groups of a solid torus with four marked disks. In this talk, we explain a method to compute the homology groups of these stabilizers and the genus three handlebody mapping class group.